Topology-preservation in 3-D image deformation and registration: Methodology and medical applications

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Abstract To obtain correct topological properties when analysing a 3-D image, a solution consists in using an *a priori* model presenting satisfying properties w.r.t. the object to analyse, and to "fit" the model on this object. We propose a methodological framework which tackles this problem by fusing two classical approaches: *image deformation*, and *image registration*, which consider *discrete* and *continuous* topology preservation, respectively. The methodology, applied on medical data, provides quite satisfactory results.
Keywords Topology preservation, image deformation, image registration, medical imaging.

1 Introduction

Obtaining correct topological properties is often an important requirement in 3-D image analysis. To deal with this issue, a solution generally consists in using an *a priori* model presenting satisfying properties w.r.t. the object to analyse, and to fit this model on the object. This can be done by: image deformation (which considers *discrete* topology preservation), or image registration (which considers *continuous* topology preservation).

1.1 Topology-preserving image deformation

Methods based on image deformation often consist in progressively modifying a discrete object $M \subset E_{\mathbb{Z}}$ (with $E_{\mathbb{Z}} \subset \mathbb{Z}^3$) in a homotopy-preserving fashion (typically by addition/removal of simple points [3]), under the guidance of *a priori* knowledge: generally a *constraint set* $K \subset E_{\mathbb{Z}}$ (*i.e.* a "geometrically correct" target to reach) and a *cost* or *priority function* f devoted to guide the deformation (see Alg. 1). The purpose of such methods is then to obtain a resulting set $X \subset E_{\mathbb{Z}}$ having the same topology as M while being "as similar as possible" to K.

This framework presents several challenges: (i) How to handle objects having a *complex* topology? (ii) How to define a cost function modelling *high-level a priori knowledge*? and (iii) How to enable a *flexible* deformation while guaranteeing its *convergence*?

Most of the methods based on this framework omit one or several of these requirements in order to deal with the other ones. In particular, many of them (i) consider objects of non-complex topology (generally simply connected) [5] or use simplifying hypotheses to lead to simpler topology [9], (ii) use cost functions based on low-level or local knowledge (distance maps, intensity, etc.) [13], and/or (iii) use a monotonic deformation process, which aims at either adding (growing processes) or removing (thinning processes) simple points [4], thus requiring that either $M \subset K$ or $K \subset M$ (forbidding, in particular, automatic generation of M for complex topologies). Note that most of these methods are devoted to cerebral or vascular structure detection from 3-D MRI or CT-data. By lack of room, we do not present here a state of the art of the methods based on these concepts. A (hopefully) complete one will be proposed and discussed in further works.

In order to develop efficient topology-preserving deformation methods, it would actually be important to authorise the use of *complex objects* M, evolving *non-monotonically* onto targets K possibly having also *complex topologies* by guidance of *high-level priority functions* f, with *low algorithmic complexities*. To our knowledge, this issue has not frequently been addressed.

Algorithm 1 Topology-preserving image deformation - general framework.

Input: $M \subset E_{\mathbb{Z}}$ ("topological" model), $K \subset E_{\mathbb{Z}}$ ("geometrical and/or topological" target), f (cost function)
Output: $X \subset E_{\mathbb{Z}}$ (final binary image)
$X = M$
repeat
  Choose $x \in \{y \in E_{\mathbb{Z}} \mid y$ is simple for $X\}$ according to f.
  if $x \in X$ **then**
    $X = X \setminus \{x\}$
  else
    $X = X \cup \{x\}$
  end if
until the process converges w.r.t. f and K.

1.2 Topology-preserving image registration

Image registration aims at estimating consistently a mapping between two (binary or grey-level) images. An important issue in nonrigid image registration is to enforce the estimated transformation to preserve the topology. The property of topology preservation is related to the continuity and invertibility of the transformation, which should be a one-to-one mapping. Enforcing such property requires to constraint the Jacobian of the transformation to be positive. This problem has already been successfully tackled for B-spline-based deformation fields in the 3-D case [11] by solving the corresponding constrained optimisation problem. An alternative to these approaches is to consider deformation fields that are solutions to Ordinary Differential Equations (ODEs) [8]. This ensures, under some conditions on the smoothness of the velocity fields, that the estimated displacement fields are diffeomorphic. A comprehensive review of the registration methods that enable to preserve topology can be found in [7]. However, it should be noticed that all these methods enforce topology preservation in the *continuous* domain (*i.e.* $E_{\mathbb{R}} \subset \mathbb{R}^3$). Thus, deforming a discrete binary image using such transformation does not ensure that the initial and the deformed images have the same *discrete* topology, because of the discretisation process that occurs during the (necessary) resampling operation. This is quite detrimental since one of the common application of image registration is atlas-based segmentation [10]. Atlas-based segmentation consists in registering a reference image $I_R : E_{\mathbb{Z}} \to \mathbb{Z}$, associated with a reference segmentation map $M \subset E_{\mathbb{Z}}$ (the "atlas"), onto the image $I_T : E_{\mathbb{Z}} \to \mathbb{Z}$ to segment, thus generating a continuous deformation field $D : E_{\mathbb{R}} \to E_{\mathbb{R}}$ (with $E_{\mathbb{R}} \cap \mathbb{Z}^3 = E_{\mathbb{Z}}$) and then to deform M using D in order to obtain the segmentation of I_T. Thus, the resulting segmentation $X = \Phi(D(M))$ (where $\Phi : \mathcal{P}(\mathbb{R}^3) \to \mathcal{P}(\mathbb{Z}^3)$ is a discretisation operator) can unfortunately not be ensured to have the same topology as M, even if it relies on a topology-preserving image registration method.

