LMI conditions for non-quadratic stabilization of T-S models with pole placement assignation

Abdelmadjid Cherifi, Kevin Guelton, Laurent Arcese

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***Abstract*— This paper presents new non-Parallel-Distributed-Compensation (non-PDC) controllers design conditions for continuous-time Takagi-Sugeno (T-S) models with pole placement assignation. Based on the $\mathcal{D}$-stability concept, a desired transient response may be obtained by placing the poles of the T-S closed-loop system in a specific region of the complex plan. After deriving standard non-quadratic $\mathcal{D}$-stability conditions for the T-S closed-loop system, new relaxed LMI-based conditions are obtained from enhanced Fuzzy Lyapunov Functions (FLF), which involve a double sum fuzzy structure. The effectiveness of the proposed result is illustrated in simulation through the benchmark of a flexible robot with single joint.**

I. INTRODUCTION

Takagi-Sugeno (T-S) models [1], also known as Quasi-LPV models [2], constitute a class of convex polytopic systems. T-S models may be of great interest in nonlinear control since they allow representing a large class of nonlinear systems, exactly on a compact set of their state space. Such ability is guaranteed when a T-S model is obtained from a sector nonlinearity approach [3].

T-S model-based control problem are usually investigated through the direct Lyapunov methodology (see e.g. [3], [4], [5], [6], [7], [8], [9]). In this context, the challenge consists on expressing the stability conditions as Linear Matrix Inequality (LMI) [10]. LMIs are interesting since they can be solved by convex optimization algorithms [11].

The pioneer results in stabilization were obtained through Quadratic Lyapunov Functions (QLF) (see e.g. [4], [3]). Nevertheless, QLF-based conditions are conservative because they require to find a common Lyapunov matrix solution for a set of LMIs (see [12] for a review of the sources of conservatism in T-S-based studies). To reduce the conservatism, alternative Lyapunov functions have been considered like piecewise Lyapunov function [13], switched Lyapunov function [14], [15] and non-quadratic Fuzzy Lyapunov Function (FLF) [16], [17], [18]. Note that, since they share the same fuzzy structure as the T-S model to be analyzed, FLF approaches appear convenient, especially when the sector nonlinearity approach is employed. However, in this non-quadratic context, the time derivatives of the membership functions occur in the stability conditions. To cope with these non convex terms, it is often assumed that the bounds of these derivatives are known before solving the LMIs, which is non trivial in most of practical case, especially in stabilization. To overcome this drawback, local conditions have been proposed [19], but with a somewhat complex formulation. In [20], the use of Line-Integral Lyapunov Function (LILF) have been proposed. According to path-independency conditions [20], [21], [22], LILF have the advantage to make the stability conditions free of the time derivatives of the membership functions. However, the stabilization results in [20] are given in terms of Bilinear Matrix Inequalities (BMI) and recent results have shown that LILF may lead to LMIs, but with a restriction to only second order systems [23], [24], [25]. Alternatively, Sum-Of-Square (SOS) approaches have been proposed but with a very restrictive modeling assumption on the system's input matrices [26], [27].

In this paper, we are interested in providing new LMI conditions with pole placement assignation in the non-quadratic framework. Indeed, ensuring the asymptotic stability does not necessarily mean ensuring a good transient response of the closed-loop system. In this context, some interesting works were carried out in order to improve the closed-loop transient response by adding pole placement constraints to the stabilization problem [28], [29], [30], [31]. Nevertheless, these results are given in the quadratic framework (or piecewise quadratic), which suffer from the above discussed concerns. Therefore, in this paper, without considering the estimation of a domain of attraction in the non-quadratic framework [19], [32], which will be the subject of a further extension, one first derives standard non-quadratic conditions for T-S closed-loop systems including $\mathcal{D}$-stability constraints [33]. Then new relaxed $\mathcal{D}$-stable LMI-based conditions are proposed from an enhanced FLF [32], which involves a double sum fuzzy structure. Finally, the effectiveness of the proposed results is illustrated in simulation through the benchmark of a flexible robot with single joint.

