Decentralized Static Output Feedback control of interconnected fuzzy descriptors

Dalel Jabri, Kevin Guelton, Noureddine Manamanni

Decentralized Static Output Feedback Control of Interconnected Fuzzy Descriptors

Dalel Jabri, *Student Member*, *IEEE*, Kevin Guelton, *Member, IEEE*, Noureddine Manamanni, *Member, IEEE*

Abstract—**This paper addresses the stabilization issue based on decentralized Static Output Feedback (SOF) non-Parallel-Distributed-Compensation (non-PDC) controllers for large scale nonlinear descriptors. The overall nonlinear plant is represented by interconnections between *n* Takagi-Sugeno (T-S) fuzzy descriptors. In order to avoid the appearance of crossing terms between local controllers and T-S subsystems, the closed-loop dynamics is written using a descriptor redundancy formulation. The stability conditions, obtained from a multiple fuzzy Lyapunov function (MFLF) candidate, are proposed in term of Linear Matrix Inequality (LMI). Finally, an academic example is provided to illustrate the efficiency of the proposed approach.**

Keywords— Takagi-Sugeno, descriptors, Large scale systems, Multiple fuzzy Lyapunov function, Decentralized control, Static output feedback controller.

I. Introduction

This paper deals with decentralized output feedback stabilization for nonlinear large-scale system represented by a network of n T-S fuzzy interconnected descriptors. Static output feedback (SOF) controller design is one of the challenging issues in control engineering. Indeed, in many practical situations, it is sometimes impossible to access to all the system's state variables. Moreover, in the case of large-scale systems, in some circumstances, it can be prohibitively expensive or impossible to collect all the process variables. Therefore, only partial information, from measured outputs, is available. In that case, SOF controllers appear as a suitable solution and lead to reduce controllers' online computational costs in practical implementations [1]-[5].

Recently, Takagi-Sugeno (T-S) fuzzy systems [6] have shown their significance in both the modelling and control of nonlinear systems. Unlike conventional modelling techniques which use a single nonlinear model to describe the behaviour of a global system, T-S system is a fuzzified set of linear polytops [6]-[7]. Therefore, based on the sector non linearity approach, T-S systems may represent exactly a wide class of nonlinear systems on a compact set of the state space [7]. In this case, some of the linear concepts, such as stability analysis through Linear Matrix Inequalities (LMI), can be extended to nonlinear systems [8]. Furthermore, Parallel Distributed Compensation (PDC) control laws [9] have shown their usefulness since they share the same membership functions as the TS system to be stabilized.

Output feedback control for TS systems has been firstly studies based on observers and estimated state feedback [10]-[12]. Other studies have focused on dynamic output feedback controllers (DOFC) design [13]-[17]. Note that a DOFC can be regarded as an extended SOF controller [1]. Therefore, it is clear that studying SOF controllers design arouse control engineers' interest. Most of the papers dealing with SOF controller design are based on quadratic Lyapunov functions, which often lead to bilinear matrix inequalities constraints. However, these latter remain conservative [18]. Non quadratic approaches have been proposed for the design of non-PDC state feedback controllers based on fuzzy Lyapunov approaches [19]-[21]. In [21], a descriptor property, called redundancy, has been employed to reduce and make easier LMI formulation of T-S control problems in the non quadratic framework. The main interest of such approache is that it allows decoupling input matrices and controller gains matrices and avoiding crossing terms in the closed-loop dynamics formulation. Following this way, the descriptor redundancy has been used to propose non quadratic SOF and DOFC LMI based controller design respectively in [5] and [17].

On the other hand, as a natural prospect to the complexity of the real systems, the stabilization of large scale systems has attracted many researches in the last decade. Using T-S modelling, the overall system is decomposed as a set of interconnected T-S subsystems [22]-[26]. Then, the stabilization of such interconnected systems can be obtained using a set of decentralized Parallel Distributed Compensation (DPDC) state feedback controllers [19]-[24]. However, to the best of authors' knowledge, decentralized SOF controller synthesis for the T-S interconnected descriptors hasn't been discussed in the literature, which motivates this study.

This paper is organised as follows: first, the considered class of the T-S interconnected descriptors and the suggested decentralized non-PDC controllers are given. Then, LMI based non quadratic stability conditions are proposed through a multiple Lyapunov function candidate using the descriptor redundancy property. Finally, an academic

This work was supported by the French Ministry of Research, the CNOUS, the French embassy in Tunisia, the GIS 3SGS within the framework of the project COSMOS2, the FEDER and the Région Champagne-Ardenne within the CPER MOSYP and NAVIMELES.

