Local D-stabilization of uncertain T-S fuzzy models via fuzzy Luapunov functions

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Local $\mathcal{D}$-stabilization of Uncertain T-S Fuzzy Models via Fuzzy Luapunov Functions

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***Abstract*—This paper deals with the non-quadratic robust $\mathcal{D}$-stabilization of uncertain Takagi-Sugeno (T-S) fuzzy systems. By considering the $\mathcal{D}$-stability concept, Linear Matrix Linearity (LMI) conditions are proposed for the design of non Parallel-Distributed-Compensation (non-PDC) controllers via non-quadratic Fuzzy Lyapunov Functions (FLF). These conditions allow local $\mathcal{D}$-stabilization. Thus, a simple way is considered to estimate the domain of attraction (DA) of the designed closed-loop dynamics. The proposed result is illustrated through a numerical example.**

I. Introduction

During the last three decades, the use of so-called "multi-model" approaches for controlling smooth nonlinear systems develops exponentially. Takagi-Sugeno (T-S) fuzzy model are made of sets of linear models blended together by convex nonlinear membership functions [22]. Among the most widely used methods, they are mainly considered since they can exactly represent a nonlinear system in a compact set of its state space by using, for example, sector nonlinearity transformations [25].

By using the direct Lyapunov method, Linear Matrix Inequality (LMI) conditions can be obtained for the stability analysis and the stabilization of T-S models (see e.g. [4, 28, 23, 14, 19, 3]). First conditions were obtained through common Quadratic Lyapunov Functions (QLF) [25, 28], which require to find common Lyapunov matrix. The use of QLF leads to conservative conditions [21]. To reduce the conservatism, the design of non-PDC controllers have been proposed in the non-quadratic framework via Fuzzy Lyapunov Functions (FLF) [15, 23, 11, 24]. In this context, the time derivatives of the membership functions appear in the obtained stability conditions. Therefore, these conditions are local and some approaches have been proposed to estimate domain of attractions [9, 10, 17].

This paper is concerned with the prescription of transient response performances. This can be achieved by considering the $\mathcal{D}$-stability concept, which was first proposed for uncertain linear systems [8]. This consists in constraining the migration of the designed closed-loop eigenvalues to belong in a prescribed LMI region. This concept has been transposed to T-S fuzzy model-based control in the quadratic framework (see e.g. [16, 12, 13, 20, 26, 1, 6, 7]), then recently in the non-quadratic framework for nominal T-S models without considering domain of attraction estimations [5]. Hence, our goal in this paper is to propose local non-quadratic LMI conditions for the robust $\mathcal{D}$-stabilization of uncertain T-S models, with the estimation of their domain of attractions.

This paper is organized as follow. After presenting some useful preliminaries, one derives robust non-quadratic conditions for uncertain T-S closed-loop system including $\mathcal{D}$-stability constraints. In this context, a simple way, inspired by [17], is considered to estimate the DA . Finally, the effectiveness of the proposed results is illustrated through a numerical example.

II. Preliminaries

Let us consider an uncertain nonlinear system given by:

$$\begin{aligned}\dot{x}(t) = &\left(A(x(t)) + \Delta A(x(t))\right) x(t) \\ &+ \left(B(x(t)) + \Delta B(x(t)) \, u(t)\right)\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $A(x(t)) \in \mathbb{R}^{n\times n}$ is the nominal state matrix, $B(x(t)) \in \mathbb{R}^{n\times m}$ is the nominal input matrix, $\Delta A(x(t)) \in \mathbb{R}^{n\times n}$ and $\Delta B(x(t)) \in \mathbb{R}^{n\times m}$ are Lesbesgue measurable bounded uncertain matrices.

From usual polytopic transformations such like the sector nonlinearity approach [25], one can define the following compact set of the state space:

$$\Omega := \left\{ x(t) \in \mathbb{R}^n \;\middle|\; \begin{array}{l} A(x(t)) = \sum_{i=1}^{r} h_i(z(t))A_i, \\ B(x(t)) = \sum_{i=1}^{r} h_i(z(t))B_i, \\ \Delta A(x(t)) = \sum_{i=1}^{r} h_i(z(t))\Delta A_i, \\ \Delta B(x(t)) = \sum_{i=1}^{r} h_i(z(t))\Delta B_i. \end{array} \right\} \tag{2}$$

where the following uncertain T-S model match exactly (1):

