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► **To cite this version:**

Abdelmadjid Cherifi, Kevin Guelton, Laurent Arcese. Non-PDC controller design for Takagi-Sugeno models via line-integral Lyapunov functions. 2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Jul 2014, Beijing, France. 10.1109/FUZZ-IEEE.2014.6891675 . hal-01736773

**HAL Id: hal-01736773**

**<https://hal.univ-reims.fr/hal-01736773v1>**

Submitted on 22 Aug 2024

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# Non-PDC Controller Design for Takagi-Sugeno Models via Line-Integral Lyapunov Functions

Abdelmadjid Cherifi, Kevin Guelton and Laurent Arcese

**Abstract**—This paper presents a new non Parallel Distributed Compensation (non-PDC) controllers design based on line-Integral Lyapunov functions for continuous-time Takagi-Sugeno (T-S) fuzzy models. The previous works are mainly based on a BMI formulation (LMI for first and second order systems only). In this paper, we show that using a property on dual system, it can be possible to formulate the design of a controller as an LMI problem for n-th order T-S systems. Two simulations are provided to show the effectiveness of the proposed approach : a numerical second order academic example and a fourth order benchmark of a single link robot with flexible joint.

## I. INTRODUCTION

Among nonlinear control theory, (T-S) fuzzy systems [1] have shown their interests since they allow extending some of the linear control concepts in the nonlinear framework [2]. Indeed, a nonlinear system can be exactly rewritten as a T-S fuzzy one on a compact set of the state space. A T-S system is a convex polytopic model, i.e. a collection of linear dynamics blended together by convex nonlinear membership functions[2].

Using the direct Lyapunov approach, various conditions have been studied for the design of fuzzy controllers and stability analysis [3], [4], [5], [6], [7]. In most of the cases such control problems are expected to be written as Linear Matrix Inequalities (LMI). Indeed LMI constraints are interesting for controller and observer design since they can be efficiently solved by convex optimisation [8]. Stability analysis and controller design have been first investigated via common quadratic Lyapunov functions. However, these methodes require to find a common Lyapunov matrix solution of a set of LMIs [2], [3]. This obviously leads to conservatism and many studies have focused on its reduction [9].

At first, LMI conditions being obtained through a double sum fuzzy structure, relaxation schemes have been introduced as sum relaxations [10], [11]. Other ways to reduce the conservatism consist on considering alternative Lyapunov functions candidates. For instance, piecewise quadratic Lyapunov functions have been employed [12]. However these are not accurate with T-S models, especially when they are obtained from the sector nonlinearity approach [2].

Other attempts considered switched approaches but introducing non obvious modeling assumption regarding to the choice of the switched regions [13], [14]. With more adequacy to the fuzzy structure of T-S models, non quadratic approaches considering fuzzy Lyapunov functions have been investigated [4], [6], [15]. In the continuous-time framework, the time derivative of the membership functions appears in the stability conditions. Therefore to lead to LMIs, the boundary of these derivative terms have been often considered [4], [6], [16]. However these bounds are difficult to obtain in practical applications, especially in stabilization, priori to solve LMI conditions. To overcome this drawback, local non quadratic stabilization have been proposed [17], but leading to complex LMI formulation.

To leave the LMI framework, some alternatives have been proposed using sum-of-squares (SOS) optimization algorithms [18], [19], [20]. Nevertheless these SOS-based conditions require strong modeling assumption which reduce their practical interest in stabilization.

Finally a less investigated way in the non quadratic framework was introduced by Rhee and Won [5], who proposed a line-integral Lyapunov function candidate. Owing to convenient path-independency conditions [21], [22], this approach presents the interest of avoiding the appearance of the time-derivative of the membership functions in the stability conditions. However, these first results were given in terms of Bilinear Matrix Inequalities (BMIs) in stabilization. Recent studies have been therefore conducted to provide LMI conditions [23], [24], [25], [26]. In [23], LMI conditions are proposed but requiring the introduction of a decision matrix with a particular form. This reduces the practical applicability of such approaches when the membership functions depend on several state variables or when the order of the system increases. In [24], a solution to this problem was proposed but requiring a two steps LMI algorithm.

More recently, non-PDC controller design has been proposed in [25] and improved in [26]. However, these results lead to LMIs for first and second orders T-S systems, BMIs for third order systems, then more and more complex as the order increases [26].

