Towards connected filtering based on component-graphs

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Abstract

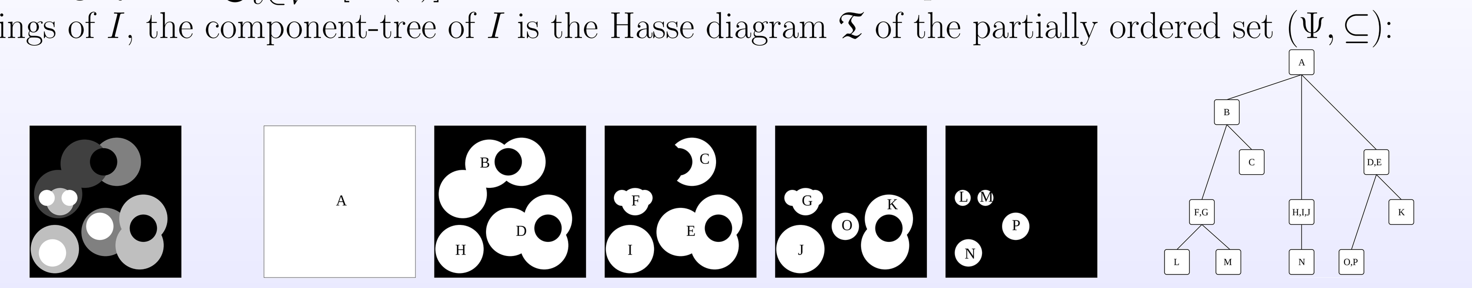
In recent works, a new notion of component-graph has been introduced to extend the data structure of component-tree –and the induced antiextensive filtering methodologies– from grey-level images to multivalued ones. In this article, we briefly recall the main structural key-points of component-graphs, and we present the initial algorithmic results that open the way to the actual development of component-graph-based antiextensive filtering procedures.

Definitions and notations

Let Ω be a nonempty finite set. For any $X \subseteq \Omega$, the set of the connected components of X is noted $\mathcal{C}[X]$. Let V be a nonempty finite set equipped with an order relation $\leqslant$. Let I be an image defined by a function $I : \Omega \to V$. For any $v \in V$, let λ_v be the thresholding function at value v, defined for any image I, by $\lambda_v(I) = \{x \in \Omega \mid v \leqslant I(x)\}$.

Component-trees

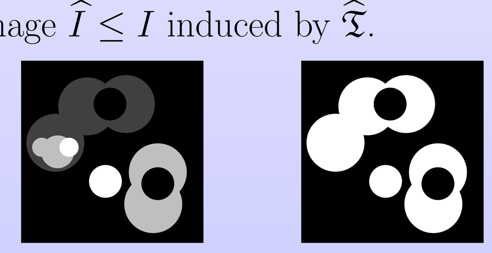
When the order relation $\leqslant$ is total, I is a grey-level image and the component-tree [2] can then be defined as follows.
Denoting by $\Psi = \bigcup_{v \in V} \mathcal{C}[\lambda_v(I)]$ the set of all the connected components obtained from all the thresholdings of I, the component-tree of I is the Hasse diagram $\mathfrak{T}$ of the partially ordered set $(\Psi, \subseteq)$:



Grey-level image I — $\lambda_0(I)$ — $\lambda_1(I)$ — $\lambda_2(I)$ — $\lambda_3(I)$ — $\lambda_4(I)$ — Component-tree $\mathfrak{T}$ of I
Thresholded images $\lambda_v(I) \subseteq \Omega$ for v varying from 0 to 4.

Antiextensive filtering framework based on component-trees [2]:

1. construction of the component-tree $\mathfrak{T}$ associated to I;
2. pruning of $\mathfrak{T}$, based on an *ad hoc* criterion and a pruning policy;
3. reconstruction of the filtered image $\widehat{I} \leq I$ induced by $\widehat{\mathfrak{T}}$.



