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Abstract— **This paper deals with decentralized stabilization of large scale switched nonlinear systems under arbitrary switching laws. A global large scale switched system can be split into a set of smaller interconnected switched Takagi Sugeno fuzzy subsystems. Then, in order to stabilize the overall closed-loop system, a set of switched non-PDC controllers is employed. The latter is designed based on Linear Matrix Inequalities (LMI) conditions obtained from a multiple switched non quadratic like-Lyapunov candidate function. A numerical example is proposed to illustrate the effectiveness of the suggested decentralized switched controller design approach.**

Keywords- switched fuzzy system, decentralized control, stabilizing non-PDC control law, arbitrary switching laws, multiple switched non quadratic like-Lyapunov functional.

I. INTRODUCTION

Among control theory, switched linear systems has grown interest since they provide a convenient modeling approach for many physical systems such as computer networks, embedded control systems, traffic control systems, automatic highway systems, chemical process [1-4]. This special class of hybrid system is represented by a family of time-invariant systems switching together in accordance to a stabilizing or an arbitrary law.

In the past decades, many studies focused on the stability analysis and stabilization issue for both linear and nonlinear switched systems [5-10]. The main challenge in treating such problems is to guaranty the stability of the whole system at the switching time. Indeed, the well-know example in [11] show that the stability of each subsystem may not involve the stability of the whole switched system. Hence, switching between subsystems may introduce instable behavior.

Attempting to solve this problem, several primary results dealing with switched linear systems propose to find a common quadratic Lyapunov function satisfying some linear matrix inequality (LMI) conditions [11]. Despite the simplicity of the obtained LMI formulation, finding a common Lyapunov function leads to conservatism. Thus, in order to reduce the conservatism, some relaxed approaches has been proposed by using piecewise quadratic Lyapunov functions [12] or multiple Lyapunov functions [13-16]. In [14], besides to the conditions ensuring the decreasing behaviors for each local Lyapunov function, an additional condition at the switching time to ensure the stability of the whole switched system. Some other works suggest verifying the decreasing behaviour of like-Lyapunov function's switching sequences, see e.g. [8] for more details.

In this study, the problem of designing decentralized switched controllers ensuring the stability of continuous time large scale switched nonlinear systems is addressed. Based on the well-known universal approximator property of Takagi-Sugeno (TS) fuzzy models for nonlinear problems [17,18], there is a growing attention on studying switched nonlinear systems based on TS fuzzy modeling, see e.g. [19]. This kind of systems, known as switched fuzzy systems, involves TS models to represent nonlinear continuous modes. This class of hybrid dynamical systems may be useful to describe precisely both continuous and discrete dynamics as well as their interactions in real-world systems [19,20]. Despite switched linear systems, few studies have been done in the switched nonlinear case. A common Lyapunov candidate function has been firstly employed to ensure the stabilization of switched fuzzy systems [11]. In [21], authors propose to employ a switched PDC controller as well as a switched fuzzy Lyapunov candidate function. However, the authors don't mention any condition to guarantee the stability of the whole system at the switching times.

According to the above described studies and since complex physical configuration and high dimension of many real systems, several works have dealt with stability and stabilization issues of large scale dynamical systems; see e.g. [22-26]. Nevertheless, few investigations can be found in the literature dealing with stability and stabilization problems of large scale switched systems [24,27,28]. In our previous works, one has proposed LMI based stabilization for large scale switched linear systems [24]. Moreover, to the best of the authors' knowledge, the stabilization issue of interconnected switched nonlinear systems hasn't yet been investigated.

Note also that, regarding to TS based approaches, the fuzzy Lyapunov function remains one of the least conservative in terms of LMI. However, the appearance of the membership function derivatives is often considered as a drawback. For more details on some recent results in TS based nonquadratic state feedback controller design, one can refer to [29-31]. Nevertheless, the meaning of this paper is not to cope with this problem. Hence, the goal is to propose a LMI based methodology, in the nonquadratic framework, for the design of decentralized switched non-PDC controllers for a class of large

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scale switched nonlinear systems under arbitrary switching laws.

This paper is organized as follows: First, the studied class of continuous-time interconnected switched fuzzy systems will be described. Then, a set of decentralized switched non-PDC controller is proposed. Hence, LMI stability conditions are provided based on a switched fuzzy like-Lyapunov function candidate. Finally, a simulation example is proposed to illustrate the efficiency of the designed approach.