Summarizing, all the problems concerned above are understood as major drawbacks for non-PDC controller design using non quadratic line-integral Lyapunov functions. These have motivated the present study. In [27], thanks to a dual system property, PDC controller design has been proposed. Following this way, the conditions

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proposed in the sequel improve the result by providing LMI conditions for non-PDC controller design using line-integral Lyapunov functions without requiring any systems order assumptions.

This paper is organized as follows : Section II introduces some useful definitions, notations and lemmas. Section III is devoted to the presentation of the new LMI conditions for non quadratic stabilization of T-S fuzzy systems. To illustrate the benefit of the proposed approach in terms of conservatism, a second order numerical example is provided and compared with previous works. Then a fourth order benchmark of a single link robot with flexible joint is considered to highlight the efficiency of the proposed approach for dynamical nonlinear systems of order than two.

## II. DEFINITIONS, NOTATIONS AND USEFUL LEMMAS

Consider the T-S fuzzy model given by :

$$\dot{x}(t) = \sum_{i=1}^r h_i(\xi(t))(A_i x(t) + B_i u(t)) \quad (1)$$

where  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^m$  is the input vector,  $\xi(t)$  is the vector of premises, for  $i \in \{1, \dots, r\}$ ,  $h_i(\xi(t)) \in [0, 1]$  are convex membership functions with  $\sum_{i=1}^r h_i(\xi(t)) = 1$ ,  $A_i \in \mathbb{R}^{n \times n}$ , and  $B_i \in \mathbb{R}^{n \times m}$  are real constant matrices defining the  $i^{\text{th}}$  vertex.

*Assumption 1* : The vector of premises  $\xi(t)$  only depends on the state variables, i.e. on the component of  $x(t)$ .

*Notations* : In the sequel, if not explicitly stated, matrices are assumed to have appropriate dimensions. Moreover, when there is any ambiguity, the time  $t$  argument will be omitted to lighten mathematical expressions.  $M \succ 0$  and  $I$  denote respectively a positive definite matrix and an identity matrix with appropriate dimensions. An asterisk (\*) denotes a transpose quantity in a matrix or the transpose of its left-hand side term for inline expressions. Consider a set of real matrices  $M_i$  and  $N_{ij}$ , for all  $(i, j) \in \{1, \dots, r\}^2$ , one denotes  $M_h = \sum_{i=1}^r h_i(\xi) M_i$ ,  $N_{hh} = \sum_{i=1}^r \sum_{j=1}^r h_i(\xi) h_j(\xi) N_{ij}$ .

Let us consider the non-PDC control law given by :

$$u(t) = \sum_{i=1}^r h_i(\xi) F_i \left( \sum_{j=1}^r h_j(\xi) H_j \right)^{-1} x(t) \quad (2)$$

where  $F_i \in \mathbb{R}^{m \times n}$  and  $H_j \in \mathbb{R}^{n \times n}$  are constant gain matrices to be synthesized.

From (1) and (2), the close-loop dynamics may be expressed, with the above defined notations as :

$$\dot{x}(t) = G(x)x(t) \quad (3)$$

with

$$G(x) = A_h + B_h F_h H_h^{-1}.$$

Therefore, the goal of this paper is to propose new LMI based conditions allowing to design  $F_i \in \mathbb{R}^{m \times n}$  and  $H_i \in \mathbb{R}^{n \times n}$  such that the closed-loop dynamics (3) is stable. Let us consider the following line-integral Lyapunov function candidate [5] :

$$v(x(t)) = 2 \int_{\Gamma(0, x(t))} g^T(\varphi) d\varphi \quad (4)$$

where  $\Gamma(0, x(t))$  is the path from the origin 0 to the current state  $x(t)$ ,  $\varphi \in \mathbb{R}^n$  is a dummy vector for the integral and assuming :

$$g(x) = P_h x(t) \quad (5)$$

with  $P_h$  satisfying the path independency conditions, such that [5], [22] :

$$P_h = P_0 + \sum_{i=1}^r h_i(x) D_i \quad (6)$$

$$\text{with } \begin{cases} P_0 = \begin{bmatrix} 0 & p_{1,2} & \cdots & p_{1,n} \\ p_{1,2} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & p_{n-1,n} \\ p_{1,n} & \cdots & p_{n-1,n} & 0 \end{bmatrix} \\ D_i = \begin{bmatrix} d_{1,1}^{\alpha_{i,1}} & 0 & \cdots & 0 \\ 0 & d_{2,2}^{\alpha_{i,2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & d_{n,n}^{\alpha_{i,n}} \end{bmatrix} \end{cases},$$

$(p_{1,2}, \dots, p_{n-1,n})$  and  $(d_{1,1}^{\alpha_{i,1}}, \dots, d_{n,n}^{\alpha_{i,n}})$  defined as in [5].

