Degarbling Mode S replies received in single channel stations with a digital incremental improvement

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Abstract

Multilateration (MLAT) and Automatic Dependent Surveillance – Broadcast (ADS-B) systems are used in air traffic control to detect, locate and identify cooperating aircraft using signals emitted by airborne transponders and received by dedicated ground stations. In areas with a high traffic density, these stations may receive simultaneously several superimposed signals. Present operational systems use only one receiving channel connected to a non-directional antenna. When the received replies are superimposed, i.e. "garbled", their detection and/or decoding are severely affected in nowadays equipment. The aim of this paper is to transform the single channel problem into a multiple channels problem in order to solve it using specific knowledge about the Mode S signals. In fact, the multiple channels problem is a typical signals separation problem applied to Mode S mixture for which several algorithms already exist. Our algorithm, named PASA, is based on the existing Projection Algorithm [13] and can be easily implemented on existing receiving stations. The effectiveness of our method is demonstrated using real data collected from our experimental receiver.

Acronyms:

ADS-B	**Automatic Dependent Surveillance - Broadcast**
ATC	**Air Traffic Control**
BSS	**Blind Source Separation**
EPA	**Extended Projection Algorithm**
FRUIT	**False Replies Unsynchronized in Time**
GPS	**Global Positioning System**
MLAT	**Multilateration**
NM	**Nautical Mile**
PA	**Projection Algorithm**
PASA	**Projection Algorithm Single Antenna**
PPM	**Pulse position Modulation**
SNR	**Signal to Noise Ratio**
SVD	**Singular Value Decomposition**
TDOA	**Time Difference Of Arrival**
WAM	**Wide Area Multilateration**

Keywords: ADS-B, MLAT, Signals Separation, single-channel signals separation

1 Introduction

1.1 Operational frame

In the air traffic control (ATC) the modern cooperating surveillance systems are the well-known Automatic Dependent Surveillance – Broadcast systems (ADS-B) and multilateration (MLAT and wide area MLAT, WAM) systems. ADS-B, MLAT and WAM use the secondary surveillance radar (SSR) Mode A/C and Mode S signals [1]-[3], for the detection, the tracking and the identification of aircraft transmitting messages at 1090 MHz with their standard transponder as well as of ground vehicles equipped with a 1090 MHz “non-transponder device”.

The SSR operates by exchanging interrogations-replies between a ground station and the targets: the ground station emits, at 1030 MHz, interrogations addressed to the targets, which receive the interrogation, and then transmit to the ground station (at 1090 MHz) a reply containing the requested data (identity or flight level). The Mode S transponder transmits either short (64 μs) or long (120 μs) replies at the standard, nominal carrier frequency of 1090 MHz [1]-[4], encoding the information by pulse position modulation (PPM). The transponders also transmit with a periodical timing, non-elicited signals called “*squitters*” in which several information about the aircraft are contained (i.e. identification code, 3D position, altitude, speed vector etc.). The “non-transponder devices” installed on a ground vehicle transmits *squitters* signals using the standard 1090 MHz Mode S signals, containing the identification code and the ground position of the vehicle.

ADS-B, MLAT and WAM systems use the SSR and the Mode S 1090 MHz signals for surveillance purposes. In MLAT and WAM, the localization is independent of the information content of the received signals, and is achieved via triangulation by measuring the signal time difference of arrival (TDOA) at a number of ground receivers at different fixed locations. The accuracy of the target localization depends on the number of receiver stations and their layout on the operational area. The MLAT/WAM ground segment is composed by a network of distributed stations (at least four measurements are needed for a 3D localization) receiving the signals and estimating their time of arrival (by timestamp), and a central processor station collecting and processing the measurements to localize the emitters. The ADS-B uses the spontaneous Mode S messages (*squitters*) discarding all the other 1090 MHz traffic: by decoding the messages, the ADS-B receiver (part of the ADS-B station) extracts identification, position and velocity of the emitter. Finally, the ground ADS-B station sends the surveillance information to the users.

1.2 State of the art and research frame

Due to the fact that ADS-B, MLAT and WAM receiver stations use a single channel receiver with a non-directional antenna, the probability of receiving Mode S signals overlapped in time is not negligible and grows up with the traffic density increasing. When replies or squitters from different sources overlap, likely the received messages are corrupted and cannot be decoded, and the emitters (either airborne or vehicular) cannot be located and identified. As remarked in [5] the interfering signals rate for a ground receiver placed in the Core Europe (Brussels) is likely from 25000 $msg\ s^{-1}$ up to 95000 $msg\ s^{-1}$ depending on different operating conditions (time of the year, number of operational interrogators etc.). With these values of the FRUIT (False Replies Unsynchronized In Time) rate, the detection probability, depending on the target range, can be less than 70% for target ranges greater than 20 NM [5].

In order to solve the problem of overlapping, source separation using array processing has been deeply investigated. There is a large literature also on the specific applications to SSR signals [6]-[18]. In [6] and [7] an adaptation of existing array processing algorithms was proposed for SSR signals. In [8]-[12] the authors presented several novel algebraic methods based on the sources property. In [13]-[17] different methods exploiting the time-domain sparsity property of the sources are proposed (paragraph 2.2 contains a detailed description). Finally, in [18] an improvement of previous algebraic methods is proposed.

In the last decade different single channel Blind source Separation (BSS) methods were proposed but none of them address specifically the SSR signals. The single-channel ICA proposed in [19] needs independent sources with disjoint spectral support. Some methods are useful for sound and speech signals [20]-[21], other are suitable in the radar and communications context [22]-[24] and in the bio-medical field [25]-[29]. A number of papers in [30] provide a wide overview of many aspects and recent advances on source separation and applications.

In this frame, we present here a single channel separation method for SSR signals suited to any single receiving station with one non-directional antenna. The algorithm is based on a data adaptation needed to use the Projection Algorithm, PA [13], and leads to a simpler and more effective method, the Projection Algorithm – Single Antenna, PASA. As explained before, the aim is to mitigate the garbling problem and to improve the channel capacity even in high traffic density situations.

This paper is organized as follows. Paragraph 2.1 shows the data model; paragraph 2.2 explains the data adaptation to transform the single channel problem into a multi-channels problem and shows the related data properties. Paragraph 2.3 links with the multi-channel separation methods and explains why the PA principle was chosen to solve the problem. Paragraph 2.4 describes PA in more detail. Paragraph 2.5 shows the practical implementation of PASA. Finally Paragraph 3 presents a case-study and the related performance analysis with recorded live signals.

A typical Mode S station for ADS-B and MLAT/WAM is composed by an antenna, an analog front-end and a digital section for signals detection and decoding. In figure 1 a general block diagram of a Mode S receiver is shown; the dotted-line box contains the extra logical functions for the PASA method.

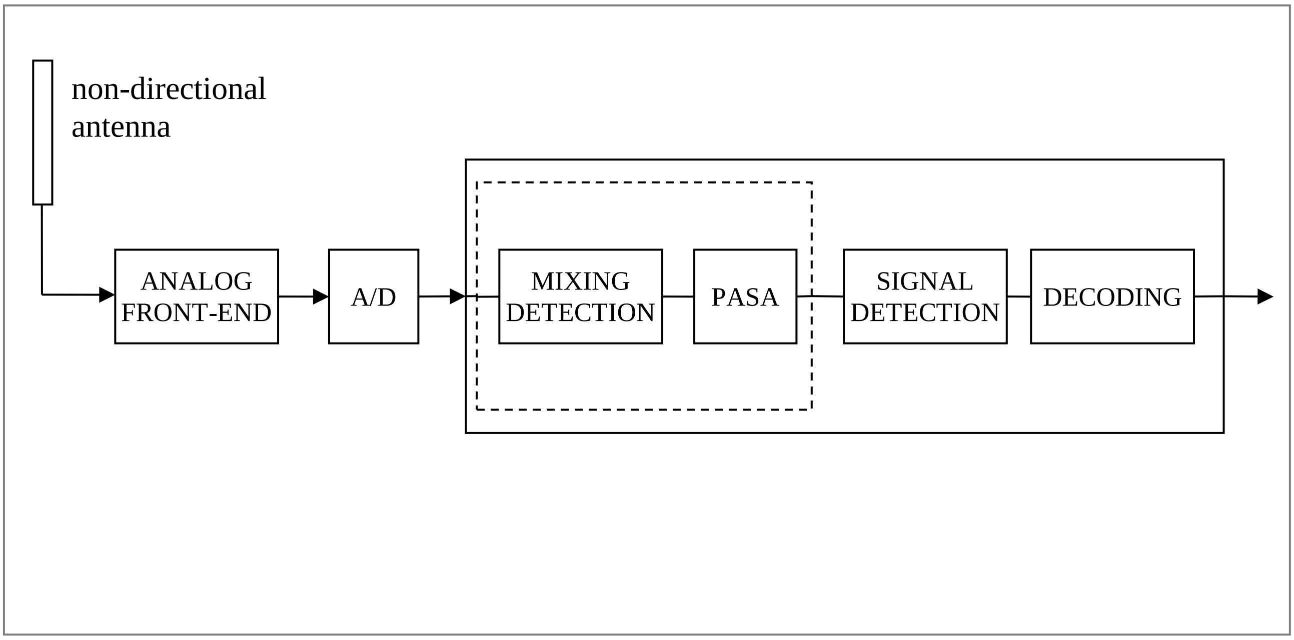

Figure 1: 1090 MHz receiver scheme for the PASA (Projection Algorithm Single Antenna)

2 Models and algorithms: PA and PASA

We first describe the single antenna data model. Next, we present its transformation to obtain an equivalent array antenna data model. The link between this model and the signals separation method is described, and after a short reminder of the PA [13] algorithm, the PASA is presented.