II. Problem Statement

Let us consider the class of nonlinear hybrid systems S composed of n continuous time switched nonlinear subsystem S_i based on TS modelling. The n state equations of the whole interconnected switched fuzzy system S are given as follows:

For $i = 1,\ldots,n$:

$$\dot{x}_i(t) = \sum_{j_i=1}^{m_i}\sum_{s_{j_i}=1}^{r_{j_i}} \xi_{j_i}(t) h_{s_{j_i}}\left(z_{j_i}(t)\right)\left[A_{s_{j_i}} x_i(t) + B_{s_{j_i}} u_i(t) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i,\alpha,s_{j_i}} x_\alpha(t)\right] \tag{1}$$

where $x_i(t)\in\mathbb{R}^{\eta_i}$, $u_i(t)\in\mathbb{R}^{\upsilon_i}$ represent respectively the state and the input vectors associated to the i^{th} subsystem; m_i is the number of switching modes of the i^{th} subsystem; r_{j_i} is the number of fuzzy rules associated to the i^{th} subsystem in the j_i^{th} mode; for $i = 1,\ldots,n$, $j_i = 1,\ldots,m_i$ and $s_{j_i} = 1,\ldots,r_{j_i}$, $A_{s_{j_i}} \in \mathbb{R}^{\eta_i\times\eta_i}$, $B_{s_{j_i}} \in \mathbb{R}^{\eta_i\times\upsilon_i}$ and $F_{i,\alpha,s_{j_i}} \in \mathbb{R}^{\eta_i\times\eta_\alpha}$ are constant matrices describing the local dynamics of each polytops; $F_{i,\alpha,s_{j_i}}$ express the interconnections between subsystems, i.e. the influence of the α^{th} subsystem on the i^{th} one; $z_{j_i}(t)$ are the premises variables and $h_{s_{j_i}}\left(z_{j_i}(t)\right)$ are positive membership functions satisfying the convex sum proprieties $\sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}}\left(z_{j_i}(t)\right) = 1$; $\xi_{j_i}(t)$ is the switching rules of the i^{th} subsystem, considered arbitrary but assumed to be real time available, these are defined such that the active system in the l_i^{th} mode lead to:

$$\begin{cases} \xi_{j_i}(t) = 1 & \text{if } j_i = l_i \\ \xi_{j_i}(t) = 0 & \text{if } j_i \neq l_i \end{cases} \tag{2}$$

In order to ensure the stabilization of the overall closed-loop fuzzy switched S, a set of decentralized state feedback switched non-PDC control laws is proposed as:

For $i = 1,\ldots,n$:

$$u_i(t) = \sum_{j_i=1}^{m_i}\sum_{k_{j_i}=1}^{r_{j_i}} \xi_{j_i}(t) h_{s_{j_i}}\left(z_{j_i}(t)\right) K_{k_{j_i}} \left(\sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}}\left(z_{j_i}(t)\right) X_{s_{j_i}}\right)^{-1} x_i(t) \tag{3}$$

where $K_{k_{j_i}}$ and $X_{s_{j_i}} = X_{s_{j_i}}^T > 0$ are the gain matrices to be synthesized.

Notations : The time t will be omitted when there is no ambiguity. However, one denotes $t_{j_i\to j_i^+}$ the switching instants of the i^{th} subsystem between the current mode j_i (at time t) and the upcoming mode j_i^+ (at time t^+), therefore:

$$\begin{cases} \xi_{j_i}(t) = 1 \\ \xi_{j_i^+}(t) = 0 \end{cases} \text{ and } \begin{cases} \xi_{j_i}(t^+) = 0 \\ \xi_{j_i^+}(t^+) = 1 \end{cases} \tag{4}$$

In order to lighten the mathematical expression, the premises entries z_{j_i} will be omitted and the following notations will be employed in the sequel:

$$G_{h_{j_i}} = \sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} G_{s_{j_i}} \text{ and } Y_{h_{j_i},h_{j_i}} = \sum_{s_{j_i}=1}^{r_{j_i}}\sum_{k_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} h_{k_{j_i}} Y_{s_{j_i},k_{j_i}} .$$

We will also distinguish, for a regular quantity $\Gamma_{s_{j_i}}$ of appropriate dimension:

$$\left(\Gamma_{h_{j_i}}\right)^{-1} = \left(\sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} \Gamma_{s_{j_i}}\right)^{-1} .$$

For matrices of appropriate dimensions we will denote :

$$\dot{X}_{h_{j_i}} = \frac{dX_{h_{j_i}}}{dt} \text{ and } \left(\dot{X}_{h_{j_i}}\right)^{-1} = \frac{d\left(X_{h_{j_i}}\right)^{-1}}{dt} .$$

As usual, a star (*) indicates a transpose quantity in a symmetric matrix.