Substituting (6) in (5), (4) is a Lyapunov function candidate and the closed-loop dynamics (3) is stable to the origine if conditions (7), (8) and (9) hold :

$$P_0 + D_i \succ 0, \quad (7)$$

$$\text{for } x(0) = 0, \quad v(x(t)) = 0, \quad (8)$$

$$\forall x(0) \neq 0, \quad \dot{v}(x(t)) < 0, \quad (9)$$

Note that (7) is satisfied if, for all  $i = \{1, \dots, r\}$ ,  $P_0 + D_i \succ 0$ . Moreover, (8) is obviously verified when considering (4). Then, LMI conditions satisfying (9) will be the subject of the main result proposed in the next section.

**Lemma 1** [28] : Let  $x \in \mathbb{R}^n$ ,  $Q = Q^T \in \mathbb{R}^{n \times n}$  and  $R \in \mathbb{R}^{m \times n}$  such that  $\text{rank}(R) < n$ . The following statements are equivalent :

$$x^T Q x < 0, \quad \forall x \in \{x \in \mathbb{R}^n : x \neq 0, Rx = 0\}, \quad (10)$$

$$\exists X \in \mathbb{R}^{n \times m} : Q + XR + R^T X^T < 0, \quad (11)$$

**Lemma 2** [10] : Let  $\Gamma_{ij}$ , for  $(i, j) \in \{1, \dots, r\}^2$ , be matrices of appropriate dimensions.  $\Gamma_{hh} < 0$  is satisfied if both following conditions hold :

$$\begin{cases} \Gamma_{ii} < 0, \quad \forall i \in \{1, \dots, r\} \\ \frac{2}{r-1} \Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \quad \forall (i, j) \in \{1, \dots, r\}^2 / i \neq j \end{cases} \quad (12)$$

**Property 1** : Let us consider the dual system of (3) given by :

$$\dot{z}(t) = G^T(z)z(t), \quad (13)$$

if (13) is stable then (3) is stable.

*Proof:* Straightforward since, if the eigenvalues of  $G^T(z)$  (whatever  $z$ ) are in the left hand-side of the complex plan, the eigenvalues of  $G(x)$  (whatever  $x$ ) also belong to the left hand-side, see [29] for other dual-based results in the LPV framework. ■

From property 1, if one can find the gain matrices  $F_i$  and  $H_i$  for  $i \in \{1, \dots, r\}$  such that (13) is stable, then (2) is a non-PDC controller stabilizing the T-S model (1). The main results are given in the following section.

### III. MAIN RESULTS

**Theorem 1** : The T-S model (1) is stabilized by the non-PDC control law (2), i.e. the closed-loop dynamics (3) is globally asymptotically stable, if there exist a scalar  $\varepsilon$ , the matrices  $P_0$  and  $D_i$  defined in (6), the gain matrices  $F_i$  and  $H_i$ , such that, for all  $i \in \{1, \dots, r\}$ ,  $P_0 + D_i \succ 0$  and conditions (12) are verified with :

$$\Gamma_{ij} = \begin{bmatrix} \Theta_{ij} & (*) \\ \Phi_{ij} & -\varepsilon(H_j + H_j^T) \end{bmatrix} \quad (14)$$

where

$$\begin{aligned} \Theta_{ij} &= A_i H_j + H_j^T A_i^T + B_i F_j + F_j^T B_i^T \\ \Phi_{ij} &= P_0 + D_i - H_j + \varepsilon(A_i H_j + B_i F_j)^T \end{aligned}$$

*Proof:* Considering the path-independency conditions, one can rewrite (9) from (4) as :

$$\begin{aligned} \dot{v}(z) &= \dot{z}^T P_h z + z^T P_h \dot{z} \\ &= \begin{bmatrix} z \\ \dot{z} \end{bmatrix}^T \begin{bmatrix} 0 & P_h \\ P_h & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} < 0 \end{aligned} \quad (15)$$