II. PRELIMINARIES

Consider the T-S fuzzy model given by [1]:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left(A_i x(t) + B_i u(t)\right) \tag{1}$$

where $x(t) \in \Omega \subseteq \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $z(t)$ is the vector of premises which may depend on the state and/or input variables, r is the number of vertices and, for $i \in \mathcal{I}_r = \{1, ..., r\}$, $h_i(z(t)) \in [0,1]$ are convex membership functions with $\sum_{i=1}^{r} h_i(z(t)) = 1$, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are real constant matrices defining the i^{th} nominal vertex.

Assumption 1 *: The T-S model* (1) *is assumed to be smooth, i.e. continuous and derivable within the time t,* $\forall x(t) \in \Omega$.

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To stabilize (1), let us consider the non-PDC control law given by [34]:

$$u(t) = \sum_{i=1}^{r} h_i(z(t))F_i \left(\sum_{j=1}^{r} h_j(z(t))H_j\right)^{-1} x(t) \quad (2)$$

where $F_i \in \mathbb{R}^{m\times n}$ and $H_j \in \mathbb{R}^{n\times n}$ are constant gain matrices to be synthesized.

Notations: *In the sequel, to lighten mathematical expressions, the time t will be omitted when there is no ambiguity. I denotes a identity matrix with appropriate dimension. An asterisk $(*)$ in a matrix denotes a transpose quantity. For any square matrix Q, $\mathcal{H}(Q) = Q + Q^T$. Consider the set of real matrices M_i and N_{ij}, with $(i,j) \in \mathcal{I}_r^2$, one denotes $M_z = \sum_{i=1}^{r} h_i(z(t))M_i$, $N_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))N_{ij}$. Finally, $\otimes$ denotes the Kronecker product.*

With the above defined notations and substituting (2) in (1), the closed-loop dynamics may be expressed as:

$$\dot{x}(t) = \tilde{A}_{zz}x(t) \quad (3)$$

with $\tilde{A}_{zz} = A_z + B_zF_zH_z^{-1}$.

Consequently, $\forall i \in \mathcal{I}_r$, if one can find the gain matrices F_i and H_i such that (3) is stable, then the T-S model (1) is stabilized by the non-PDC controller (2). The aim of this work is to propose new LMI conditions to achieve this goal in the non-quadratic framework with pole placement assignation. In this context, the following definition and lemmas are useful.

Definition 1 *(LMI region) [33]: A subset $\mathcal{D}$ of the complex plane is called an LMI region if it is defined by the matrices $L = L^T \in \mathbb{R}^{d\times d}$ and $M \in \mathbb{R}^{d\times d}$ such that:*

$$\mathcal{D} = \{\lambda \in \mathbb{C} : L + \lambda M + \bar{\lambda}M^T < 0\} \quad (4)$$

where d is called the order of the LMI region.

For more details and examples illustrating how the matrices L and M are set for a given LMI region, the reader may consult [33], [35] (an example is proposed in section IV below). LMI regions being now defined, the following lemma expresses the basic $\mathcal{D}$-stability conditions.

Lemma 1 *($\mathcal{D}$-stability)[33]. Given an LMI region defined by* (4)*, a nonlinear system (e.g. the closed-loop T-S fuzzy model* (3)*) is said to be $\mathcal{D}$-stable if there exists a Lyapunov function V(x) satisfying $\frac{1}{2}\frac{\dot{V}(x)}{V(x)} \in \mathcal{D}$, i.e.*

$$L \otimes 1 + M \otimes \frac{1}{2}\frac{\dot{V}(x)}{V(x)} + M^T \otimes \frac{1}{2}\frac{\dot{V}(x)}{V(x)} < 0 \quad (5)$$

In the sequel, the following properties of the Kronecker product will be used. For any scalars μ and matrices A, B and C with appropriate dimensions [36]:

$$A \otimes (B + \mu C) = (A \otimes B) + \mu(A \otimes C) \quad (6)$$

$$(A + \mu B) \otimes C = (A \otimes C) + \mu(B \otimes C) \quad (7)$$

$$(A \otimes C)(B \otimes D) = (AB \otimes CD) \quad (8)$$

$$(A \otimes B)^T = A^T \otimes B^T \quad (9)$$

In addition, to reduce the conservatism of the LMI conditions proposed in the sequel, the following lemma will be employed.