The basic idea is to synthesize a global decentralized (3) controller composed of n local switched non-PDC controllers ensuring the stability of each subsystem S_i regarding to the influence of the others subsystems' dynamics. Hence, substituting (3) into (1), one expresses the overall closed-loop dynamics S_{cl} described by:

For $i = 1,\ldots,n$:

$$\dot{x}_i = \sum_{j_i=1}^{m_i} \xi_{j_i} \left\{ \left[A_{h_{j_i}} + B_{h_{j_i}} K_{h_{j_i}} \left(X_{h_{j_i}}\right)^{-1}\right] x_i + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i,\alpha,h_{j_i}} x_\alpha \right\} \tag{5}$$

The following lemma will be useful in the sequel.

Lemma [32]: Let us consider two matrices A and B with appropriate dimension, the following inequality is satisfied with the scalar $\tau > 0$:

$$A^T B + B^T A \leq \tau A^T A + \tau^{-1} B^T B \tag{6}$$

III. LMI Based Decentralized Controller Design

In this section, the goal is to propose a methodology for the design of decentralized switched non-PDC controller (3) ensuring the closed-loop stability of (5). The main result is given in the following theorem.

Theorem : Assume that for each subsystem i of (1), the active mode is denoted by j_i and, for $j_i = 1,\ldots,m_i$ and $s_{j_i} = 1,\ldots,r_{j_i}$, $\dot{h}_{s_{j_i}}\left(z(t)\right) \geq \lambda_{s_{j_i}}$. The overall interconnected switched Takagi-Sugeno system (1) is stabilized by a set of n decentralized switched non-PDC control laws (3), if there exists, for all combinations of $i = 1,\ldots,n$, $j_i = 1,\ldots,m_i$ $j_i^+ = 1,\ldots,m_i$, $s_{j_i} = 1,\ldots,r_{j_i}$, $k_{j_i} = 1,\ldots,r_{j_i}$ and $l_{j_i} = 1,\ldots,r_{j_i}$, the matrices $X_{k_{j_i}} = \left(X_{k_{j_i}}\right)^T > 0$, $W_{s_{j_i},k_{j_i}}$, $K_{k_{j_i}}$ and the positive scalars, $\tau_{1,i}$, … $\tau_{i-1,i}$, $\tau_{i+1,i}$,…, $\tau_{n,i}$ (excepted $\tau_{i,i}$ which don't exist since there is no interaction between a subsystem and himself), such that the LMIs described by (7), (8) and (9)are satisfied.

$$X_{k_{j_i}} - \mu_{j_i \to j_i^+} X_{k_{j_i}} \leq 0 \tag{7}$$

$$X_{l_{j_i}} + W_{s_{j_i},k_{j_i}} > 0 \tag{8}$$

$$\left(\begin{array}{c|cccc|cccc}
\Gamma_{s_{j_i},k_{j_i}} & X_{k_{j_i}} & \cdots & \cdots & X_{k_{j_i}} & X_{k_{j_i}} & \cdots & \cdots & X_{k_{j_i}} \\
\hline
X_{k_{j_i}} & -\tau_{1,i} I & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
\vdots & 0 & \ddots & \ddots & \vdots & \vdots & \ddots & 0 & \vdots \\
\vdots & \vdots & \ddots & \ddots & 0 & \vdots & 0 & \ddots & \vdots \\
X_{k_{j_i}} & 0 & \cdots & 0 & -\tau_{i-1,i} I & 0 & \cdots & \cdots & 0 \\
\hline
X_{k_{j_i}} & 0 & \cdots & \cdots & 0 & -\tau_{i+1,i} I & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & 0 & \vdots & 0 & \ddots & \ddots & \vdots \\
\vdots & \vdots & 0 & \ddots & \vdots & \vdots & \ddots & \ddots & 0 \\
X_{k_{j_i}} & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & -\tau_{n,i} I
\end{array}\right) < 0 \tag{9}$$

with
$$\Gamma_{s_{j_i},k_{j_i}} = X_{k_{j_i}}\left(A_{s_{j_i}}\right)^T + A_{s_{j_i}} X_{k_{j_i}} - \Phi_{s_{j_i},k_{j_i}} + \left(K_{k_{j_i}}\right)^T \left(B_{s_{j_i}}\right)^T + B_{s_{j_i}} K_{k_{j_i}} + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \tau_{i,\alpha} F_{i,\alpha,s_{j_i}} \left(F_{i,\alpha,s_{j_i}}\right)^T$$

and
$$\Phi_{s_{j_i},k_{j_i}} = \sum_{l_{j_i}=1}^{r_{j_i}} \lambda_{l_{j_i}} \left(X_{l_{j_i}} + W_{s_{j_i},k_{j_i}}\right).$$

Proof: Let us define the following multiple like-Lyapunov functional candidate:

$$V\left(x_1, x_2, \ldots, x_n\right) = \sum_{i=1}^{n} \sum_{j_i=1}^{m_i} \xi_{j_i} v_{j_i}\left(x_i\right) > 0 \tag{10}$$

where $v_{j_i} = x_i^T \left(X_{h_{j_i}}\right)^{-1} x_i = x_i^T \left(\sum_{s_{j_i}=1}^{r_{j_i}} h_{s_{j_i}} X_{s_{j_i}}\right)^{-1} x_i$

with $X_{h_{j_i}} = \left(X_{h_{j_i}}\right)^T > 0$.

