Topology-preservation in 3-D image deformation and registration: Methodology and medical applications

Nicolas Passat[1], Sylvain Faisan[1], Vincent Noblet[1]

[1]LSIIT, UMR 7005 CNRS/ULP - Strasbourg

CTIC 2008 - Poitiers - 16-17/06/2008

Topology and 3D (medical) image analysis

When segmenting 3D medical images (MRI, CT, . . .), it is generally important to provide "anatomically correct" results, *i.e.* with:

- a correct morphology ("shape");
- a correct geometry (size, volume, thickness, etc.);
- a correct topology (relations, connectedness, etc.).

The issues of morphology and geometry are generally considered: it is often not the case of topology. . .

Topology and 3D (medical) image analysis

A solution to deal with topology: using an *a priori* model presenting satisfying properties w.r.t. the object to analyse, and "fitting" this model on the object.

This can be done by:

- image deformation, which considers discrete topology preservation;
- image registration, which considers continuous topology preservation.

Topology-preserving image deformation

Image deformation: progressive modification of a discrete object $M \subset E_{\mathbb{Z}} (\subset \mathbb{Z}^3)$ in a homotopy-preserving fashion (simple points), under the guidance of *a priori* knowledge:

- a *constraint set* $K \subset E_{\mathbb{Z}}$ (a "geometric" target to reach);
- a *cost* or *priority function* f devoted to guide the deformation.

Algorithm

$X = M$
repeat
  Choose $x \in \{y \in E_{\mathbb{Z}} \mid y$ *is simple for* $X\}$ *according to* f.
  if $x \in X$ **then**
    $X = X \setminus \{x\}$
  else
    $X = X \cup \{x\}$
  end if
until *the process converges w.r.t.* f *and* K.

Topology-preserving image deformation

The purpose of such methods is to obtain a result $X \subset E_{\mathbb{Z}}$ having the same topology as M while being "as similar as possible" to K.

Several challenges:

1. How to handle objects having a complex topology?
2. How to define a cost function modelling high-level *a priori* knowledge?
3. How to enable a flexible deformation while guaranteeing its convergence?

Topology-preserving image deformation

Most methods (Mangin, Poupon, Cointepas, Dokládal, Han, Daragon, Passat, Bazin, Miri, etc.) often omit one or several of these requirements.

In particular, many of them:

- consider objects of non-complex topology or use simplifying hypotheses to lead to simpler topology;
- use cost functions based on low-level or local knowledge (distance maps, intensity, etc.); and/or
- use a monotonic deformation process ($\Rightarrow$ either $M \subset K$ or $K \subset M$).

Remark

Methods exclusively applied to cerebral or vascular structure detection from 3-D MRI or CT-data.

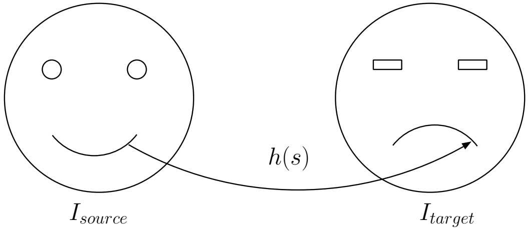
Topology-preserving image deformation

To develop efficient topology-preserving deformation methods, it would be important to authorise the use of complex objects M, evolving non-monotonically onto targets K of possibly complex topologies by guidance of high-level priority functions f, with low algorithmic complexities.

To the best of our knowledge, this issue has not frequently been addressed!

Topology-preserving image registration

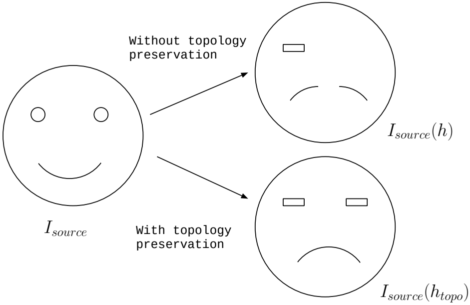
Image registration aims at estimating consistenly a mapping between two images.



$$h = \arg\min E(I_{source}, I_{target}, h)$$

Topology-preserving image registration

An important issue in nonrigid image registration is to enforce the estimated transformation to preserve the topology.



Topology-preserving image registration

Conditions for enforcing h to be a one-to-one mapping:

- h should be continuous;
- the determinant of the Jacobian of h should be strictly positive;
- h should be invariant on the boundaries of the image.

Requires to solve the following optimisation problem (Noblet, IEEE TIP, 2005):

$$h = \arg \min_{\substack{\forall s \in \Omega, J_h(s) > 0 \\ \forall s \in \partial\Omega, h(s) = s}} E(I_{source}, I_{target}, h)$$

Motivation

Image registration methods:

- $\oplus$ provide continuous topology-preserving deformation fields;
- $\ominus$ topology preservation of discrete objects deformed by such fields not considered.