Filtering $\widehat{I}$ — Segmentation

From component-trees to component-graphs

When the order relation $\leqslant$ is total or partial, we propose to introduce the component-graph as an extension of the component-tree to multivalued images.
The set Θ of all the valued connected components (X, v) of I is defined as

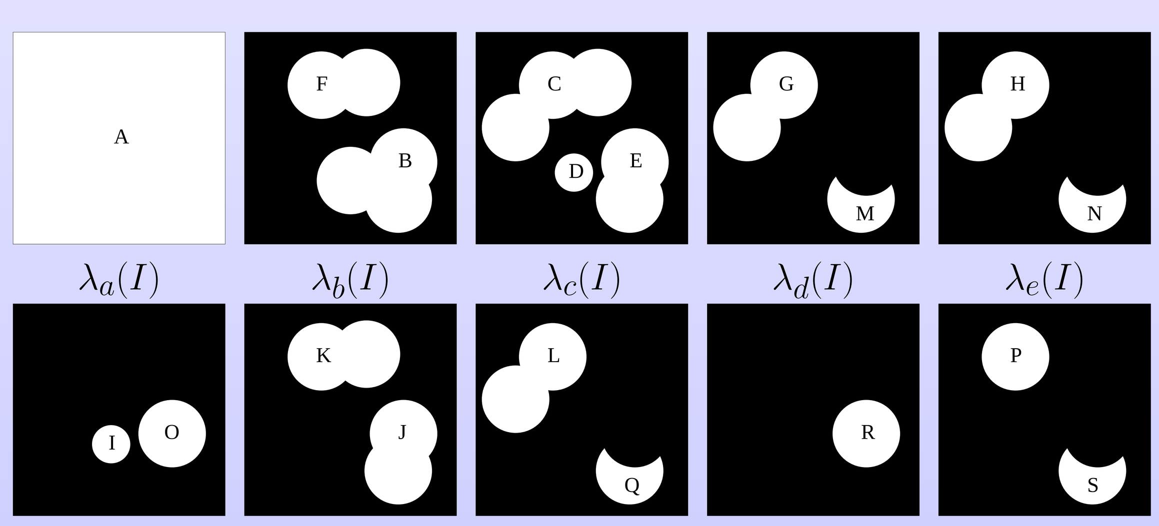
$$\Theta = \bigcup_{v \in V} \mathcal{C}[\lambda_v(I)] \times \{v\}$$

The order relation $\trianglelefteq$ on Θ is defined as

$$(X_1, v_1) \trianglelefteq (X_2, v_2) \iff (X_1 \subset X_2) \vee ((X_1 = X_2) \wedge (v_2 \leqslant v_1))$$

The *component-graph* [4] $\mathfrak{G}$ of I is based on the Hasse diagram of the ordered set $(\Theta, \trianglelefteq)$.