2.1 The Data Model

We denote scalars by italic lowercase or uppercase letters (e.g. a, A), vectors by lower case boldface letters (e.g. **x, x**[n]), vector elements with lower case letter with in-bracket position reference (i.e. x[n]), and matrices by uppercase boldface letters (e.g. **X**).

There are two formats for the 1090 MHz signals: the conventional mode (including mode A/C and military signals) and the Mode S. This work is focused on Mode S signals (reply, squitter) and on their mixtures. A squitter is a randomly self-triggered reply that follow Mode S format and contains 56 (short) or 112 (long) binary symbols b_n such as a reply. The symbol period is 1 μs, and each symbol is made up by two 0.5 μs chips. Figure 2 shows a schematic view of the time intervals for the preamble and the data block, and also shows the concept of superimposed signals.

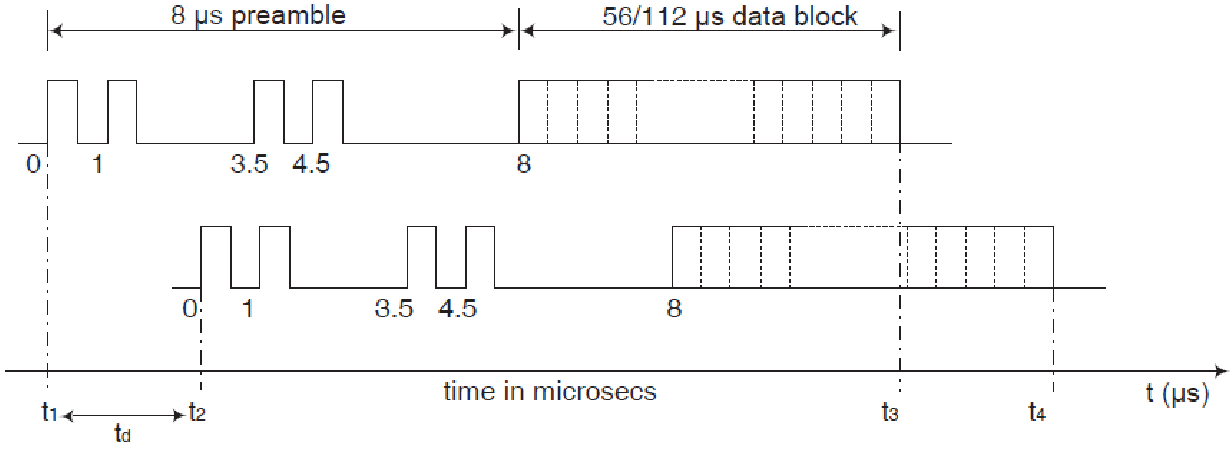

Figure 2: Schematic view of two Mode S replies with a time delay of $t_d = t_2 - t_1$

The bit of a reply or squitter are Manchester encoded, that is $b_n = 0$ is coded as $\mathbf{b}_n = [0, 1]$, and $b_n = 1$ is coded as $\mathbf{b}_n = [1, 0]$. A Mode S SSR reply/squitter is formed by an 8 μs length preamble $\mathbf{p} = [1, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0]$, followed by the encoded data: $\mathbf{b} = [\ \mathbf{p}, \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{56/112}\ ]$, with a total length of 128 (short reply) or 240 (long reply) elements (64 or 120 μs).

The Mode S reply/squitter has the form:

$$\mathbf{b}(t) = \sum_{n=0}^{127/239} \mathbf{b}[n]\, \boldsymbol{p}(t - nT) \tag{1}$$

where $p(t)$ is a rectangular pulse of width T= 0.5 μs. This signal is up-converted to the nominal transmission frequency f_o= 1090 MHz, with a ± 1 MHz tolerance as set by ICAO standards [4]. Due to this tolerance, after down-conversion to base band, a residual frequency f_r remains, adding a progressive phase rotation to the transmitted symbols. The received base band signal is:

$$\mathbf{s}\,[k] = \mathbf{b}\,[k] \exp(j\,2\,\pi\,k\,f_r\,T_s) = \mathbf{b}[k]\,\varphi^k, \qquad k = 1, 2, \ldots, N \tag{2}$$

where $\varphi = exp(j2\pi f_r T_s)$ is the elementary phase shift due to the residual carrier frequency, T_S the sampling period, and j is the imaginary unit.

Considering d signals emitted by independent sources and impinging on a single antenna, the general data model for the N samples of the received signal is:

$$\mathbf{x} = \mathbf{m}\,\mathbf{S} + \mathbf{n} \tag{3}$$

where $\mathbf{x}$ is the $[1\times N]$ data vector, $\mathbf{S}$ is the $[d\times N]$ sources matrix:

$$\mathbf{S} = \begin{bmatrix} s_1[1] & s_1[2] & \dots & s_1[N] \\ \vdots & \vdots & & \vdots \\ s_d[1] & s_d[2] & \dots & s_d[N] \end{bmatrix} \tag{4}$$

m is the mixing vector, (in this case, the sources mixing is a weighted sum) i.e. a $[1\times d]$ vector containing the complex gain of the antenna in the directions of the sources; finally, **n** is the $[1\times N]$ additive noise vector. In the present discussion we assume the two sources case, i.e. $d=2$, and N to be large enough so that the vector **x** contains both signals, with the vectors $\mathbf{s}_1[n]$ and $\mathbf{s}_2[n]$ zero-padded at their trailing and leading edges, respectively. The aim of the remaining of this paper is to describe and evaluate the method to extract the sources signals $\mathbf{s}_1[n]$ and $\mathbf{s}_2[n]$ from the observed vector **x**.

The sources (i.e. the signals) share the frequency band, are not synchronised, and have the same encoding; therefore, most of existing single-channel algorithms in the literature are not suited to the Mode S replies separation problem in the MLAT-ADS-B context. In this frame, if the leading has a free interference preamble it can be detected and decoded, but the trailing reply does suffer much more, and cannot be detected (its preamble is garbled with the data block of the leading signal), hence the need for a different solution.

2.2 Transforming the single channel problem into a multiple channel problem

Let us consider the signal emitted by one source as a stream, i.e. a vector $\mathbf{s}[n]$. Such a single source is described by the vector **s** which differs from the received vector **x** by the (complex) antenna gain and by the additive (receiver) noise. In this paragraph the noiseless case is described, therefore, being immaterial the constant antenna gain, the vector **x**, **s** (resp. the matrices **X**, **S**) are equivalent, and may be exchanged each other (when the noise is neglected) in the following formulas (see also the Appendix). A simple starting point is to *reshape* this vector $\mathbf{s}[n]$ of length N into a matrix $\mathbf{S_A}$ by stacking m elements of **s** at a time (being $m<<N$). The dimensions of the matrix $\mathbf{S_A}$ are $[m \times l]$ where l is the rounded part of N/m, and contains in the first column the first m samples of the vector **s**, in the second column the second m samples of **s** and so forth. Let **s** be the source stream:

$$\mathbf{s} = \begin{bmatrix} \mathrm{s}[1] \\ \mathrm{s}[2] \\ \vdots \\ \vdots \\ \mathrm{s}[N] \end{bmatrix} \tag{5.a}$$

and let us construct the derived data matrix $\mathbf{S_A}$:

$$\mathbf{S_A} = \begin{bmatrix} \mathrm{s}[1] & \mathrm{s}[m+1] & \cdots & \mathrm{s}[l \cdot m - m + 1] \\ \vdots & \vdots & \ddots & \vdots \\ s[m] & \mathrm{s}[2m] & \cdots & \mathrm{s}[l \cdot m] \end{bmatrix} \tag{5.b}$$

Note that we lose $N - l \cdot m$ samples in the transformation, which is smaller than m by the definition of l. From now on, we call m the reshaping factor. With the correct reshaping factor, the matrix **S** has a low-rank factorization; let us confirm that fact empirically, and then see why. Figure 3 shows a case of a received signal in field experiments sampled at 50 Megasamples per seconds that has been transformed using a reshaping factor m from 1 to 125. For each value of m, the matrix $\mathbf{S_A}$ was constructed, and its singular values were estimated. Then the eigenvalues of $\mathbf{S_A}$ were normalized by division with the value of the greatest one. Figure 3 shows these eigenvector grey-coded by amplitude; the bright white corresponds to 1, the maximum, whereas black represents values below or equal to 0.01 (-40 dB).