Lemma 2 *[37]: Let Γ_{ij}, for $(i,j) \in \mathcal{I}_r^2$, be matrices of appropriate dimensions. $\Gamma_{zz} \prec 0$ is satisfied if the following conditions hold:*

$$\Gamma_{ij} + \Gamma_{ji} \prec 0, \quad i \le j, \quad (i,j) \in \mathcal{I}_r^2, \quad (10)$$

III. MAIN RESULTS

In this section, the goal is to propose new non-quadratic LMI conditions for stabilization a class of nonlinear system (1) by non-PDC controllers (2) in a prescribed pole placement in LMI regions. First, based on the $\mathcal{D}$-stability concepts [33], non-quadratic conditions are obtained from standard FLF candidates [16]. Then, relaxed conditions are proposed via enhanced FLF involving double sum fuzzy structures [32].

A. $\mathcal{D}$-stabilizing controller design via standard FLF:

Let us first consider the standard FLF candidate given by [16]:

$$V(x(t)) = x^T(t)X_z^{-1}x(t) \quad (11)$$

In this non-quadratic context, $\mathcal{D}$-stabilization conditions for T-S models are summarized by the following theorem.

Theorem 1 *: The T-S model* (1) *being smooth, $\forall k \in \mathcal{I}_r$, $\exists \phi_k = \inf_{x\in\Omega}(\dot{h}_k(x)) \le 0$, $\phi_k \ne -\infty$. Let us assume that the ϕ_k are known and let L and M be two prescribed matrices defining a convenient LMI region (see definition 1). For $(i,j,k) \in \mathcal{I}_r^3$, if there exists the matrices $X_j = X_j^T \succ 0$, $R_{ij} = R_{ij}^T$, F_j such that the LMIs* (12) *and* (13) *are verified, then the T-S model* (3) *is $\mathcal{D}$-stabilized by the non-PDC control law* (2) *with $H_i = P_i$.*

$$\mathcal{X}_{ijk} + \mathcal{X}_{jik} \succ 0, \quad i \le j, \quad (i,j,k) \in \mathcal{I}_r^3, \quad (12)$$

$$\Psi_{ij} + \Psi_{ji} \prec 0, \quad i \le j, \quad (i,j) \in \mathcal{I}_r^2, \quad (13)$$

with:

$$\mathcal{X}_{ijk} = X_k + R_{ij},$$

$$\Psi_{ij} = L \otimes X_j + \mathcal{H}\left(M \otimes \left(A_iX_j + B_iF_j - \frac{1}{2}\Phi_{ijk}\right)\right),$$

$$\Phi_{ijk} = \sum_{k=1}^{r} \phi_k \mathcal{X}_{ijk}.$$

Proof: Consider the FLF (11). The closed-loop dynamics (3) is stable if, $\forall i \in \mathcal{I}_r$, $X_i = X_i^T \succ 0$ and [16]:

$$\begin{aligned}\dot{V}(x) &= \dot{x}^T X_z^{-1} x + x^T X_z^{-1} \dot{x} + x^T \dot{X}_z^{-1} x \\ &= 2x^T \left(\tilde{A}_{zz}^T X_z^{-1} + \tfrac{1}{2}\dot{X}_z^{-1} \right) x \\ &= 2x^T \left(X_z^{-1} \tilde{A}_{zz} + \tfrac{1}{2}\dot{X}_z^{-1} \right) x < 0\end{aligned} \tag{14}$$

According to lemma 1 and since $V(x) > 0$, the closed-loop T-S model (3) is $\mathcal{D}$-stable if:

$$L \otimes V(x) + M \otimes \frac{1}{2}\dot{V}(x) + M^T \otimes \frac{1}{2}\dot{V}(x) < 0 \tag{15}$$

One denotes:

$$Z_{zz} = X_z^{-1} \tilde{A}_{zz} + \frac{1}{2}\dot{X}_z^{-1} \tag{16}$$

Therefore, (15) can be rewritten as:

$$L \otimes x^T X_x^{-1} x + M \otimes x^T Z_{xx} x + M^T \otimes x^T Z_{xx}^T x < 0 \tag{17}$$