Moreover, from (13) one can write :

$$G(z)^T z(t) - \dot{z}(t) = 0 \quad (16)$$

which is equivalent to :

$$\begin{bmatrix} G(x)^T & -I \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} = 0 \quad (17)$$

Now, consider  $U_h$  and  $V_h$ , two slack decision matrices with appropriate dimensions, one may apply lemma 1, therefore (15) holds if the next inequality is satisfied :

$$\begin{bmatrix} 0 & P_h \\ P_h & 0 \end{bmatrix}^+ \begin{bmatrix} U_h \\ V_h \end{bmatrix} \begin{bmatrix} G(x)^T & -I \end{bmatrix} + (*) < 0 \quad (18)$$

that is to say

$$\begin{bmatrix} 0 & P_h \\ P_h & 0 \end{bmatrix}^+ \begin{bmatrix} U_h H_h^{-T} F_h^T B_h^T + U_h A_h^T & -U \\ V_h H_h^{-T} F_h^T B_h^T + V_h A_h^T & -V \end{bmatrix} + (*) < 0 \quad (19)$$

Now, let  $U_h = H_h^T$  and  $V_h = \varepsilon H_h^T$ , where  $\varepsilon$  is a positive prefixed scalar, (19) becomes :

$$\begin{bmatrix} \Theta_{hh} & (*) \\ \Phi_{hh} & -\varepsilon(H_h + H_h^T) \end{bmatrix} < 0 \quad (20)$$

with

$$\begin{aligned} \Theta_{hh} &= A_h H_h + B_h F_h + F_h^T B_h^T + H_h^T A_h^T \\ \Phi_{hh} &= P_h - H_h + \varepsilon(H_h^T A_h^T + F_h^T B_h^T) \end{aligned}$$

Now, applying lemma 2, one obtains the conditions expressed in theorem 1. ■

**Remark 1** : The conditions summarized in theorem 1 involve a parameter  $\varepsilon$ , which has to be fixed in advance. Note that, following the proof of theorem 1, this parameter is no longer required. Nevertheless, we let this parameter in the LMI conditions since it may be useful to tune the closed-loop dynamics performances. They are usually prefixed values belonging to a logarithmically spaced family of values such as  $\varepsilon \in \{10^{-6}, 10^{-5}, \dots, 10^6\}$ .

The following theorem, which improve LMI conditions given in [27], is proposed as an alternative to Finsler's relaxation without using prefixed unknown parameters.

**Theorem 2** : The T-S model (1) is stabilized by the non-PDC control law (2), i.e. the closed-loop dynamics (3) is globally asymptotically stable, if there exist the matrices  $P_0$  and  $D_i$  defined in (6), the gain matrices  $F_i$  and  $H_i$ , such that, for all  $i \in \{1, \dots, r\}$ ,  $P_0 + D_i \succ 0$  and conditions (12) are verified with :

$$\Gamma_{ij} = \begin{bmatrix} -2(P_0 + D_i) & (*) \\ \sigma_{ij} & -H_j - H_j^T \end{bmatrix} \quad (21)$$

with

$$\sigma_{ij} = P_0 + D_i + H_j^T + H_j^T A_i^T + F_j^T B_i^T.$$

*Proof:* Let  $S_1$  and  $S_2$  be any slack decision matrices with appropriate dimensions, considering the Lyapunov

candidate function (4) and the path-independency conditions (6), one can rewrite (9) as :

$$\begin{aligned}\dot{v}(z) &= \dot{z}^T P_h z + z^T P_h \dot{z} \\ &= \begin{bmatrix} z \\ \dot{z} \end{bmatrix}^T \begin{bmatrix} P_h & S_1^T \\ 0 & S_2^T \end{bmatrix} \begin{bmatrix} \dot{z} \\ 0 \end{bmatrix} + (*) < 0\end{aligned}\quad (22)$$