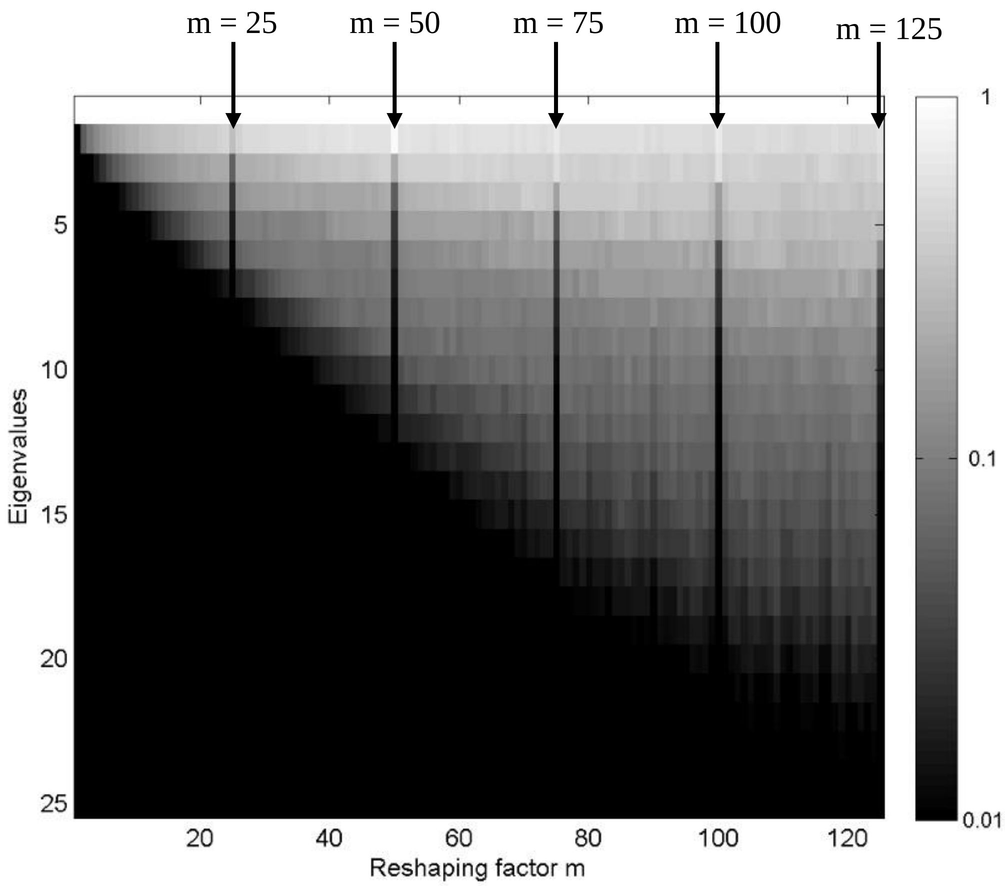

Figure 3: Eigenvalues for different reshaping factor, real short mode S reply,
N=3400, F_s = 50 MHz, SNR = 24.8 dB

According to Figure 3, for most of the reshaping factor values, the matrix $\mathbf{S_A}$ is not full rank, but still has many non-zero eigenvalues. But we found that for *m* equal to 25, 50, 75, 100, and 125, the $\mathbf{S_A}$ matrices have a low-rank factorization. At 50 Msamples/s, 25 samples correspond to half a microsecond, i.e. the duration of one elementary Mode S pulse (chip). Let us see now in detail why such simplification is possible. In the next case, for the sake of simplicity, the reply is assumed to be without any residual frequency, i.e. (equation (2)), f_r = 0 Hz, and the signal is mostly 'binary'. Also, let the sampling frequency f_s be equal to 10 MHz, i.e. 10 MSamples/s, then each bit of duration $T_b = 1\ \mu s$ will be represented by 10 samples, and each pulse (including the preamble pulses) by about 5 samples. Figure 4 describes the result of the choice $m=5$, ($m=½\, f_s\, T_b$), namely the duration of a chip.

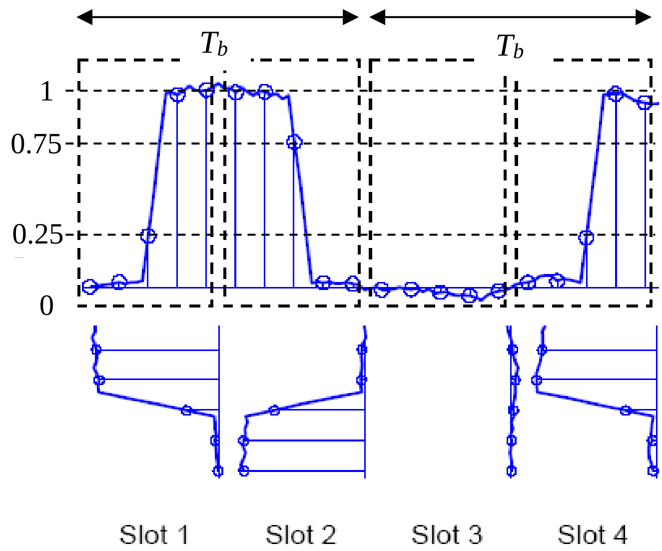

Figure 4: How to perform the transformation of x[n] into X

(f_s = 10 Msamples/s, T_b = 1 µs, m = 5 (½ f_s T_b))

The first and the last column are the same, up to noise, and the second column is nearly orthogonal to the first and to the fourth, and finally the third column contains only noise. Even if the reply and the reshaping are not synchronised together, their time lengths are commensurate, therefore the cut induced by the reshaping always happen at the same position on the chip. Considering figure 4 and denoting $\mathbf{p_u} = \begin{bmatrix} 0 \\ 0 \\ 0.25 \\ 1 \\ 1 \end{bmatrix}$, the vector from slot 1, and $\mathbf{p_d} = \begin{bmatrix} 1 \\ 1 \\ 0.75 \\ 0 \\ 0 \end{bmatrix}$, the vector from slot 2, all subsequent pulses can be described as a combination of both vectors, neglecting the noise. In this case the noiseless matrix $\mathbf{S_A}$ is written as:

$$\mathbf{S_A} = [\mathbf{p_u} \quad \mathbf{p_d}].\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \mathbf{P_A}\widetilde{\mathbf{S_A}} \qquad (6)$$

Although $\mathbf{p_u}$ and $\mathbf{p_d}$ are not orthogonal, the matrix $\mathbf{P_A}$ is full rank, so is $\widetilde{\mathbf{S_A}}$. This means that, adopting m_1= ½ f_s T_b, the rank of the noiseless matrix is two, and for the noisy version, two large singular values (above noise threshold) are expected. Note that the received reply does not have to be synchronised, as every pulse will be sliced at the same position. There are two non-trivial vectors: first half, $\mathbf{p_u}$, and second half, $\mathbf{p_d}$. Similarly, when considering values of reshaping factor integer multiples of m_1, for each additional length m_1 there are two more options: first half up or second half up; thus for $m_k = k\, m_1$, the rank is $2 \cdot k$.

Revisiting the case on Figure 4 with a non-zero residual frequency, we note that $\mathbf{p_u}$ and $\mathbf{p_d}$ are changed to reflect the phase change: $\mathbf{p}_\mathrm{u} = \begin{bmatrix} 0 \\ 0 \\ 0.25\varphi^2 \\ \varphi^3 \\ \varphi^4 \end{bmatrix}$ and $\mathbf{p}_\mathrm{d} = \begin{bmatrix} 1 \\ \varphi \\ 0.75\varphi^2 \\ 0 \\ 0 \end{bmatrix}$. So equation (6) is modified to:

$$\mathbf{S_A} = [\mathbf{p_u} \quad \mathbf{p_d}].\begin{bmatrix} 1 & 0 & 0 & \varphi^{15} \\ 0 & \varphi^5 & 0 & 0 \end{bmatrix} = \mathbf{P_A}\widetilde{\mathbf{S_A}} \tag{7}$$

Note that the $\mathbf{p_x}$'s have a residual frequency shift included in their model, therefore two different replies with different residual frequency will have a set of $\mathbf{p_x}$'s that are independent although not orthogonal.