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(\tilde{A}_i(t)x(t) + \tilde{B}_i(t)u(t)) \tag{3}$$

with r the number of vertices, $z(t) \in \mathbb{R}^p$ the vector of premises which is assumed to depend only on the state variables, $h_i(z(t)) \in [0,1]$ convex membership which satisfy $\sum\limits_{i=1}^{r} h_i(z(t)) = 1$, and:

$$\tilde{A}_i(t) = A_i + \Delta A_i(t) \in \mathbb{R}^{n\times n} \tag{4}$$

$$\tilde{B}_i(t) = B_i + \Delta B_i(t) \in \mathbb{R}^{n\times m} \tag{5}$$

where A_i and B_i are real constant matrices defining the i^{th} nominal vertex, $\Delta A_i(t)$ and $\Delta B_i(t)$ are bounded matrix uncertainties, which can be rewritten as follow [30]:

$$\Delta A_i(t) = E_i^a \delta^a(t) L_i^a \tag{6}$$

and

$$\Delta B_i(t) = E_i^b \delta^b(t) L_i^b \tag{7}$$

where E_i^a, E_i^b, L_i^a, L_i^b are real constant matrices with appropriate dimensions and $\delta^a(t)$ and $\delta^b(t)$ are uncertain matrices which verify the bounded conditions:

$$(\delta^a)^T(t)\delta^a(t) \leqslant I \tag{8}$$

and

$$\left(\delta^b\right)^T(t)\delta^b(t) \leqslant I \tag{9}$$

In order to provide relaxed controller design from a Fuzzy Lyapunov Function, let us consider the non-PDC control law given by [15, 11, 23]:

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) F_i \left(\sum_{j=1}^{r} h_j(z(t)) P_j \right)^{-1} x(t) \tag{10}$$

where $F_i \in \mathbb{R}^{m\times n}$ and $P_j \in \mathbb{R}^{n\times n}$ are constant gain matrices to be synthesized. Note that P invertible is granted as it must be positive definite.

Notations: In the sequel, the time t will be omitted in mathematical expressions when there is no ambiguity. An asterisk $(*)$ denotes a transpose quantity in a matrix and, for any real square matrices R, $\mathcal{H}(R) = R + R^T$. Consider a set of real matrices M_i and N_{ij}, for all $(i,j) \in \{1,...,r\}^2$, one denotes $M_z = \sum_{i=1}^{r} h_i(z) M_i$, $N_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z) h_j(z) N_{ij}$ and $P_z^{-1} = \left(\sum_{j=1}^{r} h_j(z(t)) P_j\right)^{-1}$.

From (3) and (10), the closed-loop dynamics may be expressed as:

$$\dot{x} = \tilde{\mathbb{G}}_{zz} x \tag{11}$$

where $\tilde{\mathbb{G}}_{zz} = \mathbb{G}_{zz} + \Delta\mathbb{G}_{zz}$, $\mathbb{G}_{zz} = A_z + B_z F_z P_z^{-1}$ and $\Delta\mathbb{G}_{zz} = \Delta A_z + \Delta B_z F_z P_z^{-1}$.

The aim of this work is to propose new LMI based conditions allowing to design a robust non-quadratic non-PDC control law (10) such that the closed-loop dynamics (11) is $\mathcal{D}-$stable whatever $\delta^a(t)$ and $\delta^b(t)$ are, with respect to (8) and (9). This will be achieved thanks to the following lemmas and definitions.

Lemma 1. *[29]: For any real matrices X, Y and $T = T^T > 0$ with appropriate dimensions, the following inequality holds:*

$$\mathcal{H}(X^T Y) < X^T T X + Y^T T^{-1} Y \tag{12}$$

Lemma 2. *[27]: Let Γ_{ij}, $(i,j) \in \{1,...,r\}^2$, be matrices of appropriate dimensions. $\Gamma_{hh} < 0$ is satisfied if both the following conditions hold:*

$$\Gamma_{ii} < 0, \forall i \in \{1,...,r\} \tag{13}$$

$$\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \forall (i,j) \in \{1,...,r\}^2 / i \neq j \tag{14}$$

Lemma 1 is dedicated to cope with uncertainties and lemma 2 is considered for conservatism reduction, since it constitutes a good compromise between complexity and conservatism improvement (see e.g. [21]). Moreover, to introduce the $\mathcal{D}$-stability concepts [8], the following definitions are considered.

