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# SOS Stability Conditions for Nonlinear Systems Based on a Polynomial Fuzzy Lyapunov Function

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**Abstract:** In this paper, a new step on fuzzy relaxation for nonlinear systems' stability analysis is addressed. Inspired from non-quadratic Lyapunov functions (NQLF), regarding to quadratic ones (QLF), a new polynomial fuzzy Lyapunov function (PFLF) is proposed as an extension to the polynomial Lyapunov function (PLF) [22]. Following the latter post-LMI challenge, the obtained stability conditions are written in terms of a sum-of-squares (SOS) optimization problem. The proposed PFLF includes the well-studied NQLF ones as a special case. Moreover, the proposed SOS based stability conditions don't require unknown parameters as well as guarantee, when a solution exists, the global stability.

**Keywords:** Polynomial fuzzy systems, Polynomial fuzzy Lyapunov function, Stability, Sum of squares (SOS), Global non-quadratic stability, Relaxed stability conditions.

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## 1. INTRODUCTION

Since their introduction in 1985 (Takagi and Sugeno, 1985), Takagi-Sugeno (TS) models have been the subject of numerous works regarding to their stability analysis and stabilization. Due to their universal approximator skills, interesting TS modeling based results have been provided for analyzing nonlinear systems. Indeed, using the well-known Sector Nonlinearity approach (Tanaka and Wang, 2001), a TS model is able to match exactly a smooth nonlinear system on a compact set of its state space. Using Quadratic Lyapunov Functions (QLF), Linear Matrix Inequality (LMI) based stability/stabilization criterion have been derived in various cases, see e.g. (Tanaka and Wang, 2001; Sala *et al.*, 2005) and references therein. Although QLF approach is still mainly employed, it is well known that it suffers from conservatism. Indeed, it requires checking the existence of some common decision matrices, which have to be solution of a set of LMI constraints, see (Sala, 2009) for a detailed review of sources of conservatism.

This work is concerned with relaxation in the sense of the choice of the most convenient Lyapunov Function candidate. Regarding to LMI based Piecewise Lyapunov Function (PWLF) approaches; some interesting results have been provided in terms of conservatism (Johansen *et al.*, 1999; Xie *et al.*, 1997), especially when the conclusion parts of the considered TS model are not simultaneously activated all together. However, TS models obtained from the sector nonlinearity approach lack this property. Complementary to the latter works and with more adequacies to the fuzzy structure of TS models, many research efforts have been recently done in the non-quadratic framework. Indeed, several works employing Non-quadratic Lyapunov Function

(NQLF) candidates have been proposed (Blanco *et al.*, 2001; Tanaka *et al.*, 2003; Guerra and Vermeiren, 2004; Feng, 2006). These ones are convenient with TS fuzzy models since NQLF shares the same membership functions (MFs). However, some drawbacks appear in the continuous time case since the time derivatives of MFs occurs when applying the direct Lyapunov method to obtain LMI based global stability conditions. Therefore, the most commonly used technique is to bound MFs time derivatives (Tanaka *et al.*, 2003). However, these bound are difficult to estimate a priori, in practice, before solving LMI conditions. To overcome this problem and to cope with the non-quadratic framework, a first way has been proposed in (Rhee and Won, 2006). The idea was to employ a Line-Integral Lyapunov Function (LILF). Nevertheless, the obtained conditions are LMIs in stability and Bilinear Matrix Inequalities (BMI) when dealing with stabilization. Moreover, as discussed in (Guelton *et al.*, 2010), the LILF requires some constraining path-independency conditions that are significantly reducing its applicability. Recently, an interesting step has been addressed with LMI based NQLF stability conditions to avoid any knowledge on MFs' dynamics (Bernal and Guerra, 2010). Nevertheless, these approaches have been obtained by reducing the global stability analysis to a local one.

Nowadays, despite the success and popularity of LMI based stability conditions for TS fuzzy models, the drawbacks described above are understood as a limit of these approaches, especially in the non-quadratic framework. Therefore, a challenging change of perspective has to be considered and this paper tends to add a step in this way. Very recently, an alternative to LMIs appears by considering polynomial approaches. Indeed, it has been shown that some convex optimization problems, such like LMI problems, can

be recasted as more general Sum Of Square (SOS) decomposition problems. Therefore, with the first SOS based stability conditions, TS model analysis has just passed an important milestone (Tanaka *et al.*, 2009; Narimani and Lam, 2010).