2.3 Link with Blind Source Separation problems

Now that we have a transformation from the initial single channel problem into a low rank multiple channels problem, let see how to transform equation 3. In a sum form, we get:

$$\mathbf{x} = \sum_{i=1}^{d} \mathrm{m[i]} \cdot \mathbf{s}_i + \mathbf{n} \tag{8}$$

by linearity of the reshaping transformation, we have:

$$\mathbf{X} = \sum_{i=1}^{d} \mathrm{m[i]} \cdot \mathbf{S}_i + \mathbf{N} \tag{9}$$

where the obtained $\mathbf{X}$ is constructed in the same fashion as $\mathbf{S_A}$ in equation (5.b):

$$\mathbf{X} = \begin{bmatrix} \mathrm{x}[1] & \mathrm{x}[m+1] & \cdots & \mathrm{x}[l \cdot m - m + 1] \\ \vdots & \vdots & & \vdots \\ \mathrm{x}[m] & \mathrm{x}[2m] & \ldots & \mathrm{x}[l \cdot m] \end{bmatrix} \tag{10}$$

from equation (9) and (7), $\mathbf{X}$ is then equal to:

$$\mathbf{X} = [\mathrm{m}[1] \cdot \mathbf{P_1} \quad \cdots \quad \mathrm{m}[d] \cdot \mathbf{P_d}] \cdot \begin{bmatrix} \widetilde{\mathbf{S_1}} \\ \vdots \\ \widetilde{\mathbf{S_2}} \end{bmatrix} + \mathbf{N} \tag{11}$$

and in reduced form:

$$\mathbf{X} = \mathbf{M}\,\tilde{\mathbf{S}} + \mathbf{N} \tag{12}$$

$\mathbf{M}$ is a [$m_k \times 2kd$] matrix, and $\tilde{\mathbf{S}}$ is [$2kd \times l$] matrix, so by definition $\mathbf{M}$ is a tall matrix, and $\tilde{\mathbf{S}}$ a wide matrix, which ranks are at most $2kd$. Thought the authors do not have a formal proof of being full rank, they never observed a case where either $\mathbf{M}$ or $\tilde{\mathbf{S}}$ was rank deficient, albeit a bad conditioning number on $\mathbf{M}$ was observed. We acknowledge that the case may happen, for instance if $f_i = f_j$ for $i \neq j$. We have now a situation that is equivalent to the instantaneous over-determined mixture that is solved with Blind Source Separation. The algorithm AZCMA [12] would not be a good candidate since the remaining frequency can be found twice for every initial source. Given that the mode S is pseudo-Gaussian by nature [9], the usage of ICA or JADE [6] seems risky. We did not check if the new sources $\widetilde{\mathbf{S_A}}$ still comply with the Manchester encoding property [12], therefore we cannot use MDA. Moreover, with any BSS algorithm, we would have to find in the set of estimated sources which $2k$ ones are paired to recreate the original d replies. The PA algorithm [13] overcomes this limitation as it will directly estimate the $2k$ derived sources together, the pairing is then trivial. In the next paragraph, we remind the PA algorithm, while in the last one, we presents its application to the reshaped equation (12).

2.4 An overview of the Projection Algorithm (PA) [13]

In [13], two algorithms were proposed to separate from an antenna array overlapping replies: either using the Mode S protocol or the older Mode A/C protocol. For the simplest case of two partially overlapping Mode S replies, the more robust Projection Algorithm (PA) has been demonstrated. The Extended Projection Algorithm (EPA) considers all other cases of mixture and hence is less robust. One particular case, i.e. a totally overlapping pair of Mode S replies, albeit with a low probability of appearance in the real world, is cumbersome as we have to resort to another algorithm: the Manchester Decoding Algorithm (MDA), [33].

The projection algorithms need the output of an antenna array. At the time sample *n*, the *i*-th element output is $x_i[n]$. After collecting *N* samples and stacking the samples from each of *m* array elements into *m* vectors **x**[n], the resulting [$m\times N$] data matrix is:

$$\mathbf{X} = \begin{bmatrix} x_1[1] & x_1[2] & \dots & x_1[N] \\ \vdots & \vdots & & \vdots \\ x_m[1] & x_m[2] & \dots & x_m[N] \end{bmatrix} \quad (13)$$

Let us consider now the reception of signals from *d* different sources by an *m*-elements antenna array. The data model is expressed as:

$$\mathbf{X} = \mathbf{M}\,\mathbf{S} + \mathbf{N} \quad (14)$$

where **M** is the [$m\times d$] mixing matrix that contains the array signatures and the complex gains of the sources, **S** is the [$d\times N$] sources signal matrix (equation (4)), and **N** is the [$m\times N$] noise matrix. In the case considered here, i.e. $d \le m$, the signals separation is a typical blind source separation (BSS) problem. The noiseless matrix **X** has a low rank factorization, with a rank of *d*.

Table 1 shows the steps of the Projection Algorithm [13]:

step	
1	Data matrix **X** acquisition
2	Sources number and timing detection, interference-free sub-matrices $\mathbf{X}_1$ e $\mathbf{X}_2$ extraction
3	Mixing matrix **M** estimation, ($\mathbf{m}_1$ estimated using $\mathbf{X}_1$ and $\mathbf{m}_2$ estimated using $\mathbf{X}_2$)
4	Beamforming matrix computation $\mathbf{W} = \mathbf{M}^\dagger$
5	Sources estimation by $\mathbf{S} = \mathbf{W}\cdot\mathbf{X}$

Table 1: Summary of PA

Let us consider the case of two (*d*=2) superimposed signals received with an *m*-elements array antenna, with $m \ge 2$. Let (t_1, t_2) and (t_3, t_4) the beginning and ending time of the leading and trailing signal respectively, see fig. 2. Three time slots are defined: [t_1 : t_2] in which only the leading signal is present, [t_2 : t_3] with signals superimposing and [t_3 : t_4] in which only the trailing signal is present. Figure 5 shows the amplitude of a signal received in field experiments, being composed by two partially overlapped Mode S signals. The PA algorithm starts with the detection of the t_i*'s* by a whiteness test based on the singular value decomposition (SVD) of the data matrix **X** using a sliding window 4 μs long over the columns of that matrix **X**. Comparing the singular values amplitude with a noise threshold, the presence and the number of sources is estimated as a function of the time. In the following $(\cdot)^{(1)}$ indicates the matrix (or the vector) composed by the columns (or the elements), related to the time interval [t_1 : t_2] and the same for $(\cdot)^{(2)}$ and the time interval [t_3 : t_4].

With this notation and the data model in equation (14), the following relations hold:

$$\mathbf{X}^{(\mathbf{1})} = \mathbf{M}\mathbf{S}^{(\mathbf{1})} + \mathbf{N}^{(\mathbf{1})} \tag{15.a}$$

$$\mathbf{X}^{(\mathbf{2})} = \mathbf{M}\mathbf{S}^{(\mathbf{2})} + \mathbf{N}^{(\mathbf{2})} \tag{15.b}$$

By definition, $\mathbf{S}^{(1)}$ contains only a part of the first source $\mathbf{s}_1$, the row $\mathbf{s}_2$ is zero-padded from the beginning till t_2, $\mathbf{S}^{(2)}$ contains only a part $\mathbf{s}_2$ and $\mathbf{s}_1$ is zero padded from t_3 to t_4. Therefore, it is possible to simplify the equations (15) in:

$$\mathbf{X}^{(\mathbf{1})} = \mathbf{m}_{\mathbf{1}}\mathbf{s}_{\mathbf{1}}^{(\mathbf{1})} + \mathbf{N}^{(\mathbf{1})} \tag{16.a}$$

$$\mathbf{X}^{(\mathbf{2})} = \mathbf{m}_{\mathbf{2}}\mathbf{s}_{\mathbf{2}}^{(\mathbf{2})} + \mathbf{N}^{(\mathbf{2})} \tag{16.b}$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are the columns of the [$m \times 2$] mixing matrix **M**, and $\mathbf{s}_i$, $i=1, 2$, are the source vectors, i.e. the rows of the [$2\times N$] matrix **S**.