According to the overview [15], the closed-loop interconnected switched system (5) is asymptotically stable if:

$$\forall t \neq t_{j_i \to j_i^+}, \quad \dot{V}\left(x_1, x_2, \ldots, x_n\right) < 0 \tag{11}$$

and

$$v_{j_i^+}\left(t_{j_i \to j_i^+}\right) \leq \mu_{j_i \to j_i^+} v_{j_i}\left(t_{j_i \to j_i^+}\right) \tag{12}$$

where $\mu_{j_i \to j_i^+}$ are positive scalars.

First, let us focus on the inequalities (12). Their aim is to ensure the global behaviour of the like-Lyapunov function (10) at the switching time $t_{j_i \to j_i^+}$. These inequalities are verified if :

For $i = 1,\ldots,n$:
$$\left(X_{h_{j_i^+}}\right)^{-1} \leq \mu_{j_i \to j_i^+} \left(X_{h_{j_i}}\right)^{-1} \tag{13}$$

which can be rewritten in its extended form as:

For $i = 1,\ldots,n$:
$$\sum_{s_{j_i}=1}^{r_{j_i}} \sum_{s_{j_i^+}=1}^{r_{j_i^+}} h_{s_{j_i}} h_{s_{j_i^+}} \left(X_{s_{j_i}} - \mu_{j_i \to j_i^+} X_{s_{j_i^+}}\right) \leq 0 \tag{14}$$

Inequality (14) is verified if (7) hold for all $i = 1,\ldots,n$, $j_i = 1,\ldots,m_i$, $j_i^+ = 1,\ldots,m_i$, $s_{j_i} = 1,\ldots,r_{j_i}$ and $s_{j_i^+} = 1,\ldots,r_{j_i^+}$.

Now, let us deal with (11), with the above defined notations, it can be rewritten as, $\forall t \neq t_{j_i \to j_i^+}$:

$$\sum_{i=1}^{n} \left[\dot{x}_i^T \left(X_{h_{j_i}}\right)^{-1} x_i + x_i^T \left(X_{h_{j_i}}\right)^{-1} \dot{x}_i + x_i^T \left(\dot{X}_{h_{j_i}}\right)^{-1} x_i\right] < 0 \tag{15}$$

Substituting (5) into (15), one can write, $\forall t \neq t_{j_i \to j_i^+}$:

$$\begin{aligned}&\sum_{i=1}^{n}\Big\{x_i^T\Big[\left(A_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}A_{h_{j_i}}+\left(\dot{X}_{h_{j_i}}\right)^{-1}\\&+\left(X_{h_{j_i}}\right)^{-1}\left(K_{h_{j_i}}\right)^T\left(B_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}B_{h_{j_i}}K_{h_{j_i}}\left(X_{h_{j_i}}\right)^{-1}\Big]x_i\\&+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Big[\left(x_\alpha\right)^T\left(F_{i,\alpha,h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}x_i+x_i^T\left(X_{h_{j_i}}\right)^{-1}F_{i,\alpha,h_{j_i}}x_\alpha\Big]\Big\}<0\end{aligned}\tag{16}$$

From (6), the inequality (16) can be bounded by, $\forall t\neq t_{j_i\to j_i^+}$:

$$\begin{aligned}&\sum_{i=1}^{n}x_i^T\Big\{\left(A_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}A_{h_{j_i}}+\left(\dot{X}_{h_{j_i}}\right)^{-1}\\&+\left(X_{h_{j_i}}\right)^{-1}\left(K_{h_{j_i}}\right)^T\left(B_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}B_{h_{j_i}}K_{h_{j_i}}\left(X_{h_{j_i}}\right)^{-1}\\&+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i,\alpha}\left(X_{h_{j_i}}\right)^{-1}F_{i,\alpha,h_{j_i}}\left(F_{i,\alpha,h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}\Big\}x_i+\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i,\alpha}^{-1}x_\alpha^Tx_\alpha<0\end{aligned}\tag{17}$$

where, for $i=1,...,n$, $\alpha=1,...,n$ and $\alpha\neq i$, $\tau_{i,\alpha}$ are positive scalars (note that, $\tau_{i,i}$ don't exist).