Definition 1. *[8]. A subset $\mathcal{D}$ of the complex plan is called an LMI region if it is defined by the matrices $L = L^T \in \mathbb{R}^{d\times d}$ and $M \in \mathbb{R}^{d\times d}$ such that:*

$$\mathcal{D} = \{\lambda \in \mathbb{C} : L + \lambda M + \bar{\lambda} M^T < 0\} \tag{15}$$

where d is called the order of the LMI region and $\bar{\lambda}$ denotes the complex conjugate of λ.

Definition 2. *[20] Given an LMI region defined by (15), a nonlinear system $\dot{x} = f(x)x$ is said to be $\mathcal{D}$-stable if there exists a Lyapunov function $V(x(t))$ satisfying $\frac{1}{2}\frac{\dot{V}(x(t))}{V(x(t))} \in \mathcal{D}$, i.e.:*

$$L \otimes V(x(t)) + M \otimes \frac{1}{2}\dot{V}(x(t)) + M^T \otimes \frac{1}{2}\dot{V}(x(t)) < 0 \tag{16}$$

where $\otimes$ denotes the Kronecker product.

With respect to the considered control objectives, several LMI regions can be designed through the matrices L and M (see e.g. [8, 2]). In this paper, the one considered, which is the most commonly used, is presented in Appendix.

III. MAIN RESULTS

The following theorem summarizes the main result of this paper.

Theorem 1. *Assume that, $\forall k \in \{1,...,r\}$, $\exists \phi_k \neq +\infty$ such that $|\dot{h}_k(z)| \leqslant \phi_k$. Let L and M be two prescribed matrices defining a convenient LMI region (see definition 1 and Appendix for more details on a specific region). The closed-loop system* (11) *is locally $\mathcal{D}$-stable, i.e the T-S uncertain model* (3) *is locally $\mathcal{D}$-stabilized by the non-PDC control law* (10)*, if there exist the matrices $P = P^T \succ 0$, F_j and the scalars τ_i^a and τ_i^b, with $(i,j) \in \{1,...,r\}^2$, such that the following conditions are verified:*

$$\tilde{\Lambda}_{ii} < 0, \quad \forall i \in \{1,...,r\}, \tag{17a}$$

$$\frac{2}{r-1}\tilde{\Lambda}_{ii} + \tilde{\Lambda}_{ij} + \tilde{\Lambda}_{ji} < 0, \quad \forall (i,j) \in \{1,...,r\}^2 / i \neq j \tag{17b}$$

$$\Xi_{iik} > 0, \quad \forall (i,k) \in \{1,...,r\}^2, \tag{18a}$$

$$\frac{2}{r-1}\Xi_{iik} + \Xi_{ijk} + \Xi_{jik} > 0, \quad \forall (i,j,k) \in \{1,...,r\}^3 / i \neq j \tag{18b}$$

where $\tilde{\Lambda}_{ij}$ is given in eq. (21) below and $\Xi_{ijk} = P_k + R_{ij}$. In that case, an estimate of the designed closed-loop system domain of attraction is obtained by maximizing c such that:

$$DA_\phi^* := \{x(0) \in \mathbb{R}^n \,|\exists \bar{c} = \max c,\, \Lambda(\bar{c}) \subseteq \Phi_\phi\}\,, \qquad (19)$$

that is to say, by finding the largest equipotential $V(x) = \bar{c}$ included in:

$$\Phi_\phi = \cap_{k=1}^r \left\{ x \in \mathbb{R}^n \,\middle|\, |\dot{h}_k(z)| \leqslant \phi_k \right\} \cap \Omega \qquad (20)$$

Proof. Let us consider the FLF given by:

$$V(x) = x^T P_z^{-1} x \qquad (22)$$

with $P_z > 0$.
From (22) and definition 2, the closed-loop system (11) is $\mathcal{D}$-stable if:

$$L \otimes x^T P_z^{-1} x + \mathcal{H}\left(M \otimes x^T \left(P_z^{-1}\tilde{\mathbb{G}}_{zz} + \frac{1}{2}\dot{P}_z^{-1} \right) x \right) < 0 \qquad (23)$$