1.3 Motivation

Surprisingly, several works have been devoted to develop registration methods providing 3-D deformation fields which preserve the *continuous* topology, while the topology preservation of discrete objects deformed by such fields has not been considered. Other works have proposed discrete topology-preserving image deformation methods generally based on "simple" hypotheses, and in particular not considering complex 3-D deformation fields.

Based on these considerations, we propose a methodological framework for deforming a binary image $M \subset E_{\mathbb{Z}}$ according to a topology-preserving deformation field $D : E_{\mathbb{R}} \to E_{\mathbb{R}}$ without altering its discrete topology. This framework can be seen as an attempt to fuse the concepts of image registration and topology-preserving deformation into an hybrid and sound methodology. This kind of approach is essentially devoted to be used in the context of atlas-based segmentation.

To our knowledge, only one other attempt has been made (in parallel to our work) to reach that goal by using a different *modus operandi*, first computing a "discrete" deformation field [1] associated to a continuous one, w.r.t. a considered image, and then using it in a deformation process [2] devoted to this image. By opposition, our approach computes the image which would be obtained by registering a binary model without considering topology, and then performs a topology-preserving deformation to make the initial model converge onto the registered one.

2 Proposed methodological framework

Following the same notations as above, the proposed methodology consists in deforming M according to a cost function f (depending on D and M) to finally obtain a result X presenting the same topology as M and a geometry as close as possible to the one of the object K which would have been obtained by a "standard" atlas-based strategy (*i.e.* without considering discrete topology preservation). It fuses the advantages of the two families of methods: it is based on an *arbitrary (possibly non-trivial)* topological model (M) which evolves in a *non-monotonic* and (discrete) *topology-preserving* fashion under the guidance of a *complex (and continuous) topology-preserving* deformation function.

The deformation of a binary image according to a topology-preserving deformation field can be stated as the following constrained optimisation problem:

$$X = arg \min_{Y \sim M} d(Y, M, D) \ , \tag{1}$$

where Y is a binary image constrained to be topologically equivalent ($\sim$) to M and $d(Y, M, D)$ is a distance between Y and the continuous deformed image $D(M)$. We introduce, in Sec. 2.1, a distance d, which actually provides a cost function (f, following the above notations). In Sec. 2.2, we explain how to constrain Y to be topologically equivalent to M during the optimisation process. The optimisation strategy is detailed in Sec. 2.3. An overview of the method is proposed in Alg. 2.

Algorithm 2 Binary image topology-preserving deformation algorithm (see text).

Input: $M \subset E_{\mathbb{Z}}$ (binary image to deform according to D), $D : E_{\mathbb{R}} \to E_{\mathbb{R}}$ (deformation field)
Output: $X \subset E_{\mathbb{Z}}$ (deformed binary image)
$X = M$
Compute $(D^{(i)})_{i=1}^{n}$ (intermediate deformation fields obtained from D).
for $D^* = D^{(1)}$ to $D^{(n)}$ **do**
  $\mathcal{L} = \{x \in E_{\mathbb{Z}} \mid x \text{ is simple for } X\}$
  repeat
    while $\mathcal{L} \neq \emptyset$ **do**
      $x = arg \max_{y \in \mathcal{L}} c(y, X, M, D^*)$
      if $x \in X$ **then**
        $X = X \setminus \{x\}$
      else
        $X = X \cup \{x\}$
      end if
      Update $\mathcal{L}$ considering the new status of points in $N_{26}(x)$ (the neighbourhood of x).
    end while
    if $\exists (x, x')$ which can be translated and which maximises $c(., X, M, D^*)$ **then**
      if $x \in X$ **then**
        $X = (X \setminus \{x\}) \cup \{x'\}$
      else
        $X = (X \setminus \{x'\}) \cup \{x\}$
      end if
      Update $\mathcal{L}$ considering the new status of points in $N_{26}(\{x, x'\})$ (the neighbourhood of $\{x, x'\}$).
    end if
  until $\mathcal{L} = \emptyset$
end for