According to property 1, one can write :

$$-\dot{z}(t) + G(x)^T z(t) = 0. \quad (23)$$

Therefore, (22) is equivalent to :

$$\begin{bmatrix} z \\ \dot{z} \end{bmatrix}^T \begin{bmatrix} P_h & S_1^T \\ 0 & S_2^T \end{bmatrix} \begin{bmatrix} -\dot{z} + G(x)^T z \\ \dot{z} \end{bmatrix} + (*) < 0 \quad (24)$$

that can be rewritten as :

$$\begin{bmatrix} z \\ \dot{z} \end{bmatrix}^T \left[ \begin{bmatrix} P_h & S_1^T \\ 0 & S_2^T \end{bmatrix} \begin{bmatrix} 0 & I \\ G(x)^T & -I \end{bmatrix} + (*) \right] \begin{bmatrix} z \\ \dot{z} \end{bmatrix} < 0 \quad (25)$$

which is satisfied, if  $\forall(z, \dot{z})$  :

$$\begin{bmatrix} P_h & S_1^T \\ 0 & S_2^T \end{bmatrix} \begin{bmatrix} 0 & I \\ G(x)^T & -I \end{bmatrix} + (*) < 0 \quad (26)$$

that is to say :

$$\begin{bmatrix} \Xi_{hh} & (*) \\ \Psi_{hh} & -S_2 - S_2^T \end{bmatrix} < 0 \quad (27)$$

with

$$\begin{aligned}\Xi_{hh} &= A_h S_1 + S_1^T A_h^T + B_h F_h H_h^{-1} S_1 + S_1^T H_h^{-T} F_h^T B_h^T \\ \Psi_{hh} &= P_h - H_h + H_h^T A_h^T + S_1^T H_h^{-T} F_h^T B_h^T\end{aligned}$$

Now, let  $S_1 = S_2 = H_h$ , (28) becomes :

$$\begin{bmatrix} A_h H_h + H_h^T A_h^T + B_h F_h + F_h^T B_h^T & (*) \\ P_h - H_h + H_h^T A_h^T + F_h^T B_h^T & -H_h - H_h^T \end{bmatrix} < 0 \quad (28)$$

By congruence transformation of (28) with  $\begin{bmatrix} I & -I \\ 0 & I \end{bmatrix}$ , one obtains :

$$\Gamma_{ij} = \begin{bmatrix} -2P_h & (*) \\ P_h + H_h^T + H_h^T A_h^T + F_h^T B_h^T & -H_h - H_h^T \end{bmatrix} \quad (29)$$

Now, applying lemma 2, one obtains the conditions expressed in theorem 2. ■

#### IV. NUMERICAL EXEMPLES

In this section, two numerical examples are proposed. The first one, based on an academic second order T-S model, is dedicated to compare the conservatism of the results proposed above regarding to the previous recent work [27]. Let us recall that, for T-S systems of order upper than two, there were no previous LMI results for non quadratic stabilization using a line-integral Lyapunov function. Hence, the second example illustrates the effectiveness of the above proposed LMI conditions on a fourth order nonlinear benchamrk of a single link robot

with flexible joint [30].

##### A. Example 1

Let us consider the T-S fuzzy model with two rules [5] :

$$\dot{x}(t) = \sum_{i=1}^2 h_i(\xi(t))(A_i x(t) + B_i u(t)) \quad (30)$$

where  $x(t) = [x_1(t) \quad x_2(t)]$  and  $\xi \equiv x_1$ . The normalized MFs are given as

$$\begin{cases} h_1(x_1) = (1 - \sin(x_1))/2 \\ h_2(x_1) = 1 - h_1(x_1) \end{cases} \quad (31)$$

The numerical values of the matrices  $A_i$  and  $B_i$  are as follows :

$$\begin{aligned}A_1 &= \begin{bmatrix} 2 & -10 \\ 2 & 0 \end{bmatrix} & B_1 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ A_2 &= \begin{bmatrix} a & -5 \\ 1 & 2 \end{bmatrix} & B_2 &= \begin{bmatrix} b \\ 2 \end{bmatrix}\end{aligned}$$

A non-PDC controller of the form 2 can be designed via a line-integral candidate Lyapunov functions with  $P_1$  and  $P_2$  having the following entries :

$$P_1 = \begin{bmatrix} d_{11}^1 & p_{12} \\ p_{12} & d_{22} \end{bmatrix}, \quad P_2 = \begin{bmatrix} d_{11}^2 & p_{12} \\ p_{12} & d_{22} \end{bmatrix} \quad (32)$$

as defined in (6) [5].