In this paper, the goal is to extend the stability conditions for polynomial fuzzy systems proposed in (Tanaka *et al.*, 2009) by the use of a new Polynomial Fuzzy Lyapunov Function (PFLF) candidate. Indeed, the Polynomial Lyapunov Function (PLF) proposed in (Tanaka *et al.*, 2009) don't take into account MFs knowledge and, similarly to the QLF approach, it requires to find a common PLF to a set of SOS conditions. Thus, similarly to the extension of QLF by NQLF, the proposed PFLF shares the same MFs structure as the TS fuzzy model to be analyzed. Moreover, it is to be pointed out that the following proposed PFLF based SOS stability conditions do not require unknown bounds of MFs dynamics and guarantee the global stability when a solution to the optimization problem holds.

## 2. PROBLEM STATEMENT

Consider the following nonlinear system:

$$\dot{x}(t) = f(x(t)) \quad (1)$$

where  $f$  is a smooth nonlinear function and  $x(t) = [x_1(t) \ \dots \ x_n(t)] \in \mathfrak{R}^n$  is the state vector.

Using the well-known sector nonlinearity approach (Tanaka and Wang, 2001), it has been shown that (1) can be rewritten (globally or semi-globally) as a polynomial fuzzy system such that (Tanaka *et al.*, 2009):

$$\dot{x} = \sum_{i=1}^r h_i(z(t)) A_i(x(t)) \hat{x}(x(t)) \quad (2)$$

where  $z(t) = [z_1(t) \ \dots \ z_p(t)] \in \mathfrak{R}^p$  is the premise vector,  $A_i(x(t))$  are polynomial matrices in  $x(t)$ ,  $\hat{x}(x(t)) = [\hat{x}_1(x(t)) \ \dots \ \hat{x}_N(x(t))] \in \mathfrak{R}^N$  is a vector of monomials in  $x(t)$  and, for  $i=1, \dots, r$ ,  $h_i(z(t))$  are positive fuzzy membership functions holding the convex sum property  $\sum_{i=1}^r h_i(z(t)) = 1$ .

In (Tanaka *et al.*, 2009), a common PLF  $V(x(t)) = \hat{x}^T(x(t)) P(x(t)) \hat{x}(x(t))$  has been employed to investigate the stability of fuzzy systems described by (2). This approach remains conservative since it requires checking the existence of a common polynomial Lyapunov matrix  $P(x(t)) \in \mathfrak{R}^{N \times N}$  regarding to the fuzzy interconnection structure of (2). Therefore, in order to provide less conservative stability criterion for the class of systems depicted by (2), one proposes the following polynomial fuzzy Lyapunov function candidate (PFLF),

which shares the same fuzzy structure as the TS model to be analyzed:

$$v(x(t)) = \hat{x}^T(x(t)) \sum_{i=1}^r h_i(z(t)) P_i(x(t)) \hat{x}(x(t)) \quad (3)$$

where  $P_i(x(t)) \in \mathfrak{R}^{N \times N}$  are polynomial fuzzy Lyapunov matrices in  $x(t)$ .

The goal is now to propose new stability conditions for the class of polynomial fuzzy systems (2) based on the PFLF candidate (3).

## 3. SOS BASED FUZZY POLYNOMIAL STABILITY CONDITIONS

In this section, one presents new Sum-of-Square based stability conditions for TS polynomial Fuzzy systems depicted by (2). To do it, one considers a PFLF candidate (3). For more clarity of further mathematical proofs, before presenting the main results, some useful preliminaries (notations, assumption and lemma) are presented.

### 3.1 Preliminaries:

*Notations:* When there is no ambiguity, the time  $t$  will be omitted as entry for simplifying mathematical expressions and  $I$  will denote an identity matrix of appropriate dimension. Moreover, let  $A_i^k(x)$  be the  $k^{\text{th}}$  row of  $A_i(x)$ , from (2) one can write, for  $k=1, \dots, n$ :

$$\dot{x}_k = \sum_{i=1}^r h_i(z) A_i^k(x) \hat{x}(x) \quad (4)$$

*Assumption 1:* For  $i=1, \dots, r$ , each  $h_i(z)$  is assumed continuously derivable within each states variables  $x_k$ ,  $k=1, \dots, n$ .