Now, with (8), one can rewrite (17) as:

$$(I \otimes x)^T \left(L \otimes X_z^{-1} + \mathcal{H}(M \otimes Z_{zz}) \right) (I \otimes x) < 0 \tag{18}$$

which holds $\forall x$ if:

$$L \otimes X_z^{-1} + \mathcal{H}(M \otimes Z_{zz}) \prec 0 \tag{19}$$

Now, multiplying (19) left and right by $(I \otimes X_z)$ and since $X_z \dot{X}_z^{-1} X_z = -\dot{X}_z$, (19) is equivalent to:

$$L \otimes X_z + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} X_z - \frac{1}{2}\dot{X}_z \right) \right) \prec 0 \tag{20}$$

Note that $\sum_{i=1}^{r} \dot{h}_i(x) = 0$, thus for any matrices $R_{ij} \in \mathbb{R}^{n \times n}$ one can write:

$$\dot{X}_z = \sum_{k=1}^{r} \dot{h}_k(z) \mathcal{X}_{ijk} \tag{21}$$

with $\mathcal{X}_{ijk} = X_k + R_{ij}$.
Now, according to assumption 1, T-S model (1) is smooth. Thus, $\forall k \in \mathcal{I}_r$, $\exists \phi_k = \inf_{x \in \Omega} (\dot{h}_k(x)) \leq 0$, $\phi_k \neq -\infty$ and, with $R_{ij} = R_{ij}^T$ and the LMI constraints $\mathcal{X}_{ijk} \succ 0$, one may write:

$$\dot{X}_z \succeq \sum_{k=1}^{r} \phi_k \mathcal{X}_{ijk} \tag{22}$$

Hence (20) is satisfied if:

$$L \otimes X_z + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} X_z - \frac{1}{2}\sum_{k=1}^{r} \phi_k \mathcal{X}_{ijk} \right) \right) \prec 0 \tag{23}$$

Finally, let $H_z = X_z$, then apply lemma 2, one obtains the conditions expressed in theorem 4. ■

Remark 1 *Note that the conditions of theorem 1 are quite conservatism regarding to the general form of the non-PDC controller* (2)*. Indeed, theorem 1 requires $H_z = X_z$ symmetric positive definite matrix.*

In the next subsection, in the same mood as the result proposed in [34] for non-PDC controller design (without pole placement), one proposes relaxed non-quadratic $\mathcal{D}$-stability conditions where H_z is no longer required to be symmetric and decoupled from the fuzzy Lyapunov matrix. Moreover, to improve the conservatism, an enhanced FLF candidate, which involves a double summation fuzzy structure [32], will be considered.

B. Relaxed $\mathcal{D}$-stabilizing conditions via enhanced FLF:

Let us consider the enhanced FLF candidate given by [32]:

$$\tilde{V}(x(t)) = x^T(t) \tilde{X}_{zz}^{-1} x(t) \tag{24}$$

In this enhanced non-quadratic context, $\mathcal{D}$-stabilization conditions for T-S models are summarized by the following theorem.

Theorem 2 *: The T-S model* (1) *being smooth, $\forall k \in \mathcal{I}_r$, $\exists \phi_k = \inf_{x \in \Omega} (\dot{h}_k(x)) \leq 0$, $\phi_k \neq -\infty$. Let us assume that the ϕ_k are known and let L and M be two prescribed matrices defining a convenient LMI region (see definition 1). For $(i,j,k) \in \mathcal{I}_r^3$, if there exists the matrices $\tilde{X}_{ij} = \tilde{X}_{ij}^T$, $\tilde{R}_{ij} = \tilde{R}_{ij}^T$, H_j, F_j and a scalar $\varepsilon > 0$, such that the LMIs* (25) , (26) *and* (27) *are verified, then the T-S model* (3) *is $\mathcal{D}$-stabilized by the non-PDC control law* (2)*.*

$$\tilde{X}_{ij} + \tilde{X}_{ji} \succ 0, \quad i \leq j, \quad (i,j) \in \mathcal{I}_r^2, \tag{25}$$