Thanks to the properties of the Kronecker product, one can rewrite (23) as:

$$\psi^T \left(L \otimes P_z^{-1} + \mathcal{H}\left(M \otimes \left(P_z^{-1}\tilde{\mathbb{G}}_{zz} + \frac{1}{2}\dot{P}_z^{-1} \right) \right) \right) \psi < 0 \qquad (24)$$

with $\psi = I \otimes x$.
Thus, (24) is verified $\forall x$ if:

$$L \otimes P_z^{-1} + \mathcal{H}\left(M \otimes \left(P_z^{-1}\tilde{\mathbb{G}}_{zz} + \frac{1}{2}\dot{P}_z^{-1} \right) \right) < 0 \qquad (25)$$

Multiplying left and right by $(I \otimes P)$ and since $P_z \dot{P}_z^{-1} P_z = -\dot{P}_z$, the inequality (25) is equivalent to:

$$L \otimes P_z + \mathcal{H}\left(M \otimes \left(\tilde{\mathbb{G}}_{zz} P_z - \frac{1}{2}\dot{P}_z \right) \right) < 0 \qquad (26)$$

Let us introduce the null term:

$$\mathcal{H}\left(MM^T \otimes D_{zz} - MM^T \otimes D_{zz} \right) = 0 \qquad (27)$$

By adding (27) to (26) and from the Kronecker product properties, one obtains:

$$\begin{array}{l} L \otimes P_z + \mathcal{H}\left(M \otimes \tilde{\mathbb{G}}_{zz} P_z \right) \\ +\mathcal{H}\left((M \otimes I)\left(M^T \otimes D_{zz} - I \otimes \frac{1}{2}\dot{P}_z \right) \right) \\ -\,(M \otimes I)\left(I \otimes \left(D_{zz} + D_{zz}^T \right) \right)\left(M^T \otimes I \right) < 0 \end{array} \qquad (28)$$

Then, let us rewrite (28) as:

$$\gamma^T \begin{bmatrix} L \otimes P_z + \mathcal{H}\left(M \otimes \tilde{\mathbb{G}}_{zz} P_z \right) & (*) \\ M^T \otimes D_{zz} - I \otimes \frac{1}{2}\dot{P}_z & I \otimes \mathcal{H}(D_{zz}) \end{bmatrix} \gamma < 0 \qquad (29)$$

with $\gamma = \begin{bmatrix} I \\ M^T \otimes I \end{bmatrix}$.
Thus, $\forall M$, the inequality (29) is verified if:

$$\tilde{\Upsilon}_{zz} + \Delta\tilde{\Upsilon}_{zz} < 0 \qquad (30)$$

with $\tilde{\Upsilon}_{zz} =$

$$\begin{bmatrix} L \otimes P_z + \mathcal{H}(M \otimes (A_z P_z + B_z F_z)) & (*) \\ M^T \otimes D_{zz} - I \otimes \frac{1}{2}\dot{P}_z & I \otimes \mathcal{H}(D_{zz}) \end{bmatrix}$$

and $\Delta\tilde{\Upsilon}_{zz} = \begin{bmatrix} \mathcal{H}(M \otimes (\Delta A_z P_z + \Delta B_z F_z)) & 0 \\ 0 & 0 \end{bmatrix}$.
From (6) and (7), one can rewrite $\Delta\tilde{\Upsilon}_{zz}$ as:

$$\Delta\tilde{\Upsilon}_{zz} = \mathcal{H}\left(\bar{\mathrm{X}}_z^T \bar{\delta} \bar{Y}_z \right) \qquad (31)$$

with $\bar{\mathrm{X}}_z^T = \begin{bmatrix} I \otimes E_z^a & I \otimes E_z^b \\ 0 & 0 \end{bmatrix}$, $\bar{Y}_z = \begin{bmatrix} M \otimes L_z^a P_z & 0 \\ M \otimes L_z^b F_z & 0 \end{bmatrix}$ and $\bar{\delta} = \begin{bmatrix} I \otimes \delta^a(t) & 0 \\ 0 & I \otimes \delta^b(t) \end{bmatrix}$ satisfying $\bar{\delta}^T \bar{\delta} \leqslant 0$.
Thus, from (31) and by applying lemma 1, the inequality (30) is satisfied if:

$$\tilde{\Upsilon}_{zz} + \bar{\mathrm{X}}_z^T \bar{T}_z \bar{\mathrm{X}}_z + \bar{Y}_z^T \bar{T}_z^{-1} \bar{Y}_z < 0 \qquad (32)$$

where $\bar{T}_z = \begin{bmatrix} I \otimes \tau_z^a & 0 \\ 0 & I \otimes \tau_z^b \end{bmatrix}$ with the scalars τ_z^a and τ_z^b.
Then, applying the Schur complement, one obtains:

$$\begin{bmatrix} \tilde{\Upsilon}_{zz} + \mathrm{X}_z^T \tilde{T}_z \mathrm{X}_z & Y_z^T \\ Y_z & \tilde{T}_z \end{bmatrix} < 0 \qquad (33)$$