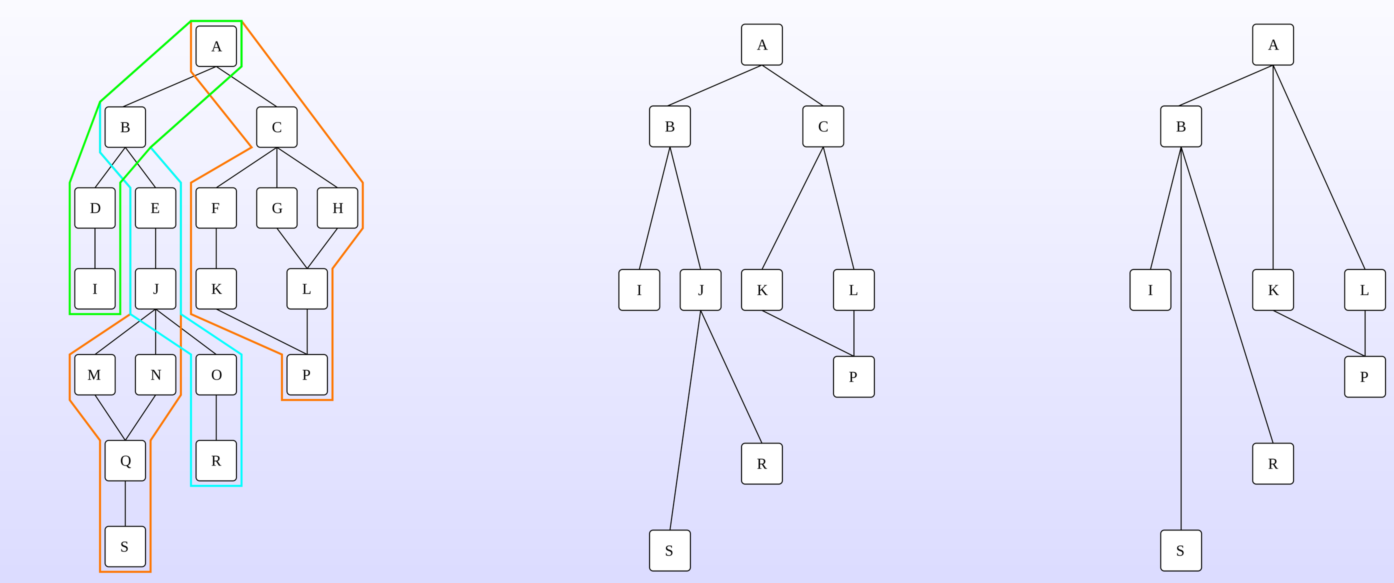
A multivalued image $I : \Omega \to V$ with $\Omega \subset \mathbb{R}^2$ and $V = \{a, b, c, d, e, f, g, h, i, j\}$. — The Hasse diagram of the ordered set $(V, \leqslant)$.



$\lambda_a(I)$ $\lambda_b(I)$ $\lambda_c(I)$ $\lambda_d(I)$ $\lambda_e(I)$
$\lambda_f(I)$ $\lambda_g(I)$ $\lambda_h(I)$ $\lambda_i(I)$ $\lambda_j(I)$
Thresholded images $\lambda_v(I)$ for $v \in V$.

Component-graphs

Three variants of component-graphs can be considered:



$\mathfrak{G}$ — $\dot{\mathfrak{G}}$ — $\ddot{\mathfrak{G}}$

- $\mathfrak{G}$ gathers all the valued connected components from Θ;
- $\dot{\mathfrak{G}}$ gathers the valued connected components of maximal values for any connected components;
- $\ddot{\mathfrak{G}}$ gathers the valued connected components associated to cylinders functions which are sup-generators of I.

Pruning the component-graph

As for the component-tree, pruning the component-graph consists of defining a subset $\widehat{\Theta} \subseteq \Theta$ based on a selection criterion ρ and a pruning policy which determines which parts of the component-graph should be preserved.
If ρ is a non-increasing criterion, several classical policies have been defined for the component-tree (for example min, direct, max, subtractive, Viterbi [2, 3]). In the case of component-graphs, the direct and max policies can be directly transposed, while the min one leads to two variants, $\min_1$ and $\min_2$, that can be axiomatically (and recursively) defined by

$$\rho(K_1) \Longrightarrow \left((\forall K_2 \blacktriangleright K_1, K_2 \in \widehat{\Theta}_{\min_1}) \Rightarrow K_1 \in \widehat{\Theta}_{\min_1}\right) \quad (1)$$

$$\rho(K_1) \Longrightarrow \left((\exists K_2 \blacktriangleright K_1, K_2 \in \widehat{\Theta}_{\min_2}) \Rightarrow K_1 \in \widehat{\Theta}_{\min_2}\right) \quad (2)$$