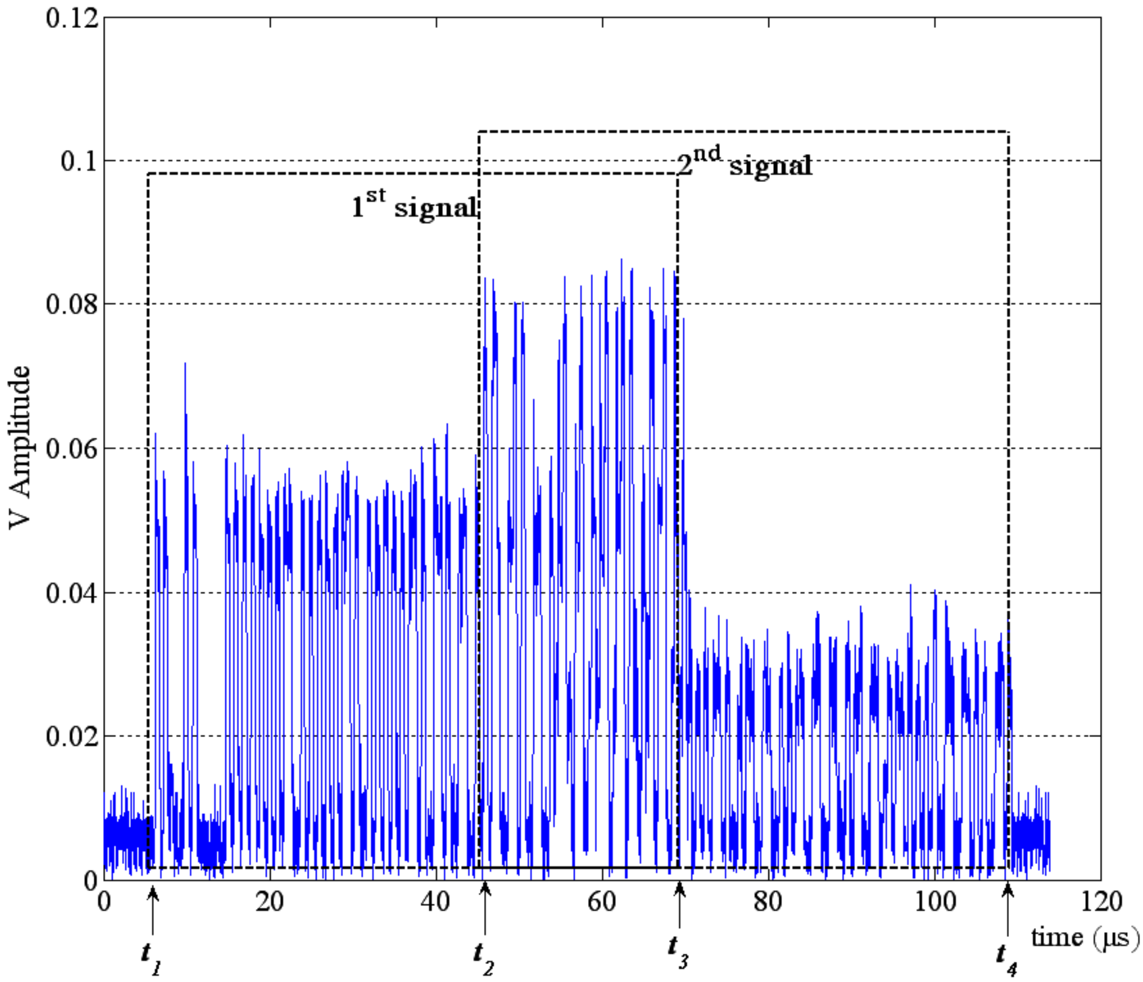

Figure 5: Amplitude of received signal composed by two Mode S overlapping sources

Note that $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are rank-one matrices in the noiseless case. The ensuing step of the algorithm consist in the estimation of the $[m \times 2]$ mixing matrix: $\widehat{\mathbf{M}} = [\widehat{\mathbf{m}}_1 \quad \widehat{\mathbf{m}}_2]$. The vectors $\widehat{\mathbf{m}}_1$ and $\widehat{\mathbf{m}}_2$ are obtained by a SVD of $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ respectively and correspond to the largest singular values. The final step is the derivation of the vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ by the rows of the Moore-Penrose pseudoinverse of $\widehat{\mathbf{M}}$. They works as spatial filters, applied to $\mathbf{X}$ to estimate the sources vectors $\mathbf{s}_i$. If the angle between the two signals impinging on the array tends to zero, $\widehat{\mathbf{M}}$ becomes ill conditioned, the noise at the output of the beamformers increase dramatically, and the signal to noise ratio (SNR) decreases. The limitation on the angle of arrival is shown in the Appendix I of [13], to which we kindly refer the reader interested to a more detailed description including the EPA algorithm.

2.5 The PASA algorithm

The concept behind the Projection Algorithm Single Antenna (PASA) consists in transforming the received single channel data stream **x** into a matrix **X** by reshuffling its elements, i.e. reshaping the vector. The low rank matrix **X** can then be processed with a modified version of the PA [13]. The application of PASA is in this paper limited to an overlap of two SSR Mode S signals. Table 2 summarize the PASA:

step	
1	Data vector **x** acquisition and reshaping to obtain data matrix **X** (using $m = ½ f_s T_b$)
2	Sources number and timing detection, sub-matrices $\mathbf{X}_1$ e $\mathbf{X}_2$ extraction
3	Mixing matrix **M** estimation, ($\mathbf{m}_1$ estimated using $\mathbf{X}_1$ and $\mathbf{m}_2$ estimated using $\mathbf{X}_2$)
4	Signature matrix computation $\mathbf{W} = \mathbf{M}^{\dagger}$
5	Sources data matrix estimation $\mathbf{S} = \mathbf{W} \cdot \mathbf{X}$
6	Matrix S inverse-reshaping to recover the separated sources $\mathbf{s}_1$ and $\mathbf{s}_2$

Table 2: Summary of PASA

Let us consider the superimposition of two signals $\mathbf{s}_1[n]$ and $\mathbf{s}_2[n]$ transmitted from two independent sources. The received data stream is collected in a row vector $\mathbf{x}[n]$ of length N, and the columns of the matrix **X** are obtained taking m elements of **x** at a time ($m << N$). The dimensions of the matrix **X** are $[m \times l]$ where l is the rounded part of N/m. This matrix contains in its first column the first m samples of the vector **x**, in its second column the second m samples of **x** and so forth:

$$\mathbf{x} = \left[\, \mathrm{x}[1] \;\; \mathrm{x}[2] \;\; \cdots \;\cdots\; \cdots \;\; \mathrm{x}[N] \right] \qquad \text{received data stream} \qquad (17.a)$$

$$\mathbf{X} = \begin{bmatrix} \mathrm{x}[1] & \mathrm{x}[m+1] & \cdots & \mathrm{x}[m+1] \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{x}[m] & \mathrm{x}[2m] & \cdots & \mathrm{x}[2m] \end{bmatrix} \qquad \text{derived data matrix} \qquad (17.b)$$

Defining a data model as in equation (14), the matrix **M** is not created by array response but, rather, by source's residual frequency *(f_r)* difference, see equation (2). The goal is to estimate **M** to derive two subspaces on which to project **X** to recover the signal vectors $\mathbf{s}_1$ and $\mathbf{s}_2$. The projectors, obtained by the pseudo-inverse of $\widehat{\mathbf{M}}$, can be called the *signature vectors*. The matrices $\mathbf{S}_1$ and $\mathbf{S}_2$, resulting from the application of the signature vectors onto **X**, shall contain the two sources, respectively. The estimated signal vectors $\hat{\mathbf{s}}_1$ and $\hat{\mathbf{s}}_2$ are then obtained by an inverse reshaping of $\mathbf{S}_1$ and $\mathbf{S}_2$ respectively.

As in the PA, the first step of PASA consists in the estimation of the sources arrival timing, to define the two subset's of the columns of **X**, in which only one source is present. For this task, the *m* value, that is the number of rows of **X**, is a fundamental parameter determining the rank of **X** and the conditioning number of $\widehat{\mathbf{M}}$. Using a too small value (like *2* or *4*) the conditioning number of $\widehat{\mathbf{M}}$ results to be too high, with the pseudo-inverse being not robust and the noise contribution after the projection too large. Moreover, with *m* incommensurable with the number of samples contained in a bit time, the rank of $\widehat{\mathbf{M}}$ is maximum and the detection of the number of sources is not possible using a whiteness test like for PA. If *m* is set equal to the number of samples contained in half bit time, (i.e. $m = ½ f_s T_b$, where f_s is the sampling frequency and T_b is the bit duration, i.e. 1 μs), each column of **X** contains either a pulse signal or noise only, or, depending on the synchronization between the observed vector **x** and the starting sample of the signal, a frame composed by a partial pulse and noise, see figure 4.

So far two signature vectors are required for each source. This means that, adopting $m = ½ f_s T_b$, performing a whiteness test on **X** to estimate the sources number and timing, two large singular values (above threshold) are expected when one source is present, and four large singular values when two sources overlap, thus making easy the estimation of the t_i*'s*. As for PA, the whiteness test is performed on a sliding window of 4 T_b, i.e. 4 μs. Once the timing estimation is done, $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$, containing only the first and only the second source respectively, are extracted and used to estimate the signature vectors (two vectors for each source). The matrix $\widehat{\mathbf{M}}$ is composed with the vectors corresponding to the first and to the second largest singular values of the decomposition of $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$. The next step consisting in the deriving of the projection matrix: $\mathbf{W} = \mathbf{M}^{\dagger} =$

$\left[\widehat{\mathbf{M}}^{\mathbf{H}}\widehat{\mathbf{M}}\right]^{-1}\widehat{\mathbf{M}}^{\mathbf{H}}$. The application of $\mathbf{W}$ onto matrix $\mathbf{X}$ gives the estimation of the sources matrix $\hat{\mathbf{S}}$ from which the separated sources are obtained by an inverse reshaping from the $\mathbf{S}$ columns into a row vector. The formulation of the algorithm steps are summarized in the Appendix. As a final remark, we remind that PASA aims at (and is able to) separate exactly two sources. This is a design choice based on the probability to have two Mode S replies overlapping, and considering the much lower probability of overlapping of three or more mode S replies. In fact, considering for example a high Mode S FRUIT rate environment (20k messages/s), the probability of receiving exactly two interfered messages (i.e. 3 %) is ten times the probability to receive three or more interfered messages (i.e. 0.3 %) [14].