Image deformation methods:

- $\oplus$ preserve discrete topology of objects;
- $\ominus$ deformation methods generally based on "simple" hypotheses.

Purpose: fusion of both approaches, and of their advantages.
$\rightarrow$ "natural" application to atlas-based segmentation.

Only similar work (in parallel to ours): Bazin *et al.* (IPMI'07, MICCAI'07).

Overview

Input:

- $M \subset E_{\mathbb{Z}}$ (topological model);
- $D : E_{\mathbb{R}} \to E_{\mathbb{R}}$ (deformation field: "$K = D(M)$").

Process: deformation of M guided by a cost function $f(D, M)$.

Output:

- $X \subset E_{\mathbb{Z}}$:
  - topologically equivalent to M;
  - geometrically "as close as possible" to K.

Method based on an arbitrary (possibly non-trivial) topological model (M) which evolves in a non-monotonic and (discrete) topology-preserving fashion under the guidance of a complex (and continuous) topology-preserving deformation function.

Overview

Deformation of a binary image according to a topology-preserving deformation field = constrained optimisation problem:

$$X = arg \min_{Y \sim M} d(Y, M, D)$$

where Y is a binary image topologically equivalent ($\sim$) to M and $d(Y, M, D)$ is a distance between Y and the continuous deformed image $D(M)$.

Discussion about:

- the distance d (which provides the "cost function" f);
- the way to preserve topology in Y;
- the optimisation strategy.

Distance / Cost function

$M \sim X \Rightarrow$ existence of a one-to-one relation between the CCs of M and the ones of X.
Let $\mathcal{N}(x, X, M)$ be the CC of M which corresponds to the CC of X containing the point x.

Definition

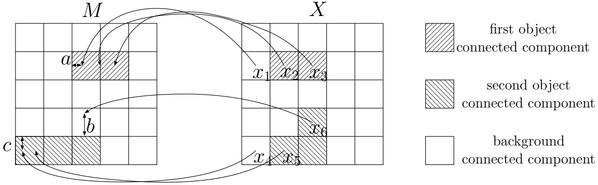
The distance $d(X, M, D)$ between X and the continuous deformed image $D(M)$ is:

$$d(X, M, D) = \sum_{x \in X} \rho(x, X, M, D)$$

where $\rho(x, X, M, D) = \min_{y \in \mathcal{N}(x,X,M)} \|y - D(x)\|$ is the distance between $D(x)$ and the CC of M which is associated to the CC that contains x in X.

Distance / Cost function

Example (2D):



Computation of $\rho(., X, M, D)$ for 6 points.

- For $i \in \{2, 3, 5\}$, $\rho(x_i, X, M, D) = 0$ ($D(x_i)$ belongs to the same CC as x_i).
- x_6 belongs to the 2nd OCC, but $D(x_6)$ belongs to the BCC: $\rho(x_6, X, M, D) = b$.
- *Idem* for $\rho(x_1, X, M, D) = a$ and $\rho(x_4, X, M, D) = c$.

Topology handling

(Classical) notion of *simple-equivalence*: iterative removal/addition of simple points (homotopy-preserving).
Strategy ill adapted when a point has to be "translated".
$\rightarrow$ Introduction of a concept of "translation".

Definition

A translation is a simultaneous modification of the status of a pair (x, x') of adjacent points such that (x, x') or $(x', x) \in S \times \overline{S}$.

Remark

To guarantee topology preservation, it is sufficient (but not necessary) to check that x (resp. x') is simple for S and x' (resp. x) is simple in S' obtained from S after the modification of x (resp. x').

Optimisation strategy

Purpose: reach the minimal value of f by topology-preserving modification of points, *i.e.*, converge onto a model topologically equivalent and geometrically similar to the continuous deformed image $D(M)$.

The selection of points to be modified requires a set $\mathcal{L}$ of all simple points of S presenting a positive cost.

Cost = "benefit" to change the label of a simple point x.
The modification of x in S enables to decrease f from:

$$c(x, S, M, D) = d(S, M, D) - d(S', M, D)$$

$$= \rho(x, S, M, D) - \rho(x, S', M, D)$$

where S' is the image obtained from S by modifying the label of x.

Optimisation strategy

When modifying a simple point x in S, there is no need to recompute $\mathcal{L}$ since:

- $\forall x' \neq x, c(x', S, M, D) = c(x', S', M, D)$;
- simpleness of points can only be modified in the 26-neighbourhood of x.

Consequently, the algorithm proceeds as follows until $\mathcal{L}$ is empty:

- the point of highest cost, denoted x_0, is removed from $\mathcal{L}$;
- the status of x_0 is then modified in S;
- points which were not simple (resp. simple) and which become simple (resp. non-simple) are added if they have a positive cost (resp. removed) in (resp. from) $\mathcal{L}$.