Note that from the convex sum property, $\sum_{i=1}^r \dot{h}_i(x) = 0$. Hence, for any matrices $R_{ij} \in \mathbb{R}^{n \times n}$, one can write:

$$\dot{P}_z = \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(z) h_j(z) \dot{h}_k(z) \left(P_k + R_{ij} \right) \qquad (34)$$

Therefore, let us assume that, $\forall k \in \{1, ..., r\}$, there exists:

$$|\dot{h}_k(z)| \leqslant \phi_k \qquad (35)$$

with $\phi_k \neq +\infty$, it yields:

$$-\dot{P}_z \leqslant \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^r h_i(z) h_j(z) \phi_k \left(P_k + R_{ij} \right) \qquad (36)$$

provided that:

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \left(P_k + R_{ij} \right) > 0,\ \forall k \in \{i, ..., r\}, \qquad (37)$$

At this stage, applying lemma 2 on the inequality (37), one obtains the condition (18a) and (18b). Moreover, after matrix expansion and majoring the inequality (33) by considering (36), then applying lemma 2, one obtains the condition (17a) and (17b) with (21).
Note that because of the assumption made in (35), one cannot guarantee the global asymptotic stabilization of the considered T-S model. Nevertheless, inspired by [17], an estimation of the domain of attraction of the designed closed-loop system can be obtained as follows.
First, let us define the compact subset related to the assumption made in (35) and given by:

$$\Pi_\phi := \cap_{k=1}^r \left\{ x(t) \in \mathbb{R}^n \,\middle|\, |\dot{h}_k(z)| \leqslant \phi_k \right\} \qquad (38)$$

Consequently, a restriction of the domain of validity of the closed-loop T-S model under the assumption (35) is given by the compact space:

$$\Phi_\phi = \Pi_\phi \cap \Omega \qquad (39)$$

$$\tilde{\Lambda}_{ij} = \begin{bmatrix} \tilde{\Lambda}_{ij}^{(1,1)} & (*) & (*) & (*) \\ M^T \otimes D_{ij} + I \otimes \left(\frac{1}{2} \sum_{k=1}^{r} \phi_k \left(X_k + R_{ij} \right) \right) & -I \otimes \mathcal{H}\left(D_{ij}\right) & 0 & 0 \\ M \otimes L_i^a \tilde{P}_j & 0 & -I \otimes \tau_i^a & 0 \\ M \otimes L_i^b F_j & 0 & 0 & -I \otimes \tau_i^b \end{bmatrix} \tag{21}$$

with $\tilde{\Lambda}_{ij}^{(1,1)} = L \otimes P + \mathcal{H}\left(\left(I \otimes A_i \right) P_j + M \otimes B_i F_j \right) + I \otimes \left(\tau_i^a E_i^a \left(E_i^a \right)^T + \tau_i^b E_i^b \left(E_{bi}^b \right)^T \right)$

Moreover, the whole DA of a dynamic model is defined by:

$$DA := \left\{ x(0) \in \mathbb{R}^n \,\middle|\, \lim_{t \to +\infty} x(t) = 0 \right\} \tag{40}$$

Furthermore, let us define the Lyapunov sublevel set given by:

$$\Lambda(c) := \left\{ x(0) \in \mathbb{R}^n \,\middle|\, V(x(t)) \leqslant c \right\} \tag{41}$$

where c is an positive scalar.
An estimate DA_ϕ^* of the closed-loop system domain of attraction DA is therefore obtained by maximizing c such that:

$$DA_\phi^* := \left\{ x(0) \in \mathbb{R}^n \,\middle|\, \exists \bar{c} = \max c,\, \Lambda(\bar{c}) \subseteq \Phi_\phi \right\} \subseteq DA, \tag{42}$$

that is to say, by finding the largest equipotential $V(x(t)) = \bar{c}$ included in Φ_ϕ. □