3 The PASA method: experimental results

In this section the practical implementation of the PASA algorithm is presented. A processing architecture to perform all the algorithm steps is described, and a case of study is shown. Finally the performance of the method is discussed, analysing the success rate with overlapping signals. To evaluate the effectiveness of the algorithm we used Mode S signals recorded by an ad-hoc experimental receiver [31]. The device is a multichannel receiver (four linear channels and one logarithmic channel), equipped with an array antenna. The logarithmic channel is mostly used as a signal detector, and the four linear channel are sampled at typical rate of 100 Msamples/s directly at the intermediate frequency (21.5 MHz). This receiver was used in single-channel mode to perform the experimental analysis as described in the following paragraphs.

3.1 Practical implementation and case of study

It has been previously assessed that, in order to detect the number of sources and their timing based on the reshaped matrix $\mathbf{X}$, it is necessary to set m equal to the number of samples in one half bit time (i.e. 0.5 μs). But, using integer multiples of the value ½ $f_s\,T_b$, like $m=f_s\,T_b$ or $m=2\,f_s\,T_b$, we have observed better performance for the sources sub-spaces estimation, thanks to a smaller conditioning number for $\widehat{\mathbf{M}}$. In order to take advantage of both configurations, a parallel type of processing has been devised and tested. This is shown in Figure 6: the processing performs two data reshaping's in parallel. The matrix obtained by a reshaping using $m = f_s\ T_b$ is suitable for the projection, as it gives better results. The data reshaped with $m = ½\ f_s\ T_b$ are the best input for the sources timing estimation. The sources timing is achieved by comparing the singular values of the matrix $\mathbf{X}$ with a CFAR threshold [32]. Once estimated the sources timing, it is possible to extract $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ from $\mathbf{X}$, they contain only the first and the second source respectively. Then, the vectors $\mathbf{m}_i$ are computed by the SVD of $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ and the matrix $\mathbf{X}$ is projected over the pseudo-inverse $\mathbf{M}^{\dagger}$ of $\widehat{\mathbf{M}}$, ($\widehat{\mathbf{M}}$=[$\mathbf{m}_1$ ... $\mathbf{m}_n$]). In practice, the matrix $\mathbf{X}$ is multiplied by $\mathbf{M}^{\dagger}$. Finally both sources are recovered with an inverse reshaping of the result.

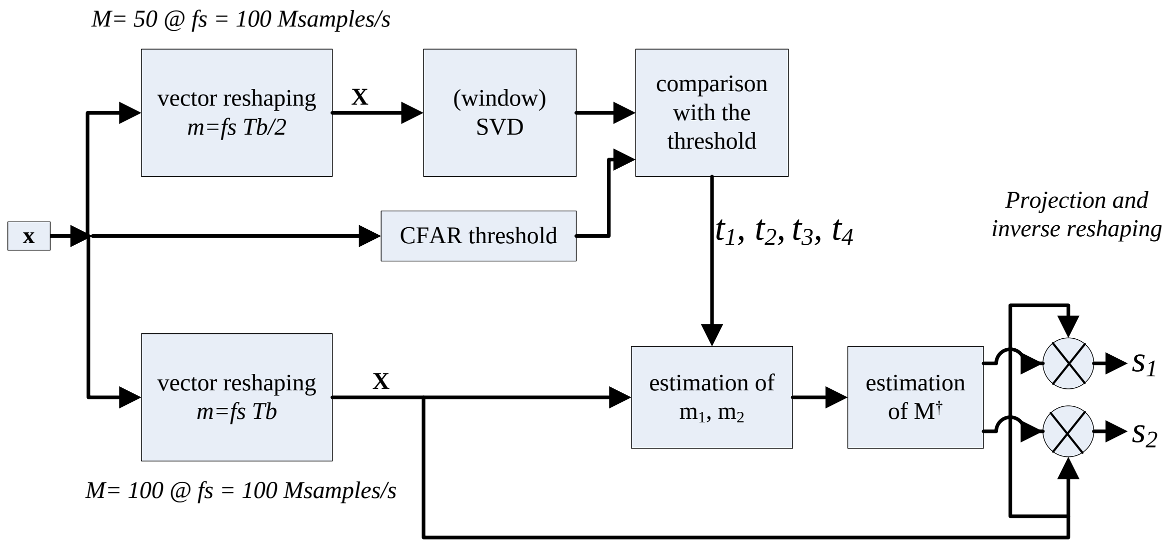

Figure 6: Proposed architecture for source's timing and extraction

The application of the PASA method with the signal shown in figure 5 is now analysed. The signal is composed by the overlapping of two Mode S "short" (64 μs) signals, sampled at f_s = 50 Msamples/s. The replies have a relative time delay of approximately 40 μs, a relative frequency shift of 0.19 MHz. Finally, the amplitude of the trailing signal is about 4.4 dB below the amplitude of the leading one. After the generation of the data matrix using $m = 25$, the SVD test is performed. Figure 7 shows the behaviour of the singular values as a function of time: the time instants $t_{1,2,3,4}$ can be easily detected by a threshold comparison.

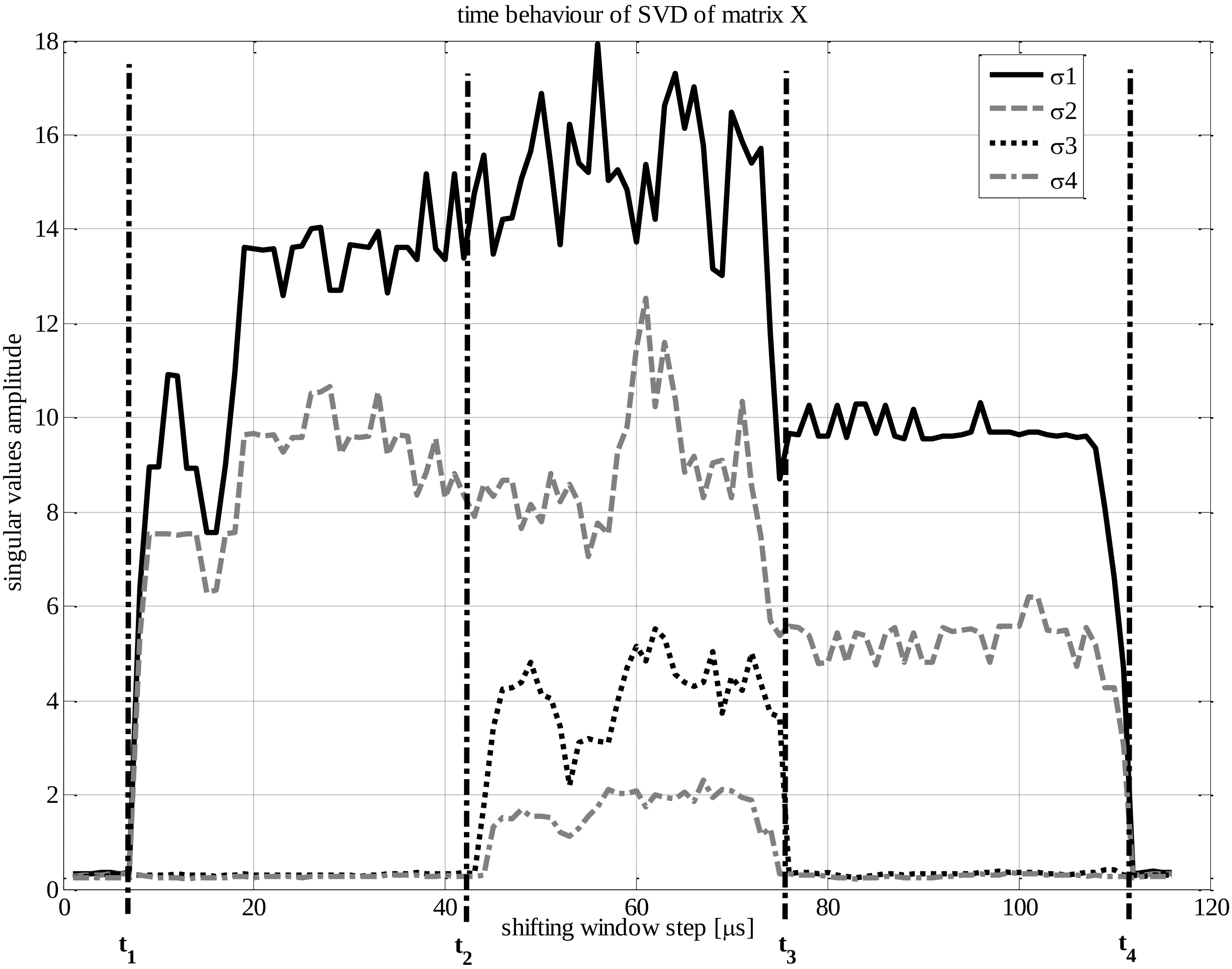

Figure 7: Singular values of X as function of time, by a sliding window over the columns of X (two superimposed Mode S signals)

Figure 8 shows the results of the separation algorithm: on the left the extracted replies are displayed, while on the right the two levels (one bit) amplitude signals, obtained by decoding these replies, are shown. A visual inspection revealed a free error decoding for both the replies, although (due to the amplitude variance) several bits are declared as "low confidence", the definition of "low confidence bit” being contained in the Appendix I of [3]. However the gain with the respect to a classical decoder is interesting since without PASA it is not possible to detect and decode the trailing reply, and the leading is decoded with several wrong bits. The PASA algorithm has worked thanks to the frequency shift between the sources (0.19 MHz). The resulting conditioning number of the matrix $\widehat{\mathbf{M}}$ is equal to 9.5.