Since $\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i,\alpha}^{-1}x_\alpha^Tx_\alpha=\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq 1}}^{n}\tau_{\alpha,i}^{-1}x_i^Tx_i$, $\forall x_i$, (17) is satisfied if, for $i=1,...,n$ and $\forall t\neq t_{j_i\to j_i^+}$:

$$\begin{aligned}&\left(A_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}A_{h_{j_i}}+\left(\dot{X}_{h_{j_i}}\right)^{-1}\\&+\left(X_{h_{j_i}}\right)^{-1}\left(K_{h_{j_i}}\right)^T\left(B_{h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\left(X_{h_{j_i}}\right)^{-1}B_{h_{j_i}}K_{h_{j_i}}\left(X_{h_{j_i}}\right)^{-1}\\&+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Big[\tau_{i,\alpha}\left(X_{h_{j_i}}\right)^{-1}F_{i,\alpha,h_{j_i}}\left(F_{i,\alpha,h_{j_i}}\right)^T\left(X_{h_{j_i}}\right)^{-1}+\tau_{\alpha,i}^{-1}I\Big]<0\end{aligned}\tag{18}$$

Left and right multiplying the inequalities (18) respectively by $X_{h_{j_i}}$ and since $-\left(\dot{X}_{h_{j_i}}\right)^{-1}=\left(X_{h_{j_i}}\right)^{-1}\dot{X}_{h_{j_i}}\left(X_{h_{j_i}}\right)^{-1}$, it yields, for $i=1,...,n$ and $\forall t\neq t_{j_i\to j_i^+}$:

$$\begin{aligned}&X_{h_{j_i}}\left(A_{h_{j_i}}\right)^T+A_{h_{j_i}}X_{h_{j_i}}-\dot{X}_{h_{j_i}}+\left(K_{h_{j_i}}\right)^T\left(B_{h_{j_i}}\right)^T+B_{h_{j_i}}K_{h_{j_i}}\\&+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Big[\tau_{i,\alpha}F_{i,\alpha,h_{j_i}}\left(F_{i,\alpha,h_{j_i}}\right)^T+\tau_{\alpha,i}^{-1}X_{h_{j_i}}X_{h_{j_i}}\Big]<0\end{aligned}\tag{19}$$

Let us now focus on the term $\dot{X}_{h_{j_i}}$. Since the inequality (19) is a double sum ($h_{j_i}h_{j_i}$) and, by extension to the relaxation scheme proposed in [20], additional slack decision matrices can be introduced. Indeed, since the membership functions holds the convex sum property, one has $\sum_{s_{j_i}=1}^{r_{j_i}}\dot{h}_{j_i}=0$ and so $\sum_{l_{j_i}=1}^{r_{j_i}}\dot{h}_{l_{j_i}}W_{h_{j_i},h_{j_i}}=0$. Therefore, one can write:

$$\dot{X}_{h_{j_i}}=\sum_{l_{j_i}=1}^{r_{j_i}}\dot{h}_{l_{j_i}}\left(X_{l_{j_i}}+W_{h_{j_i},h_{j_i}}\right)\tag{20}$$

Then, let us assume that, for $i=1,...,n$, $j_i=1,...,m_i$, $s_{j_i}=1,...,r_{j_i}$, $\lambda_{s_{j_i}}$ are the lower bound of $\dot{h}_{l_{j_i}}$, one can write :

$$-\dot{X}_{h_{j_i}}\leq-\Phi_{h_{j_i},h_{j_i}}\tag{21}$$

with $\Phi_{h_{j_i},h_{j_i}}=\sum_{s_{j_i}=1}^{r_{j_i}}\sum_{k_{j_i}=1}^{r_{j_i}}h_{s_{j_i}}h_{k_{j_i}}\sum_{l_{j_i}=1}^{r_{j_i}}\lambda_{l_{j_i}}\left(X_{l_{j_i}}+W_{s_{j_i},k_{j_i}}\right)$ and :

$$X_{l_{j_i}}+W_{s_{j_i},k_{j_i}}>0\tag{22}$$

Thus, from (21) and applying the Schur complement, (19) is satisfied if, for $i=1,...,n$ and $\forall t\neq t_{j_i\to j_i^+}$:

$$\left(\begin{array}{c|cccc|cccc}\Gamma_{h_{j_i},h_{j_i}} & X_{h_{j_i}} & \cdots & \cdots & X_{h_{j_i}} & X_{h_{j_i}} & \cdots & \cdots & X_{h_{j_i}}\\ \hline X_{h_{j_i}} & -\tau_{1,i}I & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0\\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots & \ddots & 0 & \vdots\\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots & 0 & \ddots & \vdots\\ X_{h_{j_i}} & 0 & \cdots & 0 & -\tau_{i-1,i}I & 0 & \cdots & \cdots & 0\\ \hline X_{h_{j_i}} & 0 & \cdots & \cdots & 0 & -\tau_{i+1,i}I & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & 0 & \vdots & 0 & \ddots & \ddots & \vdots\\ \vdots & \vdots & 0 & \ddots & \vdots & \vdots & \ddots & \ddots & 0\\ X_{h_{j_i}} & 0 & \cdots & \cdots & 0 & 0 & \cdots & 0 & -\tau_{n,i}I\end{array}\right)<0\tag{23}$$