2.1 Cost function

Since M and X have the same topology, there exists a one-to-one relation between the connected components (CCs) of M and the ones of X. These CCs can be background (BCCs) or object (OCCs) CCs, each one being associated to a distinct label. We define $\mathcal{N}(x, X, M)$ as the CC of M which corresponds to the CC of X containing the point x. We define the distance $d(X, M, D)$ between X and the continuous deformed image $D(M)$ as follows:

$$d(X, M, D) = \sum_{x \in X} \rho(x, X, M, D) \ , \text{ with } \rho(x, X, M, D) = \min_{y \in \mathcal{N}(x, X, M)} \|y - D(x)\| \ , \tag{2}$$

where $\rho(x, X, M, D)$ is the distance between $D(x)$ and the CC of M which is associated to the CC that contains x in X. To clarify the idea, the computation of the cost function is illustrated in a 2-D case in Figure 1. The value $\rho(x, X, M, D)$ can be approximated by computing the chamfer distance map (in M) of the CC that contains x in M and by evaluating its value at position $D(x)$ (linear interpolation is used for the computation of $\rho(x, X, M, D)$, since this approximation is largely sufficient for the algorithm convergence).

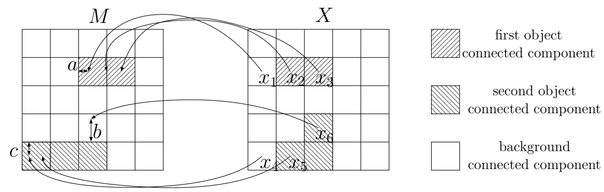

Figure 1: Computation of $\rho(., X, M, D)$ for 6 points. For $i \in \{2, 3, 5\}$, $\rho(x_i, X, M, D) = 0$, since $D(x_i)$ belongs to the same CC as x_i. However, x_6 belongs to the second OCC whereas $D(x_6)$ belongs to the BCC so that $\rho(x_6, X, M, D)$ is equal to the distance between $D(x_6)$ and the second OCC (in M), namely the value b. In the same way, $\rho(x_1, X, M, D) = a$ and $\rho(x_4, X, M, D) = c$.

2.2 Topology handling

The method starts from $S = M$, which is modified by iterative removal/addition of simple points. The label of a simple point is modified if it decreases the cost function. In case of change, the uniqueness of the CC containing x is guaranteed by the fact that x is a simple point. To determine this CC, two images representing the labels of S are used. They are updated during the whole process with S.

It has to be noticed that the removal/addition of simple points is ill adapted when a point has to be "translated". That is why a concept of translation (which is often required in a deformation process) is defined. This notion is interpreted here as the "simultaneous" modification of the status of a pair (x, x') of adjacent points such that (x, x') or $(x', x) \in S \times \overline{S}$. To guarantee topology preservation, it is sufficient (but not necessary: some possible translations may then unfortunately not be considered) to check that x (resp. x') is simple for S and x' (resp. x) is simple in S' obtained from S after the modification of x (resp. x'). The translation at point x is performed if it actually reduces the cost function. If x can be translated in different ways, the translation which minimises at best the cost function is chosen. The translation can be interpreted in terms of an addition (resp. a removal) followed by a removal (resp. an addition) of simple points. Note that the cost function is only estimated *after* the two label modifications (consequently, the first modification may increase the cost function, which would be forbidden with the classical removal/addition of simple points).

2.3 Optimisation strategy

The purpose of the optimisation strategy is to reach the minimal value of the cost function by iterative removal/addition of simple points or by translations preserving the topology, *i.e.*, to converge to a model topologically equivalent and geometrically similar to the continuous deformed image $D(M)$. The selection of points to modify requires a set $\mathcal{L}$ of all simple points of S presenting a positive cost. The cost is defined as the benefit to change the label of a simple point x. More precisely, the modification of x in S enables to decrease the cost function from the value:

$$c(x, S, M, D) = d(S, M, D) - d(S', M, D) = \rho(x, S, M, D) - \rho(x, S', M, D) , \tag{3}$$

where S' is the image obtained from S by modifying the label of x.

During the dynamical scheme, when modifying a simple point x in S to obtain a new image S', there is no need to recompute $\mathcal{L}$ since (i) $c(x', S, M, D) = c(x', S', M, D)$ for all points $x' \neq x$, and (ii) simpleness of points can only be modified in the 26-neighbourhood of x. Consequently, the algorithm proceeds as follows until $\mathcal{L}$ is empty. The point of highest cost, denoted x_0, is removed from $\mathcal{L}$. The label of x_0 is then modified in S. This may change the simple points which are in the 26-neighbourhood of x_0: points which were not simple (resp. simple) and which become simple (resp. non-simple) are added if they have a positive cost (resp. removed) in (resp. from) $\mathcal{L}$.