Optimisation strategy

Large displacements $\Rightarrow$ possible convergence onto local minima.
A solution: deformation performed in a "smooth" way by estimating *intermediate* deformation fields $D^{(i)}$ computed from D.

$$D^{(0)} = Id, \;\; D^{(n)} = D$$

$$\forall j \in [0, n-1], \;\forall x, \;\; \|D^{(j+1)}(x) - D^{(j)}(x)\| < 1$$

Remark

Lower bound for n: $\max_{x \in M} \|D(x) - x\|$.

The deformation fields $D^{(i)}$ $(0 < i < n)$ are finally defined by:

$$\forall x, (D^{(i)}(x) - x) = \frac{i}{n}(D(x) - x)$$

Starting from $S = M$, the dynamical process is iteratively carried out with $D^{(1)}, \ldots, D^{(n)}$ using the currently deformed image S.

Algorithm

Input: $M \subset E_{\mathbb{Z}}$, $D : E_{\mathbb{R}} \longrightarrow E_{\mathbb{R}}$
Output: $X \subset E_{\mathbb{Z}}$
$X = M$
Compute $(D^{(i)})_{i=1}^{n}$ (intermediate deformation fields obtained from D).
for $D^* = D^{(1)}$ to $D^{(n)}$ **do**
  $\mathcal{L} = \{x \in E_{\mathbb{Z}} \mid x \text{ is simple for } X\}$
  repeat
    while $\mathcal{L} \neq \emptyset$ **do**
      $x = \mathit{arg}\max_{y \in \mathcal{L}} c(y, X, M, D^*)$
      if $x \in X$ **then**
        $X = X \setminus \{x\}$
      else
        $X = X \cup \{x\}$
      end if
      Update $\mathcal{L}$ considering the new status of points in $N_{26}(x)$.
    end while
    if $\exists(x, x')$ which can be translated and which maximises $c(., X, M, D^*)$ **then**
      if $x \in X$ **then**
        $X = (X \setminus \{x\}) \cup \{x'\}$
      else
        $X = (X \setminus \{x'\}) \cup \{x\}$
      end if
      Update $\mathcal{L}$ considering the new status of points in $N_{26}(\{x, x'\})$.
    end if
  until $\mathcal{L} = \emptyset$

end for

Application to skull segmentation

Application of the proposed methodological framework to atlas-based segmentation (MICCAI'08).

"Deforming a pre-processed skull template (M) associated to a reference CT image (R) according to a deformation field (D) generated by the non-rigid registration between R and a CT image (T) to be segmented (X)".

- Skull structure segmentation (partial: "external" parts);
- CT data (256^3, millimetric);
- 15 images;
- non-rigid registration: Noblet (IEEE TIP, 2005).

Topological model (the "altas")

Template (M):

- generated by segmentation and "manually" corrected;
- binary;
- geometrically and topologically correct ($b_0 = 1$, $b_1 = 10$, $b_2 = 0$);
- only models the parts of the skull which have to be segmented.

Validations

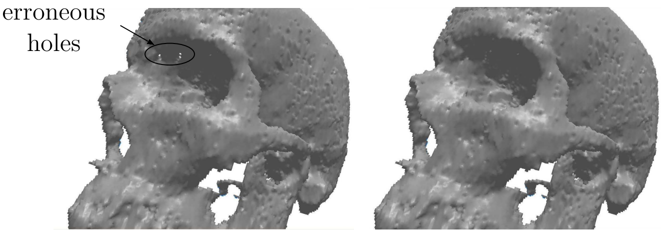
Validations from:

- geometrical point of view;
- topological point of view;

by comparison to:

- nearest interpolation;
- linear interpolation followed by a thresholding.

Validations



	d_μ	$d_{\max}$	b_0	b_1	b_2
M_0	$3.44.10^{-3}$	1.06	1.00	10.0	0.00
M_1	$3.70.10^{-3}$	0.87	1.06	24.8	7.20
M_2	$3.44.10^{-3}$	0.50	1.33	109	150

Contribution

Proposed method: one of the first enabling to deform a 3-D object of arbitrary topology by guiding the deformation process in order to geometrically converge onto the result obtained by continuous topology-preserving image registration.

It is not so complex, and it works well in real applications!

But it is (unfortunately) still not perfect. . .

Perspectives

Some points still have to be carefully considered (non-exhaustive list):

- How to correctly deal with *all* deformation fields (non-linear decomposition)?
- How to avoid - as much as possible - deadlocks caused by topological constraints?
- How to deal with label images (current solutions are not really correct. . .)?
- *Ad lib.* . .

Thank you for your attention.

Contacts

- Nicolas Passat: passat@dpt-info.u-strasbg.fr
  https://dpt-info.u-strasbg.fr/~passat
- Sylvain Faisan: sylvain.faisan@ensps.u-strasbg.fr
  http://lsiit-miv.u-strasbg.fr/miv/fiche.php?id=22
- Vincent Noblet: vincent.noblet@ensps.u-strasbg.fr
  http://lsiit-miv.u-strasbg.fr/miv/fiche.php?id=5