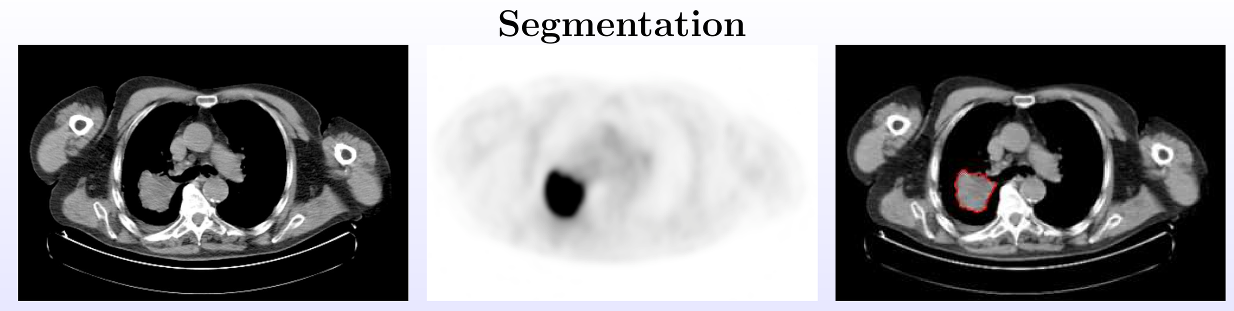
where $\blacktriangleleft$ denotes the cover relation associated to the order relation $\trianglelefteq$ on Θ.

Reconstructing an image from a pruned component-graph

The filtered image $\widehat{I} : \Omega \to V$ should be obtained from the cylinder functions $\{C_K \mid K \in \widehat{\Theta}\}$. However, contrary to the component-tree case, the expression of $\widehat{I}$ is not necessarily well-defined. Indeed, there is no guarantee that for any $x \in \Omega$, the set $\{C_K(x) \mid K \in \widehat{\Theta}\} \subseteq V$ admits a maximum (or even a supremum) for $\leqslant$. Therefore, specific strategies should be used to recover a well-defined image.

Experiments

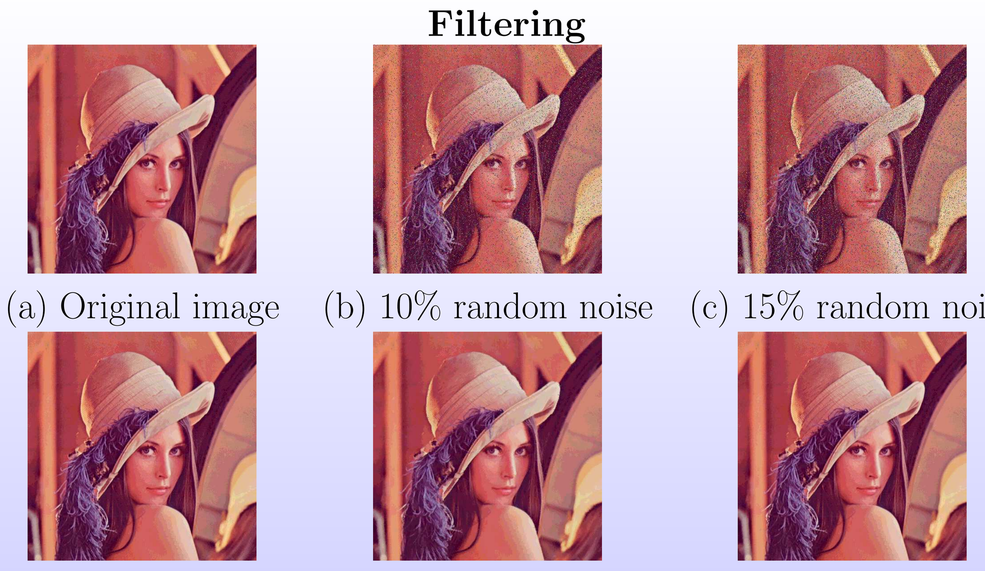
Segmentation



(a) CT image. (b) PET image. (c) Detected components.
(Image courtesy of D. Papathanassiou, Institut Jean-Godinot, France.)

- PET (Positron Emission Tomography) and standard CT (Computed Tomography) X-ray data.
- CT image provides homogeneous zones corresponding to specific tissues.
- PET image provides local intensity minima where tumours are active, but with lower spatial accuracy.
- Computation of the component-graph $\ddot{\mathfrak{G}}$ and detection of tumors based on non-increasing criterion using attributes "area" and "height".

Filtering



(a) Original image (b) 10% random noise (c) 15% random noise
(d) Original image (e) Filtering of (b) Filtering of (c)

- Denoising based on area opening: removal of connected components of size smaller than 10 pixels.

References

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