Remark 1. *Recall that the non-quadratic conditions proposed above do not guarantee the global asymptotic stability of the considered T-S model. Indeed, they involve the bounds of the time derivatives of the membership functions* $|\dot{h}_k(z)| \leqslant \phi_k$*. Thus, the parameters* ϕ_k *must be known a priori to apply the LMI conditions presented in theorem 1. However, it is difficult (or impossible) to estimate them before synthesizing the closed-loop dynamics. Recent results have been proposed to allow the estimation of the domain of attraction simultaneously to the resolution of the LMI problem (see e.g. [10, 18]). Nevertheless, these results being relatively complex for practitioners, one have adopted an alternative way inspired by the recent work of [17]. As a practical consequence, the* ϕ_k *must be chosen as large as possible in order to guarantee a local stabilization with the greatest possible domain of attraction. This can be achieved by linear search to maximize the* ϕ_k*.*

IV. Numerical example

Consider the following uncertain T-S model with two rules given by:

$$\dot{x} = \sum_{i=1}^{2} h_i(z) \left(\left(A_i + \Delta A_i \right) x + \left(B_i + \Delta B_i \right) u \right) \tag{43}$$

with $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A_1 = \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & -5 \\ 1 & 2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1.6 \\ 2 \end{bmatrix}$. The uncertain matrices ΔA_i and ΔB_i are decomposed as in (6) and (7) with $E_1^a = E_2^a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $E_1^b = E_2^b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $L_1^a = L_2^a = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $L_1^b = L_2^b = 0.03$ and $\delta(t)$ scalar. $h_1(z(t)) = \frac{1-\sin(x_1)}{2}$ and $h_2(z(t)) = \frac{1+\sin(x_1)}{2}$ are the convex membership functions.

Note that, in this example, the considered T-S fuzzy model is an academic one and is not derived from a specific nonlinear model (1) via the sector nonlinearity approach. Thus, one assumes that $\Omega = \mathbb{R}^2$.

The following gain matrices of a non-PDC controller (10) have been obtained from theorem 1 with $\phi_1 = \phi_2 = 60$ and the LMI region given in Appendix (see Figure 3), parametrized by $s = 21$, $q = -17$, $\beta = 10$, $\gamma = 9$ and $\theta = \pi/3$.

$$F_1 = \begin{bmatrix} -88.50 & -75.03 \end{bmatrix},\ F_2 = \begin{bmatrix} -50.67 & -49.58 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 7.00 & 4.96 \\ 4.96 & 3.67 \end{bmatrix},\ P_2 = \begin{bmatrix} 6.66 & 5.42 \\ 5.42 & 4.59 \end{bmatrix}.$$

Figure 1 shows the estimation of the designed closed-loop system's domain of attraction DA_ϕ^*, which is delimited by the largest equipotential of the Lyapunov function included in Φ_ϕ.

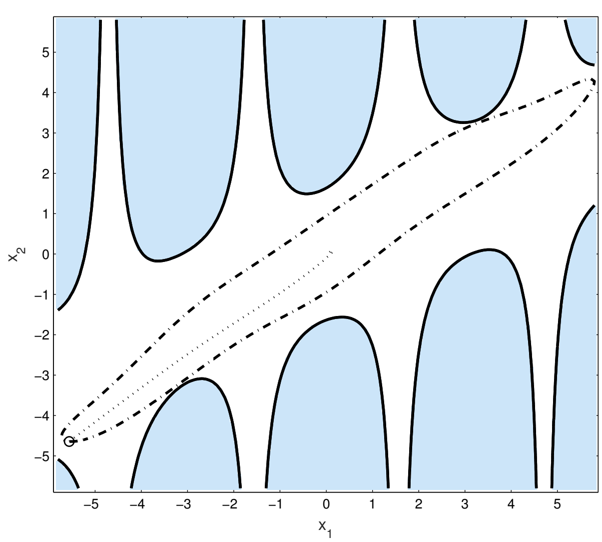

Fig. 1. Estimation of the domain of attraction DA_ϕ^*: Restricted state space Φ_ϕ (white) delimited by the bounds $|\dot{h}_k| = \phi_k$ (solid line), largest equipotential of the Lyapunov function included in Π_ϕ (dash-dot line), closed loop trajectory with initial conditions at the border of DA_ϕ^* (dash line).