Figure 1 shows the feasibility fields computed, using the MATLAB LMI Toolbox [31], from theorem 1 (with  $\varepsilon = 0.1$ ), theorem 2 proposed above and corollary 3 in [27]. As one can see, the results proposed in theorem 1 and theorem 2 are always outperforming the ones proposed in [27].

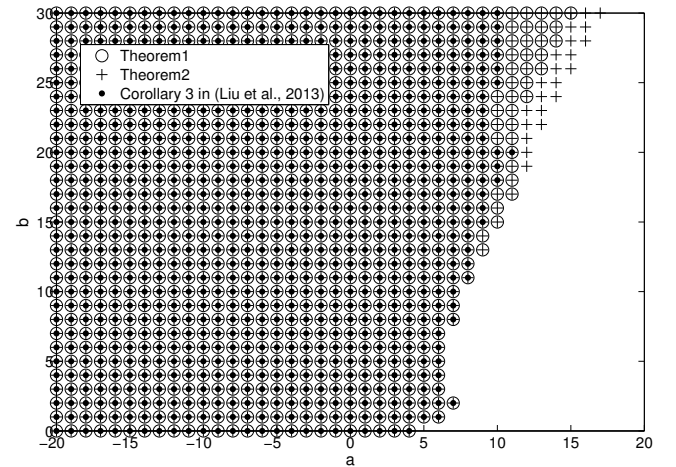


Fig. 1. Comparison of the feasibility fields obtained through theorem 1, theorem 2 and corollary 3 in [27].

Let us illustrate the particular case  $a = 12$ ,  $b = 27$  and  $\varepsilon = 0.1$  where no solution exist with corollary 3 in [27]. The solution obtained from theorem 1 is given by

the following matrices :

$$\begin{aligned}
P_1 &= \begin{bmatrix} 95.32 & 7.66 \\ 7.66 & 2.16 \end{bmatrix} \\
P_2 &= \begin{bmatrix} 674.99 & 7.66 \\ 7.66 & 2.16 \end{bmatrix} \\
H_1 &= \begin{bmatrix} 21.38 & -4.96 \\ 6.26 & 1.47 \end{bmatrix} \\
H_2 &= \begin{bmatrix} 37.22 & -12.34 \\ 9.83 & 0.51 \end{bmatrix} \\
F_1 &= \begin{bmatrix} -52.5 & 5.69 \end{bmatrix} \\
F_2 &= \begin{bmatrix} -39.12 & 4.80 \end{bmatrix}
\end{aligned}$$

Figure 2 shows the open-loop phase portrait of the T-S fuzzy system (30). As one can see, this open-loop T-S system is unstable. Figure 3 shows the closed-loop phase portrait of the T-S system (30) stabilized by the designed non-PDC controller. Let us notice that the closed-loop system is now globally asymptotically stable.

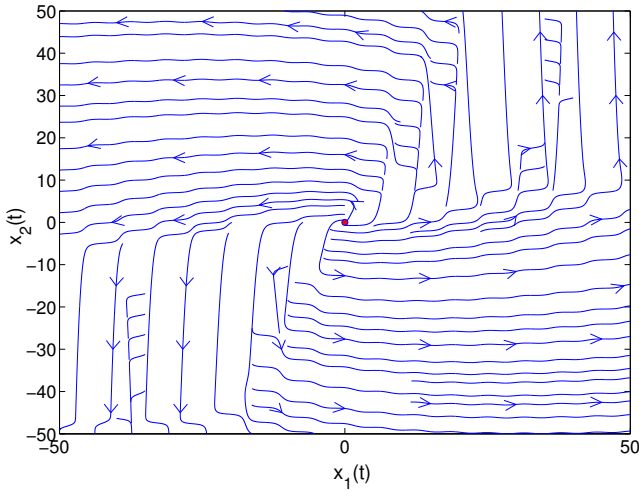


Fig. 2. Open-loop phase portrait.