with

$$\begin{aligned}\Gamma_{h_{j_i},h_{j_i}}&=X_{h_{j_i}}\left(A_{h_{j_i}}\right)^T+A_{h_{j_i}}X_{h_{j_i}}-\Phi_{h_{j_i},h_{j_i}}\\&+\left(K_{h_{j_i}}\right)^T\left(B_{h_{j_i}}\right)^T+B_{h_{j_i}}K_{h_{j_i}}+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i,\alpha}F_{i,\alpha,h_{j_i}}\left(F_{i,\alpha,h_{j_i}}\right)^T.\end{aligned}$$

Then, (22) and (23) are satisfied if respectively (8) and (9) hold. That ends the proof. ■

Remark: When $\mu_{j_i\to j_i^+}$ are unknown, conditions of theorem above are not LMI. In order to obtain LMI conditions, one may choose the positive decreasing rates $\mu_{j_i\to j_i^+}$ according to :

$$\prod_{\substack{j_i=1 \\ j_i^+ \neq j_i}}^{m_i} \mu_{j_i \to j_i^+} \leq 1 \tag{24}$$

IV. NUMERICAL EXAMPLE

This section is dedicated to illustrate the efficiency of the proposed approaches. We consider the following system composed of two interconnected switched takagi-sugeno subsystems given by:

Subsystem 1:

$$\dot{x}_1 = \sum_{j_1=1}^{2} \sum_{s_{j_1}=1}^{2} \xi_{j_1} h_{s_{j_1}} \left[A_{s_{j_1}} x_1 + B_{s_{j_1}} u_1 + F_{1,2,s_{j_1}} x_2 \right] \tag{25}$$

with

$$x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} A_{1_{1_1}} = \begin{bmatrix} -2 & 1 \\ 0.1 & -2.1 \end{bmatrix},\ A_{2_{1_1}} = \begin{bmatrix} -2 & 1 \\ 0.1 & -1.1 \end{bmatrix},$$

$$A_{1_{2_1}} = \begin{bmatrix} -1 & 1 \\ 0.1 & -2 \end{bmatrix},\ A_{2_{2_1}} = \begin{bmatrix} -1 & 1 \\ 0.1 & -3 \end{bmatrix},\ B_{1_{1_1}} = \begin{bmatrix} -1.2 \\ 0.2 \end{bmatrix},$$

$$B_{2_{1_1}} = \begin{bmatrix} -1.2 \\ 1.2 \end{bmatrix},\ B_{1_{2_1}} = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix},\ B_{2_{2_1}} = \begin{bmatrix} -1.5 \\ -1.5 \end{bmatrix},$$

$$F_{1,2,1_{1_1}} = \begin{bmatrix} 0.01 & 0.001 & 0.1 \\ 0.01 & 0.01 & 0.1 \end{bmatrix},\ F_{1,2,2_{1_1}} = \begin{bmatrix} 0.01 & 0.01 & 0.1 \\ 0.01 & 0.01 & 0.1 \end{bmatrix},$$

$$F_{1,2,1_{2_1}} = \begin{bmatrix} 0.01 & 0.01 & 0.01 \\ 0.07 & 0.2 & 0.2 \end{bmatrix},\ F_{1,2,2_{2_1}} = \begin{bmatrix} 0.01 & 0.001 & 0.01 \\ 0.08 & 0.02 & 0.2 \end{bmatrix}$$

and the membership functions:

$$h_{1_{1_1}}\left(x_1(t)\right) = \sin^2\left(x_{11}(t)\right),\ h_{2_{1_1}}\left(x_1(t)\right) = \sin^2\left(x_{12}(t)\right),$$

$$h_{1_{2_1}}\left(x_1(t)\right) = 1 - h_{1_{1_1}}\left(x_1(t)\right), \text{ and } h_{2_{2_1}}\left(x_1(t)\right) = 1 - h_{2_{1_1}}\left(x_1(t)\right).$$