When $\rho(x, S, M, D) = 0$, the point x belongs to the correct CC. A cost $\rho(x, S, M, D) > 0$ can result from the fact that $D(x)$ is at the interface of objects, or from topological constraints. However, it may also result from the convergence of the method to a local minimum. To deal with this issue, we check for all points x verifying $\rho(x, S, M, D) > 0$ if it is possible to translate x to reduce the cost function without topology modification.

In order to avoid convergence onto local minima (resulting from geometrical or topological constraints) which can appear with large displacements, the deformation is performed in a "smooth" way by estimating $n+1$ intermediate deformation fields $D^{(i)}$ computed from D such that:

$$\begin{cases} D^{(0)} = Id,\ D^{(n)} = D\ , & (i) \\ \forall j \in [0, n-1],\ \forall x,\ \|D^{(j+1)}(x) - D^{(j)}(x)\| < 1\ . & (ii) \end{cases} \tag{4}$$

Constraint (ii) provides a lower bound for n: $n \geq \max_{x \in M} \|D(x) - x\|$. The deformation fields $D^{(i)}$ $(0 < i < n)$ are finally defined by:

$$\forall x, (D^{(i)}(x) - x) = \frac{i}{n}(D(x) - x)\ . \tag{5}$$

Starting from $S = M$, simple points are then added/removed to/from S until convergence of the dynamical scheme w.r.t. the cost function based on $D^{(1)}$. The same process is then iteratively carried out with $D^{(2)}$, ..., $D^{(n)}$ using the currently deformed image S.

3 Experiments and discussion

The proposed methodology has been considered for skull segmentation from 3-D CT data. In this context, the atlas M is a skull template (see Figure 2, left) associated to a reference CT image I_R. This template has a complex topology: it is composed of one connected component without cavities but with several holes (10, of various "sizes") corresponding to anatomical structures (foramina, etc.). This binary atlas M is then deformed according to a continuous 3-D field D (generated by registering [11] I_R onto the image I_T from which the skull structures have to be segmented). The method has been applied on 15 CT images of millimetric resolution ($E_{\mathbb{Z}} = [0, 255]^3$).

The obtained results have been analysed from topological and geometrical points of view, by comparison to "classical" atlas-based interpolation methods (nearest interpolation technique (M_1), and linear interpolation followed by a thresholding (M_2): see Figure 2, right, and Table 1). A more detailed analysis can be found in [6], from which we can conclude that the geometry of the results is similar for the different methods, but only the proposed one correctly handles topology.

The proposed method is one of the first enabling to deform a 3-D object of arbitrary topology by guiding the deformation process in order to geometrically converge onto the result obtained by continuous topology-preserving image registration. In further works, some points will be more

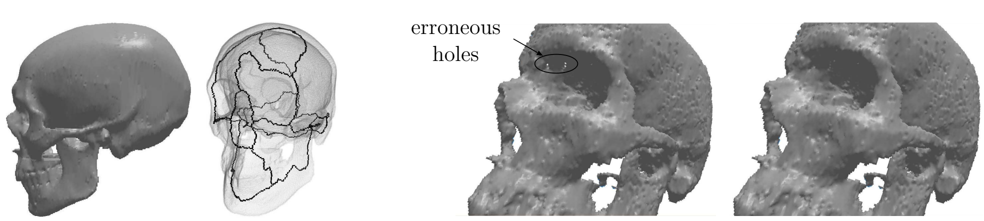

Figure 2: Left: topological skull template (whole, and with its topological skeleton). Right: segmentations obtained with M_1 and M_2 (left, see text) and the proposed method (right).

	d_μ	$d_{\max}$	b_0	b_1	b_2
M_0	$3.44.10^{-3}$	1.06	1.00	10.0	0.00
M_1	$3.70.10^{-3}$	0.87	1.06	24.8	7.20
M_2	$3.44.10^{-3}$	0.50	1.33	109	150

Table 1: Comparison of the method (M_0) with two other ones (M_1, M_2: see text): d_μ (resp. $d_{\max}$): mean (resp. maximal) value of $\rho(x, X, M, D)$; b_i: Betti numbers of X (mean values).

specifically considered: How to correctly deal with *all* deformation fields (especially, how to efficiently decompose them into successive intermediate fields)? How to avoid deadlocks caused by topological constraints (considering other strategies than translations, for example, the use of minimal simple pairs [12])? etc. The case of label image deformation will also be carefully considered, since it remains - to the best of our knowledge - an open problem.

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