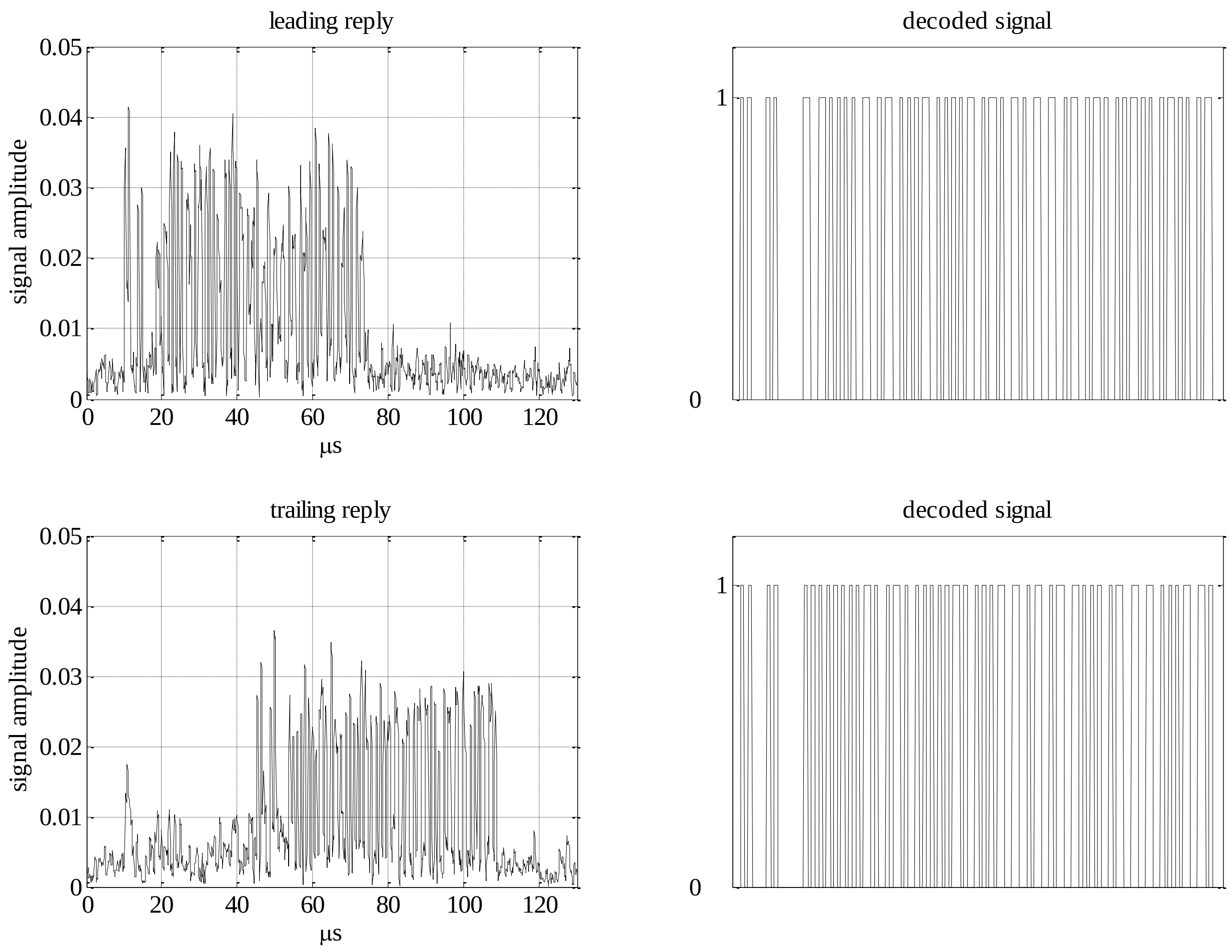

Figure 8: Results of PASA application, garbled signal shown in figure 5

3.2 Evaluation of the PASA with multiple live signals

The evaluation of PASA has been carried with real, multiple recorded signals, selecting the particular records showing interference between two Mode S replies. All the suitable signals (105 records) have been processed with PASA. A case is considered a success if downstream the application of PASA it is possible to correctly detect the preamble of the trailing reply. The preamble detection is performed with the standard routine in compliance to the recommendation of appendix I of [2]. Since we did not have ‘a priori’ information about the messages code and the transponders unique address, it was not possible to analyze the decoding errors. Therefore, for each case we have analyzed: the time delay (t_d), the power ratio between the signals (P_R), the frequency shift (f_{sh}) and the conditioning of the matrix $\widehat{\mathbf{M}}$ (*cond*). A post processing analysis has been done to investigate any correlation between the success of the method and these characteristics.

In table 3 the estimated characteristics on the involved signals are shown: the *min* and *max* values, the *mean* and the *standard deviation* of each parameter have been evaluated. High values of the *standard deviation* related to the mean values show that the characteristics of the considered signals are rather randomly distributed in their own ranges.

	min value	mean value (μ)	max value	std deviation (σ)
relative time delay (td)	0.7 μs	36 μs	110 μs	25 μs
relative frequency shift (fsh)	0.02 MHz	0.32 MHz	1.35 MHz	0.24 MHz
signal power ratio (PR)	0.012 dB	3.07 dB	13.6 dB	2.7 dB
M conditioning number (η)	2.3	17	62	9.8

Table 3: Characteristics of the superimposed signals involved in PASA evaluation

The application of the PASA method permitted the trailing reply detection in 63 cases, i.e. with a success rate of 60 %, where a failure case is defined when downstream the application of the PASA method it is not yet possible to detect the trailing reply, because of the missed preamble detection.

In order to analyze how the performance are conditioned by the signals characteristics, the parameters of table 1 were evaluated separately on success and on failure cases.

In Figure 9 the histograms and the mean and standard deviation values of the estimated parameters are shown.