### B. Example 2

Let us consider the well-known benchmark of a single link robot with flexible joint depicted in Figure 4 and described by the fourth order nonlinear state space model [32] :

$$\begin{cases} \dot{x}_1(t) = x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = \frac{1}{J_1} [k(x_2(t) - x_1(t)) - mgLf(x_1(t))x_1(t)] \\ \dot{x}_4(t) = \frac{1}{J_2} [u - k(x_2(t) - x_1(t))] \end{cases} \quad (33)$$

where  $J_1$ ,  $J_2$  are respectively the actuator and arm inertials,  $m$  the arm mass,  $L$  the arm length,  $k$  the spring

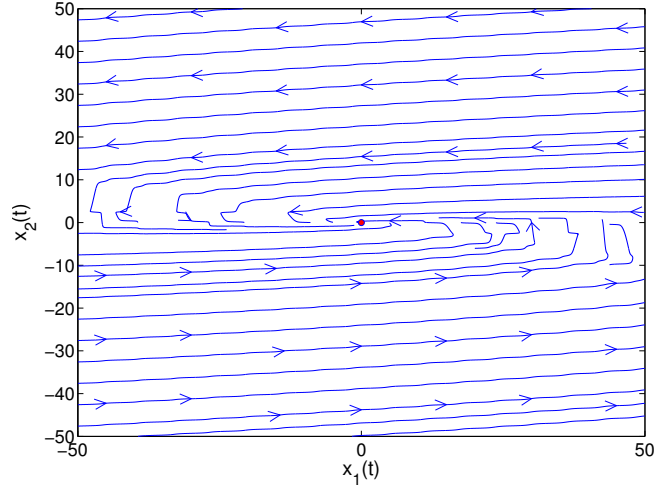


Fig. 3. Closed-loop phase portrait.

stiffness and  $f(x_1(t)) = \sin(x_1(t))/x_1(t) \in [\rho; 1]$ .  $x_1$  and  $x_2$  are respectively the angular position of the arm and the actuator.

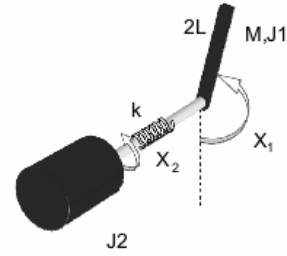


Fig. 4. Single link robot with a flexible joint [30].

Using the well-known sector nonlinearity approach, an exact T-S model of (33) may be expressed as [2] :

$$\dot{x}(t) = \sum_{i=1}^2 h_i(x_1(t)) A_i x(t) + B u(t) \quad (34)$$

with  $x(t) = (x_1(t) \ x_2(t) \ x_3(t) \ x_4(t))^T$ ,  $h_1(x_1(t)) = (f(x_1(t)) + \rho)/(1 + \rho)$ ,  $h_2(x_1(t)) = 1 - h_1(x_1(t))$ ,  $\rho = \min(\sin x_1(t)/x_1(t))$  and :

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k-mgL}{J_1} & \frac{k}{J_1} & 0 & 0 \\ \frac{k}{J_2} & \frac{k}{J_2} & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k-mgL\rho}{J_1} & \frac{k}{J_1} & 0 & 0 \\ \frac{k}{J_2} & \frac{k}{J_2} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2} \end{bmatrix}$$

The controller design have been done through theorem 1 using the MATLAB LMI toolbox [31]. The model parameters are assumed as  $k = 100N.m.rd^{-1}$ ,  $m = 1kg$ ,

$g = 9.81m.s^{-2}$  and  $L = 1m$ . The solution of theorem 2 with  $\varepsilon = 0.1$  are given by :

$$H_1 = \begin{bmatrix} 98,95 & 100,38 & -147,27 & -173,52 \\ 100,59 & 109,53 & -164,11 & -224,09 \\ -41,06 & -61,48 & 864,86 & 986,34 \\ -14,94 & -60,36 & 463,59 & 920,47 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 109,17 & 113,80 & -180,04 & -177,41 \\ 98,21 & 108,35 & -177,95 & -218,88 \\ -47,62 & -109,02 & 852,68 & 1103,53 \\ 52,71 & -25,49 & 305,99 & 935,15f \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 100,82 & 100,92 & -89,22 & -67,77 \\ 100,92 & 106,35 & -134,92 & -131,52 \\ -89,22 & -134,92 & 834,59 & 679,60 \\ -67,77 & -131,52 & 679,60 & 996,73 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 101,38 & 100,92 & -89,22 & -67,77 \\ 100,92 & 106,35 & -134,92 & -131,52 \\ -89,22 & -134,92 & 834,59 & 679,60 \\ -67,77 & -131,52 & 679,60 & 996,73 \end{bmatrix}$$