$$\tilde{\mathcal{X}}_{ijk} + \tilde{\mathcal{X}}_{jik} \succ 0, \quad i \leq j, \quad (i,j,k) \in \mathcal{I}_r^3, \tag{26}$$

$$\Upsilon_{ij} + \Upsilon_{ji} \prec 0, \quad i \leq j, \quad (i,j) \in \mathcal{I}_r^2, \tag{27}$$

with:

$$\tilde{\mathcal{X}}_{ijk} = \tilde{X}_{kj} + \tilde{X}_{ik} + \tilde{R}_{ij},$$

$$\Upsilon_{ij} = \begin{bmatrix} \Upsilon_{ij}^{(1,1)} & (*) \\ \Upsilon_{ij}^{(1,2)} & -\varepsilon I \otimes \mathcal{H}(H_j) \end{bmatrix},$$

$$\Upsilon_{ij}^{(1,1)} = L \otimes \tilde{X}_{ij} + \mathcal{H}\left(M \otimes \left(A_i H_j + B_i F_j - \tilde{\Phi}_{ij} \right) \right),$$

$$\Upsilon_{ij}^{(1,2)} = M \otimes (\tilde{X}_{ij} - H_j) + \varepsilon I \otimes (H_j^T A_i^T + F_j^T B_i^T),$$

$$\tilde{\Phi}_{ij} = \frac{1}{2}\sum_{k=1}^{r} \phi_k \tilde{\mathcal{X}}_{ijk}.$$

Proof: By following the same path as the proof of theorem 1 until equation (20), but with the enhanced FLF (24), the closed-loop dynamics (3) is stable if (25) holds (to guarantee $V(x(t)) \succ 0$) and:

$$L \otimes \tilde{X}_{zz} + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} \tilde{X}_{zz} - \frac{1}{2}\dot{\tilde{X}}_{zz} \right) \right) \prec 0 \tag{28}$$

Inequality (28) can be rewritten as:

$$L \otimes \tilde{X}_{zz} - \mathcal{H}\left(M \otimes \frac{1}{2}\dot{\tilde{X}}_{zz} \right) + \mathcal{H}\left(M \otimes \tilde{A}_{zz} \tilde{X}_{zz} \right) \prec 0 \tag{29}$$

Let us now consider an arbitrary scalar ε, one may introduce the null terms:

$$\mathcal{H}(M \otimes \tilde{A}_{zz} H_z) - \mathcal{H}(M \otimes \tilde{A}_{zz} H_z) = 0 \tag{30}$$

and

$$\mathcal{H}(\varepsilon I \otimes \tilde{A}_{zz} H_z^T \tilde{A}_{zz}^T) - \mathcal{H}(\varepsilon I \otimes \tilde{A}_{zz} H_z \tilde{A}_{zz}^T) = 0 \qquad (31)$$

By summing (29) with (30) and (31), it yields:

$$\begin{array}{c} L \otimes \tilde{X}_{zz} - \mathcal{H}\left(M \otimes \frac{1}{2}\dot{\tilde{X}}_{zz}\right) + \mathcal{H}(M \otimes \tilde{A}_{zz} H_z) \\ +\mathcal{H}(M \otimes \tilde{A}_{zz}\tilde{X}_{zz} - M \otimes \tilde{A}_{zz} H_z + \varepsilon I \otimes \tilde{A}_{zz} H_z^T \tilde{A}_{zz}^T) \\ -\varepsilon I \otimes \mathcal{H}(\tilde{A}_{zz} H_z \tilde{A}_{zz}^T) \prec 0 \end{array} \qquad (32)$$

or equivalently:

$$\begin{array}{c} L \otimes \tilde{X}_{zz} + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} H_z - \frac{1}{2}\dot{\tilde{X}}_{zz}\right)\right) \\ +\mathcal{H}((I \otimes \tilde{A}_{zz})(M \otimes (\tilde{X}_{zz} - H_z) + \varepsilon I \otimes H_z^T \tilde{A}_{zz}^T)) \\ (I \otimes \tilde{A}_{zz})(-\varepsilon I \otimes \mathcal{H}(H_z))(I \otimes \tilde{A}_{zz}^T) \prec 0 \end{array} \qquad (33)$$