*Remark 1:* In previous non-quadratic studies, the bounds of the MFs derivatives  $|\dot{h}_i(z)| \leq \theta_i$  are required priori to solve LMI conditions, see e.g. (Tanaka *et al.* 2003, Guelton *et al.*, 2009). This is often point out as a criticism of LMI based non-quadratic approaches. However, let us recall that:

$$\dot{h}_i(z) = \sum_{k=1}^n g_i^k(z) \dot{x}_k \quad (5)$$

with  $g_i^k(z) = \frac{\partial h_i(z)}{\partial x_k}$ . Therefore, despite the bounds of  $\dot{h}_i(z)$  which are difficult to be obtained in practice, under assumption 1, the bounds of  $g_i^k(\cdot) \in [\alpha_{i1}^k, \alpha_{i2}^k]$  may be easily obtained on a compact set  $\Omega$ . For example, consider

$h_1(z) = \cos x_1$ , the MFs derivatives are  $\dot{h}_1(z) = \dot{x}_1 \sin x_1$ , depending on the state dynamics  $\dot{x}_1$  and so making difficult obtaining the bound  $|\dot{h}_1(z)| \leq \theta_1$  without restrictive assumptions on the whole T-S model dynamics. However, the bounds of  $g_1^1(z) = \frac{\partial h_1(z)}{\partial x_1} = \sin x_1$  are always known.

This property will be used in the sequel to obtain convex SOS based conditions.

*Lemma 1* (Tuan *et al.*, 2001): The inequality  $\sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \Gamma_{ij} < 0$  is satisfied if, for all combinations of  $i, j = 1, 2, \dots, r$ ,  $\Gamma_{ii} < 0$  hold and, for  $1 \leq i \neq j \leq r$ ,  $\frac{1}{r-1} \Gamma_{ii} + \frac{1}{2} (\Gamma_{ij} + \Gamma_{ji}) < 0$  hold.

### 3.2 Main result:

*Theorem 1:* The polynomial fuzzy system (2) is globally asymptotically stable if there exists, for all combinations of  $i = 1, \dots, r$ ,  $j = 1, \dots, r$ ,  $j \neq i$ ,  $p = 1, \dots, r$ ,  $q = 1, \dots, n$  and  $s = 1, 2$ , the symmetric polynomial matrices  $P_i(x) \in \mathfrak{R}^{N \times N}$  and  $R_i(x) \in \mathfrak{R}^{N \times N}$  such that (6), (7) and (8) are satisfied, with the polynomials  $\varepsilon_{1i}(x) > 0$  for  $x \neq 0$ ,  $\varepsilon_{2i}(x) \geq 0$  and  $\varepsilon_{3ij}(x) \geq 0$  for all  $x$ .

$$\hat{x}^T(x) (P_i(x) - \varepsilon_{1i}(x) I) \hat{x}(x) \text{ is SOS} \quad (6)$$

$$-\hat{x}^T(x) (\Upsilon_{ips}^q(x) + \varepsilon_{2i}(x) I) \hat{x}(x) \text{ is SOS} \quad (7)$$

$$-\hat{x}^T(x) \left( \frac{1}{r-1} \Upsilon_{ips}^q(x) + \frac{1}{2} (\Upsilon_{ips}^q(x) + \Upsilon_{jps}^q(x)) + \varepsilon_{3ij}(x) I \right) \hat{x}(x) \text{ is SOS} \quad (8)$$

$$\text{with } \Upsilon_{ips}^q(x) = \frac{1}{nr} (A_i^T(x) T^T(x) P_j(x) + P_j(x) T(x) A_i(x) + \sum_{k=1}^n \frac{\partial P_j(x(t))}{\partial x_k(t)} A_i^k(x) \hat{x}(x)) + \varphi_{ips}^q,$$

$$\varphi_{ips}^q = \alpha_{ps}^q A_i^q(x(t)) \hat{x}(x) (P_p(x) + R_j(x)) \quad \text{and} \quad \text{where} \\ T(x) \in \mathfrak{R}^{N \times n} \text{ is a polynomial matrix in } x \text{ such that} \\ \hat{\dot{x}}(x) = T(x) \dot{x}, \text{ i.e. whose } (i, j)^{\text{th}} \text{ entry is given by} \\ T_{(i,j)}(x) = \frac{\partial \hat{x}_i(x)}{\partial x_j}.$$