Figure 2 shows the simulation of the closed-loop state response with the initial condition $x(0) = \begin{bmatrix} -5.56 & -4.64 \end{bmatrix}^T$, i.e. at the border of DA^*_ϕ. As one can notice in Figure 2(a), the synthesized robust non-PDC control law stabilizes the considered uncertain system (43) with an uncertainty $\delta(t)$, plotted in Figure 2(b). Moreover, as shown in Figure 2(c), the initial condition being inside DA^*_ϕ, the time derivatives of membership functions never exceed the prescribed bounds $\phi_k = 60$. Finally, Figure 2(d) shows that the migration of closed-loop eigenvalues stay inside the defined LMI region despite the presence of uncertainties. This confirms the effectiveness of the conditions proposed in theorem 1.

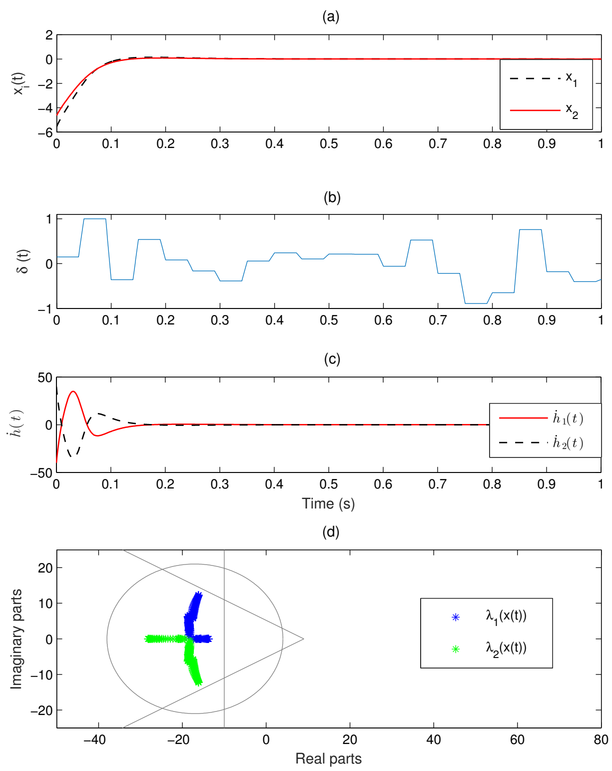

Fig. 2. Closed-loop simulation: (a) Closed-loop state response, (b) Uncertain signal $\delta(t)$, (c) Evolution of the time derivatives of the membership functions, (d) Migration of the eigenvalues.

V. Conclusion

In this paper, the problem of the local non-quadratic robust $\mathcal{D}$-stabilization of uncertain T-S systems has been considered. LMI conditions have been derived for the design of robust non-PDC controllers with $\mathcal{D}$-stability constraints with a simple method allowing to estimate of the closed-loop domain of attraction. The effectiveness of the proposed approach have been illustrated through a numerical example. Further works will focus on conservatism reduction and the optimization of the domain of attraction throughout LMI processing.

Acknowledgment

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Appendix

In this paper, an usual LMI region for $\mathcal{D}$-stability purpose is considered [8, 2] (see Figure 3). It is defined by the combination of the following subregion:

1) the left half plan defined by $\mathcal{R}e(\lambda) < \beta$,
2) a conic sector defined by its apex at $(\gamma, 0)$ and an inner angle $\pi/2 - \theta$,
3) a circle centered at $(q, 0)$ with a radius s.

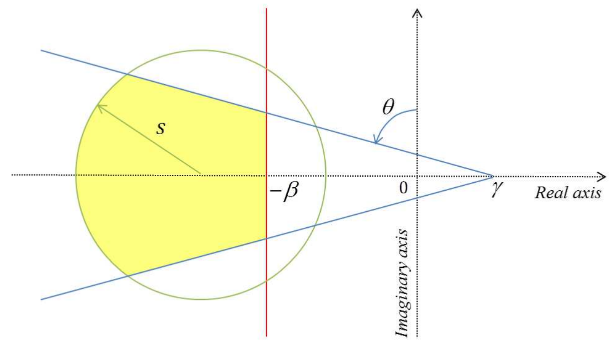

Fig. 3. Usual LMI region considered for $\mathcal{D}$-stability purpose.

In that case and with regards to definition 1, the following matrices the considered LMI region [8]:

$$L = \begin{bmatrix} 2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\gamma\cos\theta & 0 & 0 & 0 \\ 0 & 0 & -2\gamma\cos\theta & 0 & 0 \\ 0 & 0 & 0 & -s & -q \\ 0 & 0 & 0 & -q & -s \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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