Subsystem 2:

$$\dot{x}_2 = \sum_{j_2=1}^{2} \sum_{s_{j_2}=1}^{2} \xi_{j_2} h_{s_{j_2}} \left[A_{s_{j_2}} x_2(t) + B_{s_{j_2}} u_2(t) + F_{2,1,s_{j_2}} x_1(t) \right] \tag{26}$$

with

$$x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix} A_{1_{1_2}} = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 0 \\ 1 & 0.1 & -1.1 \end{bmatrix},\ A_{2_{1_2}} = \begin{bmatrix} -2 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0.1 & -2.1 \end{bmatrix},$$

$$A_{1_{2_2}} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 1.1 & -1 \end{bmatrix},\ A_{2_{2_2}} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0.1 & -1 \end{bmatrix},\ B_{1_{1_2}} = \begin{bmatrix} -0.1 \\ 0.5 \\ 0.1 \end{bmatrix},$$

$$B_{2_{1_2}} = \begin{bmatrix} -0.1 \\ 0.5 \\ 1.1 \end{bmatrix},\ B_{1_{2_2}} = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.2 \end{bmatrix},\ B_{2_{2_2}} = \begin{bmatrix} 0.1 \\ 0.5 \\ 1.2 \end{bmatrix},\ F_{2,1,1_{1_2}} = \begin{bmatrix} 0.01 & 0.6 \\ 0.3 & 0.2 \\ 0.2 & 0.1 \end{bmatrix},$$

$$F_{2,1,2_{1_2}} = \begin{bmatrix} 0.01 & 0.6 \\ 0.3 & 0.2 \\ 0.02 & 0.1 \end{bmatrix},\ F_{2,1,1_{2_2}} = \begin{bmatrix} 0.01 & 0.5 \\ 0.1 & 0.4 \\ 0.2 & 0.2 \end{bmatrix},$$

$$F_{2,1,2_{2_2}} = \begin{bmatrix} 0.01 & 0.5 \\ 0.1 & 0.04 \\ 0.2 & 0.2 \end{bmatrix}$$

and the membership functions

$$h_{1_{1_2}}\left(x_2(t)\right) = \sin^2\left(x_{21}(t)\right),\ h_{2_{1_2}}\left(x_2(t)\right) = \sin^2\left(x_{22}(t)\right),$$

$$h_{1_{2_2}}\left(x_2(t)\right) = 1 - h_{1_{1_2}}\left(x_2(t)\right),\ h_{2_{2_2}}\left(x_2(t)\right) = 1 - h_{2_{1_2}}\left(x_2(t)\right).$$

Let us assume that each subsystem switches under within the frontier defined by: $H_{11} = 0.9x_{11} + x_{12}$, $H_{12} = -0.2x_{11} + 9x_{12}$, $H_{21} = -x_{21} + x_{22}$ and $H_{22} = x_{21} - 2x_{22}$.

A set of decentralized switched controllers (3) is synthesized based on theorem 1 via the Matlab LMI toolbox. To do so, the decreasing rates are chosen as $\mu_{1_1 \to 2_1} = 0.4$, $\mu_{2_1 \to 1_1} = 2$, $\mu_{1_2 \to 2_2} = 0.4$, $\mu_{2_1 \to 1_1} = 2$ according to (24) and the lower bounds of membership functions as $\lambda_{1_{1_1}} = -4$, $\lambda_{1_{2_1}} = -1$, $\lambda_{1_{1_2}} = -6.5$ and $\lambda_{1_{2_2}} = -1.5$.

The close-loop subsystem dynamics are shown in Figure 1 for the initial states $x_1(0) = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$ and $x_2(0) = \begin{bmatrix} -1 & 1.5 & -1 \end{bmatrix}^T$.

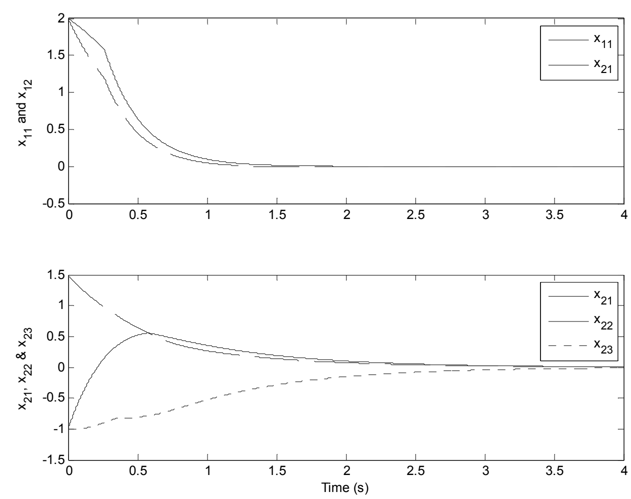

Figure 1. States dynamics of the overall closed-loop interconnected switched Takagi-Sugeno system.

Figure 2 shows the control signals as well as the switching modes' evolution. As expected, the synthesized decentralized switched controller stabilizes the overall large scale switched system composed of (25) and (26).