*Proof:* Let us consider the PFLF candidate (3). The polynomial T-S system (2) is stable if:

$$\dot{v}(x) = \sum_{i=1}^r h_i(z) (\dot{x}^T(x) P_i(x) \hat{x}(x) + \hat{x}^T(x) P_i(x) \dot{x}(x)) \\ + \sum_{i=1}^r h_i(z) \hat{x}^T(x) \dot{P}_i(x) \hat{x}(x) + \sum_{i=1}^r \dot{h}_i(z) \hat{x}^T(x) P_i(x) \hat{x}(x) < 0 \quad (9)$$

Let, for  $i = 1, \dots, r$  and  $k = 1, \dots, n$ ,  $g_i^k(z) = \frac{\partial h_i(z)}{\partial x_k}$ . Under assumption 1,  $\forall t, g_i^k(z) \in [\alpha_{i1}^k, \alpha_{i2}^k]$ . Therefore, using the sector nonlinearity decomposition one can write:

$$g_i^k(z) = v_{i1}^k(z) \alpha_{i1}^k + v_{i2}^k(z) \alpha_{i2}^k \quad (10)$$

with  $v_{i1}^k \geq 0$ ,  $v_{i2}^k \geq 0$  and  $v_{i1}^k(z) + v_{i2}^k(z) = 1$ .

Now, from (4) and (10) one can write, for  $i = 1, \dots, r$ :

$$\dot{h}_i(z) = \sum_{k=1}^n \frac{\partial h_i(z)}{\partial x_k} \dot{x}_k \\ = \sum_{j=1}^r \sum_{k=1}^n \sum_{s=1}^2 h_j(z) v_{is}^k(z) \alpha_{is}^k A_j^k(x) \hat{x}(x) \quad (11)$$

Then, the inequality (9) can be rewritten as:

$$\sum_{i=1}^r h_i(z) (\dot{x}^T T^T(x) P_i(x) \hat{x}(x) + \hat{x}^T(x) P_i(x) T(x) \dot{x}) \\ + \sum_{i=1}^r h_i(z) \hat{x}^T(x) \dot{P}_i(x) \hat{x}(x) \\ + \sum_{i=1}^r \dot{h}_i(z) \hat{x}^T(x) P_i(x) \hat{x}(x) < 0 \quad (12)$$

Considering (2), it yields:

$$\sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \hat{x}^T(x) (A_i^T(x) T^T(x) P_j(x) + P_j(x) T(x) A_i(x)) \hat{x}(x) \\ + \sum_{j=1}^r h_j(z) \hat{x}^T(x) \dot{P}_j(x) \hat{x}(x) \\ + \sum_{j=1}^r \dot{h}_j(z) \hat{x}^T(x) P_j(x) \hat{x}(x) < 0 \quad (13)$$

Moreover, using the above defined notations, one can write:

$$\sum_{j=1}^r h_j(z) \dot{P}_j(x) = \sum_{j=1}^r h_j(z) \sum_{k=1}^n \frac{\partial P_j(x)}{\partial x_k} \dot{x}_k(t) \\ = \sum_{i=1}^r \sum_{j=1}^r h_i(z) h_j(z) \sum_{k=1}^n \frac{\partial P_j(x)}{\partial x_k} A_i^k(x) \hat{x}(x) \quad (14)$$

By extension to the way shown in (Mozelli *et al.*, 2009) in the LMI based non-quadratic context, since the membership functions hold the convex sum property  $\sum_{i=1}^r h_i(z) = 1$ , one has

$\sum_{i=1}^r \dot{h}_i(z) = 0$  and so, for any fuzzy polynomial matrices  $\sum_{j=1}^r h_j(z) R_j(x)$  in  $x$ , one can write:

$$\begin{aligned} & \sum_{j=1}^r \dot{h}_j(z) \hat{x}^T(x) P_j(x) \hat{x}(x) \\ &= \sum_{j=1}^r \sum_{p=1}^r h_j(z) \dot{h}_p(z) \hat{x}^T(x) (P_p(x) + R_j(x(t))) \hat{x}(x) \quad (15) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{p=1}^r \sum_{q=1}^r \sum_{s=1}^2 h_i(z(t)) h_j(z) v_{ps}^q(z) \hat{x}^T(x) \varphi_{ijps}^q \hat{x}(x) \end{aligned}$$

with  $\varphi_{ijps}^q = \alpha_{ps}^q A_i^q(x(t)) \hat{x}(x) (P_p(x) + R_j(x))$