Prof. N. Manamanni and Dr. K. Guelton are with the CReSTIC EA 3804, Université de Reims Champagne-Ardenne, Moulin de la Housse BP1039, 51687 Reims Cedex 2, France.

Miss D. Jabri is both with CReSTIC, Université de Reims Champagne-Ardenne, France, and MACS, Université de Gabès, Tunisia.

Corresponding author: K. Guelton; tel:+33326913261; fax: +33326913106; e-mail: kevin.guelton@univ-reims.fr.

example is given to prove the effectiveness of the designed SOF non-PDC controller.

II. Problem Statement

Consider the class of large-scale nonlinear descriptors S composed of n T-S fuzzy interconnected descriptors S_i described as follows:

For $i = 1,...,n$,

$$\begin{cases} \sum_{j=1}^{l_i} v_i^j\left(z_i(t)\right) E_i^j \dot{x}_i(t) = \sum_{k=1}^{r_i} h_i^k\left(z_i(t)\right)\left\{ A_i^k x_i(t) + B_i^k u_i(t) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha}^k x_\alpha(t) \right\} \\ y_i(t) = \sum_{k=1}^{r_i} h_i^k\left(z_i(t)\right)\left(C_i^k x_i(t) + D_i^k u_i(t)\right) \end{cases} \quad (1)$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, $y_i(t) \in \mathbb{R}^{q_i}$, $z_i(t) \in \mathbb{R}^{p_i}$ represent respectively the state, the input, the output and the premises vectors associated to the i^{th} descriptor. $x_\alpha(t) \in \mathbb{R}^{n_\alpha}$ denotes the state vector of the α^{th}, $\alpha \neq i$, descriptor. l_i is the number of fuzzy rules associated to the left-hand side of the state equation (1). So, for $j = 1,...,l_i$, $E_i^j \in \mathbb{R}^{n_i \times n_i}$ are constant matrices, if necessary singular, and $v_i^j(z_i) \geq 0$ are the left-hand side membership functions verifying the convex sum property $\sum_{j=1}^{l_i} v_i^j\left(z_i(t)\right) = 1$. In the same way, r_i is the number of fuzzy rules associated to the right-hand term in (1). Thus, for $k = 1,...,r_i$ $A_i^k \in \mathbb{R}^{n_i \times n_i}$, $B_i^k \in \mathbb{R}^{n_i \times m_i}$, $F_{i\alpha}^k \in \mathbb{R}^{n_i \times n_\alpha}$, $C_i^k \in \mathbb{R}^{q_i \times n_i}$, $D_i^k \in \mathbb{R}^{q_i \times m_i}$ are constant matrices and $h_i^k(z_i) \geq 0$ are the membership functions associated to the right hand side fuzzy rules satisfying the convex sum propriety $\sum_{k=1}^{r_i} h_i^k\left(z_i(t)\right) = 1$. Note that $F_{i\alpha}^k$ are interconnection matrices expressing the influence of the α^{th} subsystems on the i^{th} one.

To ensure the stabilization of the overall descriptor S, a decentralized non-PDC SOF controller is proposed. The basic idea is to synthesize a global controller composed of n local SOF controllers assuming that each local controller is able to ensure the stability of the subsystem S_i regarding to the interconnections among the others subsystems. For more convenience, the local non-PDC SOF controller shares the same fuzzy sets with the subsystem S_i. The i^{th} decentralized non-PDC SOF controller is given by:

$$u_i(t) = \left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j\left(z_i(t)\right) h_i^s\left(z_i(t)\right) K_i^{js} \right)\left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j\left(z_i(t)\right) h_i^s\left(z_i(t)\right) W_{11i}^{js} \right)^{-1} y_i(t) \quad (2)$$

for all $i = 1,...,n$, where K_i^j are non-PDC gain matrices and W_{11i}^{js} are Lyapunov dependant non singular gain matrices to be synthesized.

Remark 1: One assumes that the premises vectors $z_i(t)$ are only depending on the inputs $u_i(t)$, the output $y_i(t)$ or measurable state variables.