Since $\sum_{i=1}^{r} \dot{h}_i(z) = 0$, for any symmetric matrices $\tilde{R}_{ij} \in \mathbb{R}^{n \times n}$ one has:

$$\dot{\tilde{X}}_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z) \sum_{k=1}^{r} \dot{h}_k(z)(\tilde{X}_{kj} + \tilde{X}_{ik} + \tilde{R}_{ij}) \qquad (34)$$

Now, according to assumption 1, the T-S model (1) is smooth. Thus, $\forall k \in \mathcal{I}_r$, $\exists \phi_k = \inf_{x \in \Omega}(\dot{h}_k(x)) \leq 0$, $\phi_k \neq -\infty$ and (33) is satisfied if:

$$\begin{array}{c} L \otimes \tilde{X}_{zz} + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} H_z - \frac{1}{2}\tilde{\Phi}_{zz}\right)\right) \\ +\mathcal{H}((I \otimes \tilde{A}_{zz})(M \otimes (X_{zz} - H_z) + \varepsilon I \otimes H_z^T \tilde{A}_{zz}^T)) \\ (I \otimes \tilde{A}_{zz})(-\varepsilon I \otimes \mathcal{H}(H_z))(I \otimes \tilde{A}_{zz}^T) \prec 0 \end{array} \qquad (35)$$

with $\tilde{\Phi}_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z) \sum_{k=1}^{r} \phi_k(\tilde{X}_{kj} + \tilde{X}_{ik} + \tilde{R}_{ij})$ and:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)(\tilde{X}_{kj} + \tilde{X}_{ik} + \tilde{R}_{ij}) \succ 0 \qquad (36)$$

Applying lemma 2, (36) is satisfied if (26) holds. Now, let us rewrite (35) as:

$$\begin{bmatrix} I \\ I \otimes \tilde{A}_{zz} \end{bmatrix}^T \begin{bmatrix} \Upsilon_{zz}^{(1,1)} & (*) \\ \Upsilon_{zz}^{(1,2)} & -\varepsilon I \otimes \mathcal{H}(H_z) \end{bmatrix} \begin{bmatrix} I \\ I \otimes \tilde{A}_{zz} \end{bmatrix} \prec 0 \qquad (37)$$

where $\Upsilon_{zz}^{(1,1)} = L \otimes \tilde{X}_{zz} + \mathcal{H}\left(M \otimes \left(\tilde{A}_{zz} H_z - \frac{1}{2}\tilde{\Phi}_{zz}\right)\right)$ and $\Upsilon_{zz}^{(1,2)} = M \otimes (X_{zz} - H_z) + \varepsilon I \otimes H_z^T \tilde{A}_{zz}^T$.
Inequality (34) holds if:

$$\Upsilon_{zz} = \begin{bmatrix} \Upsilon_{zz}^{(1,1)} & (*) \\ \Upsilon_{zz}^{(1,2)} & -\varepsilon I \otimes \mathcal{H}(H_z) \end{bmatrix} \prec 0 \qquad (38)$$

Applying lemma 2, (38) is satisfied if (27) holds. ■

Remark 2 *The conditions of theorem 2 include the ones of theorem 1. Indeed, from theorem 2, the conditions of theorem 1 can be recovered by considering $\tilde{X}_{ij} = \tilde{X}_{ji} = X_j$, $H_j = X_j$ and $\tilde{R}_{ij} = R_{ij} - X_k$, then applying the Schur complement and considering ε arbitrarily small. Therefore the conditions expressed in theorem 2 are less conservative than the ones of theorem 1.*

Remark 3 *The enhanced FLF conditions without pole placement proposed theorem 7 in [32] constitute a special case of theorem 2. Indeed, they can be recovered from theorem 2 by considering $L = 0$ and $M = 1$, i.e. the LMI region defined as the left hand-side of the complex plane.*