Therefore, from (14) and (15), inequality (13) is satisfied if:

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{p=1}^r \sum_{q=1}^r \sum_{s=1}^2 h_i(z) h_j(z) v_{ps}^q(z) \hat{x}^T(x) \Upsilon_{ijps}^q(x) \hat{x}(x) < 0 \quad (16)$$

$$\begin{aligned} \Upsilon_{ijps}^q(x) &= \frac{1}{nr} (A_i^T(x) T^T(x) P_j(x) + P_j(x) T(x) A_i(x) \\ &\text{with } + \sum_{k=1}^n \frac{\partial P_j(x(t))}{\partial x_k(t)} A_i^k(x) \hat{x}(x)) + \varphi_{ijps}^q \end{aligned}$$

Finally, since for all  $p, q$  and  $s$  one has  $v_{ps}^q(z) \geq 0$ , applying lemma 2, one obtains the conditions of theorem 1.  $\square$

*Remark 2:* From (6), (7) and (8), choosing as special case  $P_j(x) = P(x)$  a common polynomial Lyapunov matrix as well as  $R_j(x) = -P(x)$ , one obtains the PLF based stability conditions proposed in (Tanaka *et al.*, 2009). Thus, theorem 1's conditions are less conservative.

*Remark 3:* As quote in (Tanaka *et al.*, 2009), the second order PLF remains to the quadratic Lyapunov function  $V(x) = x^T P x$  with  $P \in \mathfrak{R}^{n \times n}$ . Similarly, the PFLF (3) is equivalent, at the second-order, to the well-known Non-quadratic  $V(x) = \sum_{i=1}^r h_i(z) x^T P_i x$  (Tanaka *et al.*, 2003).

Let us recall that the main drawback of previous LMI based fuzzy Lyapunov results was that, to study the global stability of a T-S model, some unknown parameters (e.g. lower bounds of membership function derivatives) have to be known in advance (to solve LMI problem). A LMI based alternative to this problem has been proposed in (Benal and Guerra, 2010) but it leads to complex LMI formulation and necessitate reducing the global stability analysis goal to a local point of view. What is important to highlight with the proposed SOS based approach is that, the conditions of theorem 1 provide, as a special case, an alternative to the above quoted problems of non-quadratic approaches since it

stands for global stability analysis of T-S systems without requiring unknown parameters regarding to the MFs dynamics.

### 3. NUMERICAL EXAMPLE

In order to compare and illustrate the benefit of the proposed approach in terms of conservatism regarding to previous results, let us consider the following nonlinear system corresponding to example 2 in (Tanaka *et al.*, 2009) :

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -2x_1(t) - x_2(t) - g(t)x_1(t) \end{aligned} \quad (17)$$

where  $g(t) \in [0, k]$  for all  $t$  and which can be exactly represented by the following T-S fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^2 h_i(z(t)) A_i x(t) \quad (18)$$

$$\begin{aligned} \text{where } z(t) &= g(t), \quad A_1 = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -2-k & -1 \end{bmatrix}, \\ h_1(z(t)) &= \frac{k-g(t)}{k} \quad \text{and} \quad h_2(t) = \frac{g(t)}{k}. \end{aligned}$$

As quoted in (Tanaka *et al.*, 2009), the quadratic stability conditions (Tanaka and Wang, 2001) guarantees the stability of (18) for  $k \leq 3.82$ , the piecewise quadratic approach (Xie *et al.*, 1997) guarantees the stability for  $k \leq 4.7$ .

Let us now consider  $\hat{x} = x$ , we can apply the proposed PFLF approach (theorem 1) on the fuzzy system (18) as well as the PLF based ones (Tanaka *et al.*, 2009). The results, summarized in Table 1 for the second and the sixth order polynomial Lyapunov functions, are obtained using SOSTOOLS for Matlab (Prajna, 2009) from theorem 1 with the following tuning:

- The order of polynomials in  $R(x)$  is the same as the ones of  $P(x)$  (respectively 0 and 4).
- For all  $i=1, \dots, r$  and  $j=1, \dots, r$ ,  $\varepsilon_{1i}(x)$ ,  $\varepsilon_{2i}(x)$  and  $\varepsilon_{3ij}(x)$  are set as second-orders positive polynomials to be optimized by SOSTOOLS algorithm.