A. Notations

To clarify the mathematical expression, the following notations will be used in the sequel:

$E_i^{v_i} = \sum_{j=1}^{l_i} v_i^j E_i^j$, $Y_i^{h_i h_i} = \sum_{s=1}^{r_i}\sum_{k=1}^{r_i} h_i^s h_i^k Y_i^{sk}$, $T_i^{v_i h_i} = \sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j h_i^s T_i^{js}$, $X_i^{v_i h_i} = T_i^{v_i h_i}\left(R_i^{v_i h_i}\right)^{-1}$, $Z_i^{v_i h_i h_i} = \sum_{j=1}^{l_i}\sum_{s=1}^{r_i}\sum_{k=1}^{r_i} v_i^j h_i^s h_i^k Z_i^{jsk}$, and so on…

A star (*) in a matrix indicates a transpose quantity. For space convenience, the time t as well as the premises $z_i(t)$ will be omitted when there is no ambiguity.

In [5], the design of a SOF controller has been proposed for classical T-S systems based on a descriptor redundancy approach. Following this way, to take advantage of a descriptor redundancy formulation in the case of decentralized SOF design for interconnected T-S descriptors, (1) and (2) can be rewritten with the above defined notations respectively as:

For $i = 1,...,n$:

$$\begin{cases} E_i^{v_i} \dot{x}_i(t) = A_i^{h_i} x_i(t) + B_i^{h_i} u_i(t) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha}^{h_i} x_\alpha(t) \\ 0 = -y_i(t) + C_i^{h_i} x_i(t) + D_i^{h_i} u_i(t) \end{cases} \quad (3)$$

For $i = 1,...,n$:

$$0 = -u_i(t) + \left(K_i^{v_i h_i}\right)\left(W_{11i}^{v_i h_i}\right)^{-1} y_i(t) \quad (4)$$

Let's now consider the extended state vector $\tilde{x}_\beta = \begin{bmatrix} x_\beta^T & \dot{x}_\beta^T & y_\beta^T & u_\beta^T \end{bmatrix}^T$, with $\beta = i$ or α. Substituting (4) into (3), the overall closed-loop dynamics may be written as:

For all $i = 1,...,n$,

$$\tilde{E}_i \dot{\tilde{x}}_i = \tilde{G}_i^{v_i h_i} \tilde{x}_i + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} \tilde{F}_{i\alpha}^{h_i} \tilde{x}_\alpha \quad (5)$$

with $\tilde{E}_i = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\tilde{F}_{i\alpha}^{h_i} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ F_{i\alpha}^{h_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

and $\tilde{G}_i^{v_i h_i} = \begin{bmatrix} 0 & I & 0 & 0 \\ A_i^{h_i} & -E_i^{v_i} & 0 & B_i^{h_i} \\ C_i^{h_i} & 0 & -I & D_i^{h_i} \\ 0 & 0 & K_i^{v_i h_i}\left(W_{11i}^{v_i h_i}\right)^{-1} & -I \end{bmatrix}$.

III. DECENTRALIZED SOF CONTROLLER DESIGN

The main purpose of this paper is to provide a design methodology for a decentralized SOF controller in order to stabilize nonlinear interconnected descriptors described by (1). The following lemmas will be used in the sequel.

Lemma 1 [27]**:**

Let us consider A and B two matrices of appropriate dimensions and a positive constant $\varepsilon > 0$:

$$A^T B + B^T A \le \varepsilon AA + \varepsilon^{-1} BB \quad (6)$$

The main result is summarized in the following theorem.

Theorem 1: *Assume that, for* $i = 1,\dots,n$, $j = 1,\dots,l_i$ *and* $s = 1,\dots,r_i$, $\dot{h}_i^s\left(z(t)\right) \ge \varpi_i^s$ *and* $\dot{v}_i^j\left(z(t)\right) \ge \rho_i^j$. *The overall descriptor* S *composed of* n *interconnected T-S descriptors* S_i *described by (1) is stabilized by the network of* n *non-PDC decentralized control laws described by (2) if there exists, for all combinations of* $i = 1,\dots,n$, $j = 1,\dots,l_i$, $k = 1,\dots,r_i$, $s = 1,\dots,r_i$, $\zeta = 1,\dots,l_i - 1$ *and* $\xi = 1,\dots,r_i - 1$, *the matrices* $W_{i1}^{js} = \left(W_{i1}^{js}\right)^T > 0$, W_{5i}^{sk}, W_{6i}^{sk} *non singular,* W_{7i}^{sk}, W_{8i}^{sk}, W_{11i}^{js} *non singular,* W_{13i}^{js}, W_{14i}^{js}, W_{15i}^{js}, W_{16i}^{js} *non singular, and the positive scalars* $\tau_\alpha^i > 0$, *for* $\alpha = 1,\dots,n$ *and* $\alpha \ne i$, *such that the following LMIs are satisfied:*