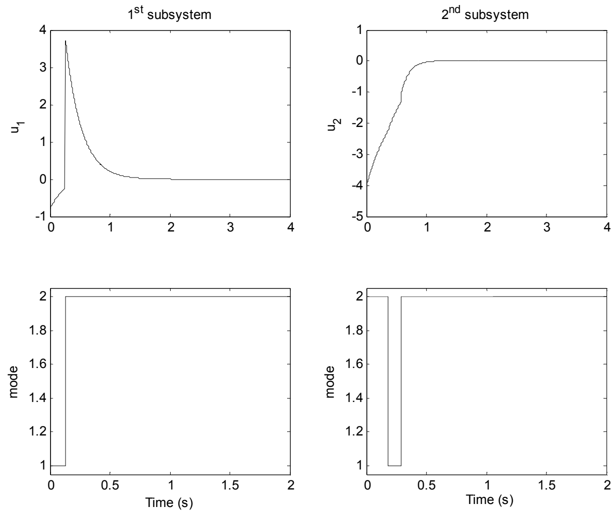

Figure 2. Control signals and switched laws' evolutions.

Figure 3 shows that like-Lyapunov functions $v_1(x_1)$ and $v(x_2)$ of both the subsystems, as well as the global like-Lyapunov function $V(x_1, x_2, ..., x_n)$, have a global decreasing behavior along the systems' trajectories.

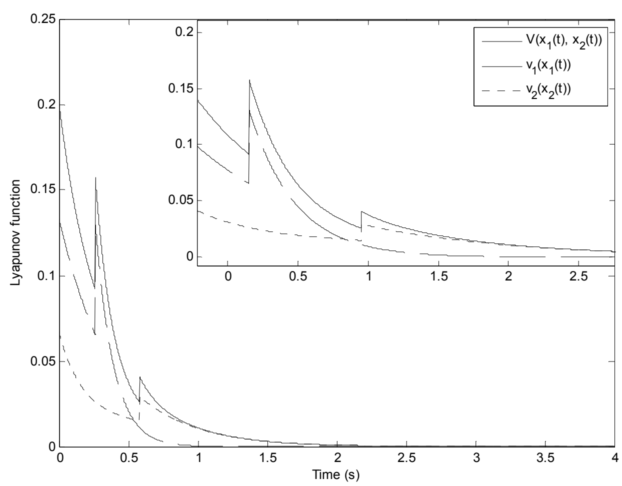

Figure 3. Behaviour of the like-Lyapunov functions.

Figure 4 provides the time evolution of the membership functions derivatives $\dot{h}_{1_{1_1}}(x_1(t))$, $\dot{h}_{2_{1_1}}(x_1(t))$, $\dot{h}_{1_{1_2}}(x_2(t))$ and $\dot{h}_{2_{1_2}}(x_2(t))$. This shows that the assumed lower bounds are verified in simulation for the considered initial conditions.

V. Conclusion

This study has focused on large scale switched nonlinear systems where, due to their well-known universal approximator properties, each nonlinear mode has been represented by a fuzzy Takagi-Sugeno system. Hence, the considered class of hybrid dynamical systems is composed by a set of interconnected switched Takagi-Sugeno subsystems. To ensure the stability of the whole system in closed-loop, a set of decentralized switched non-PDC controllers has been proposed. Therefore, LMI based conditions for the decentralized controller design has been obtained through the consideration of a candidate multiple switched non quadratic like-Lyapunov functional. Finally, a numerical example has been proposed to show the effectiveness of the proposed approach.

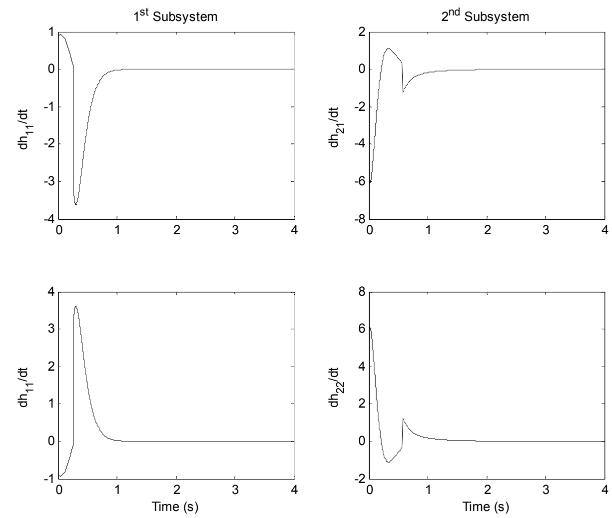

Figure 4. Evolution of membership functions derivatives.

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