As expected, regarding to table 1, the proposed PFLF sum-of-square based stability conditions are outperforming the other approaches in terms of conservatism. Moreover, for each of these different values, the obtained PFLF has the form:

$$V(x) = \sum_{i=1}^2 h_i(z) p_i(x) \quad (19)$$

where  $p_1(x)$  and  $p_2(x)$  are given for the second order by:

$$p_1(x) = 33.452x_1^2 + 8.688x_1x_2 + 8.826x_2^2$$

$$\text{and } p_2(x) = 35.71x_1^2 + 8.142x_1x_2 + 7.771x_2^2,$$

and for the sixth order by:

$$p_1(x) = 323.119x_1^6 + 110x_1^5x_2 + 269.29x_1^4x_2^2 + 121.9x_1^3x_2^3 + 76.63x_1^2x_2^4 + 100x_1x_2^5 + 3.1533x_2^6$$

$$\text{and } p_2(x) = 323.79x_1^6 + 103x_1^5x_2 + 269.806x_1^4x_2^2 + 126.406x_1^3x_2^3 + 77.27x_1^2x_2^4 + 18x_1x_2^5 + 3.1533x_2^6.$$

Table 1. Maximal value of  $k$

	2 <sup>nd</sup> order polynomials	6 <sup>th</sup> order polynomials
PFLF (Theorem 1)	$k \leq 4.48$	$k \leq 6.30$
PLF (Tanaka <i>et al.</i> , 2009)	$k \leq 3.82$	$k \leq 6.17$
PWLF (Xie <i>et al.</i> , 1997)	$k \leq 4.7$	
QLF (Tanaka and Wang, 2001)	$k \leq 3.82$	

Fig. 1 shows the behaviors of the obtained PFLF (19) at their respective feasible limits. These numerical simulations have been done for an initial condition  $x^T(0) = [8 \ 0]$ . Moreover, for scale compatibility the results have been normalized before being plotted. As expected, the obtained PFLF are monotonously decreasing.

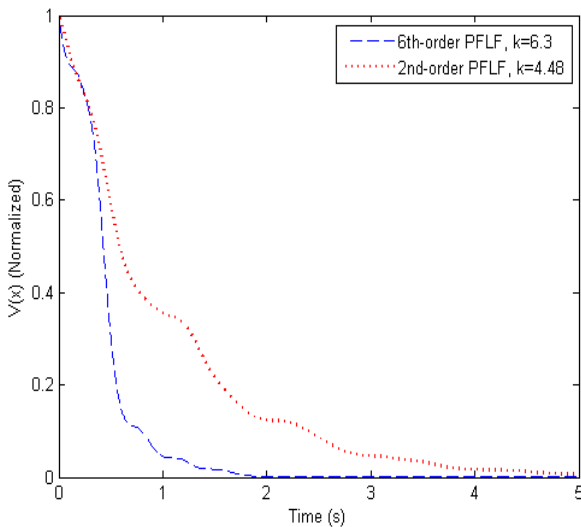


Fig. 1. Time transient of the obtained PFLF (example 1).

## 6. CONCLUSIONS

In this paper, a new step on fuzzy relaxation for nonlinear systems stability analysis has been addressed. Inspired from non-quadratic Lyapunov functions regarding to quadratic ones, a new polynomial fuzzy Lyapunov function has been proposed extending the polynomial Lyapunov function (Tanaka *et al.*, 2009). Following the latter post-LMI challenge, the obtained stability conditions have been written in terms of a sum-of-squares optimization problem. The proposed polynomial fuzzy Lyapunov function includes the well-studied non-quadratic ones as a special case. Moreover, the proposed SOS based stability analysis doesn't require unknown parameters to be known in advance as well as guarantee, when a solution to the SOS based optimization problem hold, a global asymptotical stability. Therefore, some drawbacks of classical LMI based non-quadratic approaches are overcome. Further prospect will naturally be to provide sum-of-squares controller design conditions based on polynomial fuzzy Lyapunov function candidates (work in progress).

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