$$W_{i1}^{j\xi} - W_{i1}^{jr_i} > 0 \quad (7)$$

$$W_{i1}^{\zeta s} - W_{i1}^{l_i s} > 0 \quad (8)$$

$$\left(\begin{array}{c|cccccc} \psi_i^{jks} & (*) & \cdots & \cdots & \cdots & \cdots & (*) \\ \hline \tilde{W}_i^{jks} & -\tau_1^i I & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & & & \vdots \\ \vdots & \vdots & \ddots & -\tau_{i-1}^i I & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & -\tau_{i+1}^i I & \ddots & \vdots \\ \vdots & \vdots & & & \ddots & \ddots & 0 \\ \tilde{W}_i^{jks} & 0 & \cdots & \cdots & \cdots & 0 & -\tau_{ni} I \end{array}\right) < 0 \quad (9)$$

with ψ_i^{jks} *defined below and*

$$\tilde{W}_i^{jks} = \begin{bmatrix} W_{1i}^{js} & 0 & 0 & 0 \\ W_{5i}^{ks} & W_{6i}^{ks} & W_{7i}^{ks} & W_{8i}^{ks} \\ 0 & 0 & W_{11i}^{js} & 0 \\ W_{13i}^{js} & W_{14i}^{js} & W_{15i}^{js} & W_{16i}^{js} \end{bmatrix}.$$

Proof: Let the multiple fuzzy Lyapunov function candidate be:

$$V(t) = \sum_{i=1}^{n} V_i\left(x_i(t)\right) \ge 0 \quad (10)$$

with $V_i\left(x_i(t)\right) = \tilde{x}_i^T(t)\,\tilde{E}\left(\tilde{W}_i^{v_i h_i h_i}\right)^{-1}\tilde{x}_i(t) \ge 0$

As usual, (10) needs $\tilde{E}\left(\tilde{W}_i^{v_i h_i h_i}\right)^{-1} = \left(\tilde{W}_i^{v_i h_i h_i}\right)^{-T}\tilde{E} \ge 0$ leading to condition the Lyapunov matrix such that :

$$\tilde{W}_i^{v_i h_i h_i} = \begin{bmatrix} W_{1i}^{v_i h_i} & 0 & 0 & 0 \\ W_{5i}^{h_i h_i} & W_{6i}^{h_i h_i} & W_{7i}^{h_i h_i} & W_{8i}^{h_i h_i} \\ W_{9i}^{h_i h_i} & W_{10i}^{h_i h_i} & W_{11i}^{v_i h_i} & W_{12i}^{h_i h_i} \\ W_{13i}^{v_i h_i} & W_{14i}^{v_i h_i} & W_{15i}^{v_i h_i} & W_{16i}^{v_i h_i} \end{bmatrix} \quad (11)$$

$$\psi_i^{jks} = \begin{bmatrix} \begin{pmatrix} \left(W_{5i}^{ks}\right)^T + W_{5i}^{ks} - \sum_{s=1}^{r_i-1}\varpi_i^s\left(W_{i1}^{js} - W_{i1}^{jr_i}\right) \\ -\sum_{j=1}^{l_i-1}\rho_i^j\left(W_{i1}^{js} - W_{i1}^{l_i s}\right) \end{pmatrix} & (*) & (*) & (*) \\ \left(W_{6i}^{ks}\right)^T + A_i^k W_{1i}^{js} - E_i^j W_{5i}^{ks} + B_i^k W_{13i}^{js} & \begin{pmatrix} -\left(W_{6i}^{ks}\right)^T\left(E_i^j\right)^T + \left(W_{14i}^{js}\right)^T\left(B_i^k\right)^T \\ -E_i^j W_{6i}^{ks} + B_i^k W_{14i}^{js} + \sum_{\substack{\alpha=1 \\ \alpha\ne i}}^{n}\tau_{i\alpha}F_{i\alpha}^k\left(F_{i\alpha}^k\right)^T \end{pmatrix} & (*) & (*) \\ \left(W_{7i}^{ks}\right)^T + C_i^k W_{1i}^{js} + D_i^k W_{13i}^{js} & -\left(W_{7i}^{ks}\right)^T\left(E_i^j\right)^T + \left(W_{15i}^{js}\right)^T\left(B_i^k\right)^T + D_i^k W_{14i}^{js} & -\left(W_{11i}^{js}\right)^T + \left(W_{15i}^{js}\right)^T\left(D_i^k\right)^T - W_{11i}^{js} + D_i^k W_{15i}^{js} & (*) \\ \left(W_{8i}^{ks}\right)^T - W_{13i}^{js} & -\left(W_{8i}^{ks}\right)^T\left(E_i^j\right)^T + \left(W_{16i}^{js}\right)^T\left(B_i^k\right)^T - W_{14i}^{js} & \left(W_{16i}^{js}\right)^T\left(D_i^k\right)^T + K_i^{js} - W_{15i}^{js} & -\left(W_{16i}^{js}\right)^T - W_{16i}^{js} \end{bmatrix}$$

with $\left(W_{1i}^{v_ih_i}\right)^T = W_{1i}^{v_ih_i} > 0$.