Remark 4 *As usual in the non-quadratic framework, when FLFs are employed, the stability conditions depend on the bounds of the time derivatives of the membership functions $\phi_k = \inf_{x \in \Omega}(\dot{h}_k(z)) \leq 0$. Even if under assumption 1 these bounds always exist, they are difficult to estimate in practice, especially in stabilization, before having synthesized the closed-loop dynamics. Since these bounds are required to solve the LMI problems expressed in theorem 1 and 2, it is assumed that they are known, i.e. $\dot{h}_k(z) \geq \phi_k$ (similarly to several non-quadratic studies, e.g. [16], [18], [38]). Moreover, in this case, it is not correct to tell about global asymptotical stabilization since it is somewhat hypothetic to say that these bounds arises for every initial conditions $x(0)$. To deals with this concerns, one may consider the estimation of a domain of attraction [19], [32]. Nevertheless, for space reason and since it is not the main focus of the proposed results, such estimation is omitted in the present study and will be the subject of further prospects.*

Remark 5 *Similarly to other results based on the Finsler's lemma (see e.g. [39], [34] or [25]), the conditions summarized in theorem 2 involve a prefixed scalar parameter $\varepsilon > 0$. In practice, such parameters are usually prefixed or optimized by linear programming inside a logarithmically spaced family of values such as $\varepsilon \in \{10^{-6}, 10^{-5}, \ldots, 10^{6}\}$. As quote in [34], this logarithmically spaced family avoids an exhaustive linear search. Moreover, in [39], the authors showed that for thousands of LPV models and comparing with numerous results (classical Q approach, Finsler's application, and several other variations), this way of doing was outperforming the existing results in a large way.*

IV. Simulation results

In this section, the fourth order nonlinear benchmark of a single link robot with flexible joint, depicted in Figure 1, is considered [40], [41].

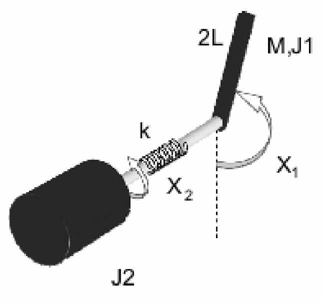

Fig. 1. Single link robot with a flexible joint [41].

From its nonlinear model [41] and applying the well-known sector nonlinearity approach [3], the dynamics of this robot can be exactly described by the following fourth order T-S model [42]:

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(x_1(t))A_i x(t) + Bu(t) \tag{39}$$

where $x = [x_1\, x_2\, x_3\, x_4]^T$ is the state vector, x_1 and x_2 are respectively the angular position of the arm and the actuator, $x_3 = \dot{x}_1$ and $x_4 = \dot{x}_2$, u is the motor control input, $h_1(x_1) = 1 - h_2(x_1)$ with $h_2(x_1) = (1-f(x_1))/(1+\rho)$ and $f(x_1(t)) = \sin(x_1(t))/x_1(t) \in [\rho\,; 1]$ with $\rho = \min(\sin x_1(t)/x_1(t)) \approx -0,2172$. The vertices matrices are given by:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k-mgL}{J_1} & \frac{k}{J_1} & 0 & 0 \\ \frac{k}{J_2} & \frac{k}{J_2} & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k-mgL\rho}{J_1} & \frac{k}{J_1} & 0 & 0 \\ \frac{k}{J_2} & \frac{k}{J_2} & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix}$$

where $J_1 = J_2 = 1\,kg.m^2$ are respectively the actuator and arm inertias, $m = 1\,kg$ is the mass of the arm, $L = 1\,m$ is the length of the arm, $k = 100\,N.m.rad^{-1}$ is the spring stiffness, $g = 9.81\,m.s^{-2}$ is the acceleration due to gravity.

A non-PDC controller (2) have been design through the LMI conditions of theorem 2, using the MATLAB LMI toolbox [11], with the LMI region is defined as follows [33]:

1) the left half plan defined by $\mathcal{R}e(\lambda) < \beta$,
2) a conic sector defined by its apex at $(\gamma, 0)$ and an inner angle $\pi/2 - \theta$,
3) a circle centered at $(q, 0)$ with a radius s,

leading to the following LMI region matrices:

$$L = \begin{bmatrix} -2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\gamma\cos\theta & 0 & 0 & 0 \\ 0 & 0 & -2\gamma\cos\theta & 0 & 0 \\ 0 & 0 & 0 & -s & -q \\ 0 & 0 & 0 & -q & -s \end{bmatrix},$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For $s = 8$, $q = 7$, $\beta = 3$, $\theta = \pi/5$, $\gamma = 3$ and $\phi_1 = \phi_2 = -2$, the solution with $\varepsilon = 0.008$ is given by the following non-PDC gain matrices (the matrices $\tilde{X}_{ij}$ and $\tilde{R}_{ij}$, for $(i,j) \in \mathcal{I}_r^2$, are omitted for space reasons):

$$H_1 = \begin{bmatrix} 0.0147 & 0.0155 & -0.0545 & -0.0355 \\ 0.0156 & 0.0183 & -0.0751 & -0.0689 \\ -0.0561 & -0.0761 & 0.4068 & 0.4139 \\ -0.0360 & -0.0692 & 0.4174 & 0.6392 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.0197 & 0.0203 & -0.0661 & -0.0490 \\ 0.0202 & 0.0227 & -0.0831 & -0.0802 \\ -0.0669 & -0.0837 & 0.4103 & 0.3933 \\ -0.0489 & -0.0805 & 0.3959 & 0.6333 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -0.1507 & 0.2897 & -2.7528 & -8.0718 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -0.1353 & 0.3169 & -2.0624 & -8.0128 \end{bmatrix}.$$

These gains provide that the closed-loop system is $\mathcal{D}$-stable. It is confirmed by Figure 2, which shows the closed-loop trajectories, the time derivatives of the membership functions and the poles location, for the initial conditions $x(0) = [-\pi/5 \;\; \pi/5 \;\; 0 \;\; 0]^T$. Note that the assumption $\dot{h}_k(z) \geq \phi_k = -2$ is verified in this simulation.

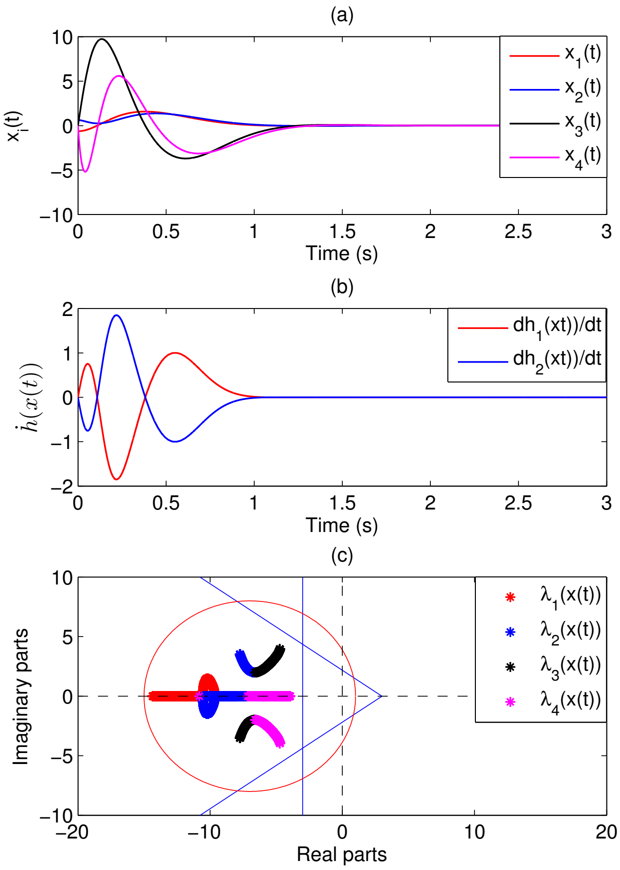

Fig. 2. Closed-loop simulation: (a) closed-loop trajectories, (b) time derivatives of the membership functions, (c) poles location in LMI region.

V. Conclusion

In this paper, non-PDC controller design conditions with pole placement assignation for T-S fuzzy systems using non quadratic Lyapunov function are proposed through the $\mathcal{D}$-stability concept. Relaxed results have been provided via enhanced Fuzzy Lyapunov Functions involving a double sum fuzzy structure. An example with the fourth order benchmark of a flexible robot with single joint has shown the effectiveness of the proposed results. Further works will be done to deal with the estimation of the domain of attraction in this non-quadratic $\mathcal{D}$-stabilization context.

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