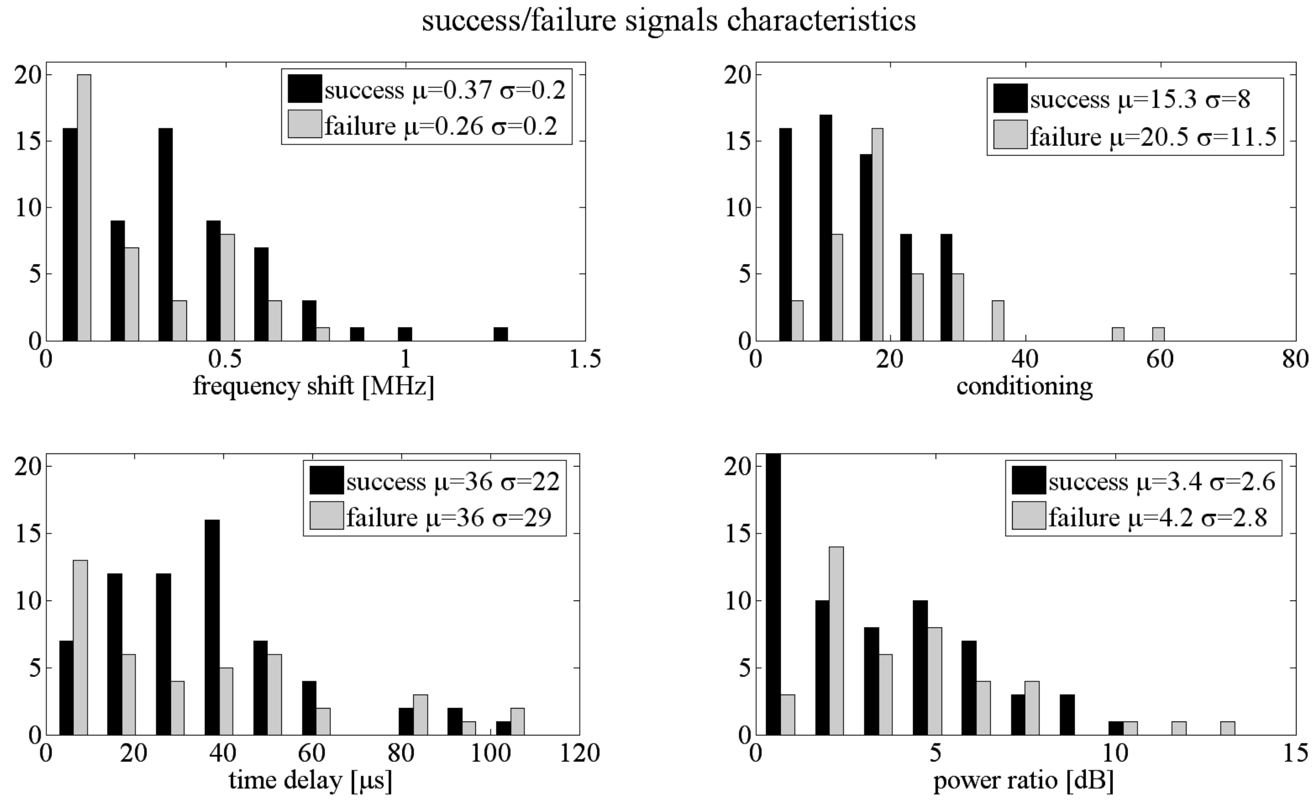

Figure 9: Signal characteristics evaluated on the success and failure groups

It seems that the effectiveness of the PASA is not strongly affected by the relative time delay of the interfering signals, although it was not practically possible to test the method with very small time delays cases. The PASA seems to behave better at low power ratio between replies, order of 0 dB, anyway its behaviour is not significantly correlated with power ratio. The effectiveness of the method seems to be significantly dependent on the frequency relative shift and the conditioning number, as expected. An analysis was done on the mixed signals with an high relative frequency shift (more than 0.15 MHz) but identified as failure cases: they are characterized either by a high power difference between the two sources (the trailing signals are over 10 dB above the leading one), or a poor time delay (less than 10 μs). A typical situation generating failure is the following: if there is a high power difference (more than 3 dB), the projection of the data matrix onto the subspace of the smaller source is not efficient, and the components of the most powerful signal persist on the reconstructed signal that is not detectable. The Mode S receiver in compliance with the requirements of [3] is able to handle this situation: if during the data-block decoding a preamble more than 3 dB greater than the earlier is validated, then the earlier signal is rejected so that the data-block demodulation of the new signal can proceed. By design, the robustness of the PPM permits a free-error decoding (also thanks to the cyclic redundancy check) in the presence of a low-amplitude interference. Another identified cause arises when overlapped signals have a too small time difference, leading to two main complications: *i*) if the relative time delay is smaller than the window length of the whiteness test (4 μs) it is not possible to recognize the sources timing and the algorithm fails; *ii*) if the relative time delay is low, but sufficient to estimate the sources timing, there are a few columns of the matrix **X** on which to estimate the signals signature vectors, then the projection matrix does not permit a good estimate of the two sources sub-spaces and the output SNR is too low to recognize the signals. Summing up: the higher the relative frequency shift between the mixing sources, the smaller the **M** conditioning and better the PASA performance. A limiting factor is represented by other parameters: when the relative time delay is less than 10 μs, and/or the signal level difference is above 3 dB, the effectiveness of the method is generally insufficient.

The experimental results demonstrate that PASA is useful as signals separation algorithm between replies from similar ranges, which should be a typical scenario of an approach area or of a large airport.

3.3 Concluding remarks

In this paper we presented a method called PASA to separate overlapped Mode S signals using a single channel receiver, in order to enhance the Mode S signal-based surveillance of a busy terminal area or a big airport. The method is partly based on the array processing algorithm named PA, presented in [13]. A measurement and test campaign with a dedicated receiver allowed us to test the method with live cases and to analyze its effectiveness and its limits. It has been noticed that PASA permits a significant improvement of the success rate, while there isn’t any other reasonably option to solve the problem of signal overlapping. Concerning the experimented failure cases, it was verified that they are due to some characteristics of the superimposed signals, like frequency shift, time delay and amplitude levels ratio. A direction for future research on this topic are tests with a large databases to analyze the correlation between the failures and the signals parameters, to finalize future algorithmic improvements.

Appendix: formulation of the algorithm steps

Let $\mathbf{x}[n]$ the vector of length N of the sampled data stream with two partially overlapped Mode S signals. For the sake of simplicity, let N be large enough to contain both signals and to be an integer multiple of $m= \frac{1}{2} f_s\ T_b$. We collect from $\mathbf{x}[n]$ the following vectors:

$$\mathbf{x}_n = \begin{bmatrix} \mathrm{x}[m \cdot n - m + 1] \\ \vdots \\ \mathrm{x}[m \cdot n] \end{bmatrix}$$

of dimension m, with $n=1, \ldots, L$, and $L=N/m$.

Let $\mathbf{X} \triangleq [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \cdots \quad \mathbf{x}_L]$, a $[m\times L]$ data matrix composed by vectors $\mathbf{x}_n$.

The sampled signal $\mathbf{x}[n]$ contains the sum of the two sources ($\mathbf{s}_1$ and $\mathbf{s}_2$) samples and the noise ($\mathbf{n}$):

$$\mathbf{x}[n]= \mathbf{s}_1[n] + \mathbf{s}_2[n] + \mathbf{n}[n].$$

Since the transformation is linear, we have:

$$\mathbf{X} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{N},$$

where $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{N}$ are $[m\times L]$ matrices obtained by $\mathbf{s}_1[n]$, $\mathbf{s}_2[n]$ and $\mathbf{n}[n]$ respectively, with the same method as $\mathbf{X}$.

In the noiseless case $\mathbf{S}_1$ and $\mathbf{S}_2$ are low rank matrices (i.e. with rank 2 or 3 depending by the initial point of the stacking from $\mathbf{s}_1$ and $\mathbf{s}_2$).

Considering a rank factorization, $\mathbf{S}_1$ and $\mathbf{S}_2$ may be written as:

$$\mathbf{S}_1 = \mathbf{M}_1\ \mathbf{V}_1,$$

$$\mathbf{S}_2 = \mathbf{M}_2\ \mathbf{V}_2,$$

where M_i is a $[m\times r]$ matrix and V_i is a $[r\times L]$ and r is the rank of S_i.

Using the introduced factorization we obtain $\mathbf{X} = \mathbf{M}_1\ \mathbf{V}_1 + \mathbf{M}_2\ \mathbf{V}_2 + \mathbf{N}$**,** which can be written as:

$$\mathbf{X} = [\mathbf{M}_1 | \mathbf{M}_2] \begin{bmatrix} \mathbf{V}_1 \\ - \\ \mathbf{V}_2 \end{bmatrix} + \mathbf{N} \triangleq \mathbf{M}\ \mathbf{V} + \mathbf{N}.$$

The idea of PASA is the application of PA to this data model, using the pseudo-inverse of $\mathbf{M}$ to extract the sources: $\mathbf{W} = \mathbf{M}^{\dagger} = [\mathbf{M}^{\mathbf{H}}\mathbf{M}]^{-1}\mathbf{M}^{\mathbf{H}}$. The used projectors are:

$$\mathbf{W_1} = \begin{bmatrix}\mathbf{M_1} & \mathbf{0_{m\times d_1}}\end{bmatrix}\mathbf{W},$$

$$\mathbf{W_2} = \begin{bmatrix}\mathbf{0_{m\times d_1}} & \mathbf{M_2}\end{bmatrix}\mathbf{W},$$

where d_1 and d_2 are the number of signature vectors used for each source, respectively.

Using the projectors on $\mathbf{X}$ the sources are extracted. For the first source we have:

$$\hat{\mathbf{S}}_1 = \mathbf{W}_1\mathbf{X} = \mathbf{W}_1\mathbf{MV} + \mathbf{W}_1\mathbf{N} = \begin{bmatrix}\mathbf{M_1} & \mathbf{0_{m\times d_1}}\end{bmatrix}\mathbf{WMV} + \mathbf{W}_1\mathbf{N} = \begin{bmatrix}\mathbf{M_1} & \mathbf{0_{m\times d_1}}\end{bmatrix}\mathbf{M}^{\dagger}\mathbf{MV} + \mathbf{W}_1\mathbf{N} =$$

$$= \begin{bmatrix}\mathbf{M_1} & \mathbf{0_{m\times d_1}}\end{bmatrix}\begin{bmatrix}\mathbf{V_1} \\ - \\ \mathbf{V_2}\end{bmatrix} + \mathbf{W}_1\mathbf{N} = \mathbf{M}_1\mathbf{V_1} + \mathbf{W}_1\mathbf{N} = \mathbf{S_1} + \mathbf{W}_1\mathbf{N}$$

The same for the second source.

Last step is the easy reconstruction of $\mathbf{s}_i[n]$ from $\hat{\mathbf{S}}_i = [\mathbf{s}_1 \quad \mathbf{s}_2 \quad \cdots \quad \mathbf{s}_L]$, ($i = 1, 2$).

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