$$F_1 = [ -849,5 \quad -325,1 \quad 1754,2 \quad -5159,7 ]$$

$$F_2 = [ -2184,7 \quad -1460,2 \quad 4762,4 \quad -4238,4 ]$$

Figure 5 shows the closed-loop trajectories of the stabilized nonlinear system (33) and its dual. As one can notice, the non-PDC controller being designed through the dual system, the stabilization of the latter leads to the stabilization the original nonlinear system. To confirm this dual property, the eigenvalues (over the system trajectories) of the original nonlinear system and its dual are plotted on Figure 6. Of course, eigenvalues of both systems belong to the left hand-side of the complex plan.

## V. CONCLUSION

In this paper, non quadratic stability of T-S system has been considered. To overcome the drawback of classical non quadratic approach, i.e. the occurrence of the time derivatives of the membership functions in the stability conditions, a line-integral Lyapunov function has been considered [5]. The limits of previous non quadratic works using such Lyapunov functions remain the BMI formulation of controller design condition (LMI for first and second order systems only). To unlock this important problem, the results proposed in this paper have been obtained through a dual system property without any order assumption. Hence, from now, the way is open for more complex control problems using line-integral Lyapunov functions such as performance specification, robustness, and so on.

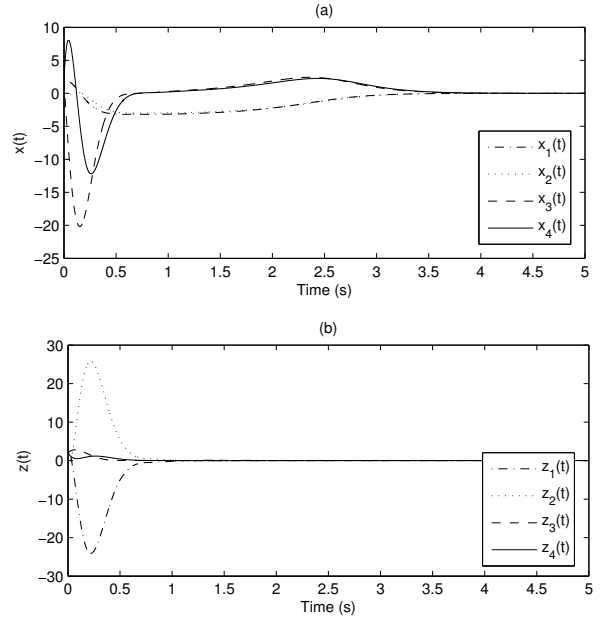


Fig. 5. Closed-loop trajectories : (a) Original nonlinear system (33) and (b) dual system.

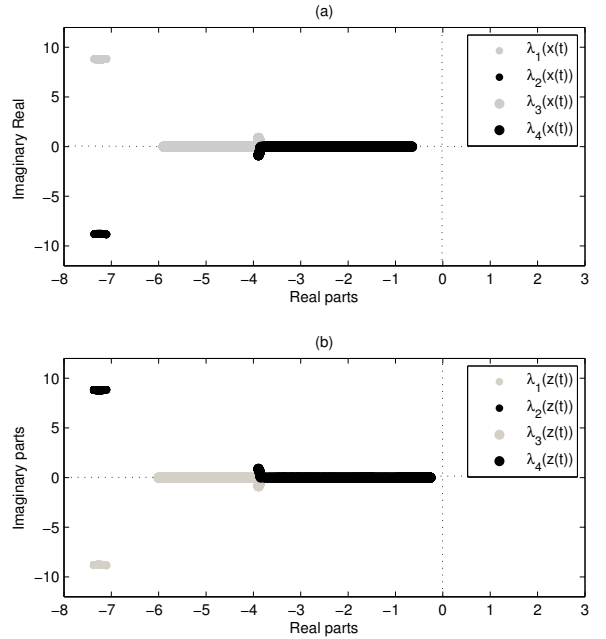


Fig. 6. Eigenvalues over the systems trajectories : (a) Original nonlinear system (33) and (b) dual system.

## VI. ACKNOWLEDGEMENT

The authors would like to thank Miss Aline Tegraal for her valuable comments regarding to this study.

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