From (10), the closed-loop system (5) is stable if:

$$\sum_{i=1}^{n}\left(\dot{\tilde{x}}_i^T \tilde{E}\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}\tilde{x}_i + \tilde{x}_i^T \tilde{E}\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}\dot{\tilde{x}}_i + \tilde{x}_i^T \tilde{E}\left(\dot{\tilde{W}}_i^{v_ih_ih_i}\right)^{-1}\tilde{x}_i\right) < 0 \quad (12)$$

That is to say if:

$$\sum_{i=1}^{n}\begin{pmatrix} \tilde{x}_i^T\left(\left(\tilde{G}_i^{v_ih_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1} + \left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{G}_i^{v_ih_i} + \tilde{E}\left(\dot{\tilde{W}}_i^{v_ih_ih_i}\right)^{-1}\right)\tilde{x}_i \\ + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{x}_\alpha^T\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}\tilde{x}_i + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{x}_i^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{F}_{i\alpha}^{h_i}\tilde{x}_\alpha \end{pmatrix} < 0 \quad (13)$$

Using lemma 1, (13) can be bounded by:

$$\sum_{i=1}^{n}\left(\tilde{x}_i^T\left(\left(\tilde{G}_i^{v_ih_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1} + \left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{G}_i^{v_ih_i} + \tilde{E}\left(\dot{\tilde{W}}_i^{v_ih_ih_i}\right)^{-1}\right)\tilde{x}_i\right)$$
$$+\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tau_\alpha^i\tilde{x}_i^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{F}_{i\alpha}^{h_i}\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}\tilde{x}_i + \left(\tau_\alpha^i\right)^{-1}\tilde{x}_\alpha^T\tilde{x}_\alpha\right) < 0 \quad (14)$$

Note that $\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tau_\alpha^i\right)^{-1}\tilde{x}_\alpha^T\tilde{x}_\alpha = \sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tau_i^\alpha\right)^{-1}\tilde{x}_i^T\tilde{x}_i$, (14) holds $\forall \tilde{x}_i$ if, for all $i = 1,...,n$:

$$\left(\tilde{G}_i^{v_ih_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1} + \left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{G}_i^{v_ih_i} + \tilde{E}\left(\dot{\tilde{W}}_i^{v_ih_ih_i}\right)^{-1}$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tau_\alpha^i\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-T}\tilde{F}_{i\alpha}^{h_i}\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1} + \left(\tau_i^\alpha\right)^{-1}I\right) < 0 \quad (15)$$

Left and right multiplying (15) respectively by $\left(\tilde{W}_i^{v_ih_ih_i}\right)^T$ and $\tilde{W}_i^{v_ih_ih_i}$, it yields, for $i = 1,...,n$:

$$\left(\tilde{W}_i^{v_ih_ih_i}\right)^T\left(\tilde{G}_i^{v_ih_i}\right)^T + \tilde{G}_i^{v_ih_i}\tilde{W}_i^{v_ih_ih_i} + \tilde{E}\left(\tilde{W}_i^{v_ih_ih_i}\right)^T\left(\dot{\tilde{W}}_i^{v_ih_ih_i}\right)^{-1}\tilde{W}_i^{v_ih_ih_i}$$
$$+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_\alpha^i\tilde{F}_{i\alpha}^{h_i}\left(\tilde{F}_{i\alpha}^{h_i}\right)^T + \left(\tilde{W}_i^{v_ih_ih_i}\right)^T\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tau_i^\alpha\right)^{-1}\tilde{W}_i^{v_ih_ih_i} < 0 \quad (16)$$

Note that $-\overbrace{\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}}^{\cdot} = \left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}\dot{\tilde{W}}_i^{v_ih_ih_i}\left(\tilde{W}_i^{v_ih_ih_i}\right)^{-1}$ [24], then applying the Schur complement, (16) can be rewritten as, for all $i = 1,...,n$:

$$\left(\begin{array}{c|cccccc} \psi_i^{v_ih_ih_i} & (*) & \cdots & \cdots & \cdots & \cdots & (*) \\ \hline \tilde{W}_i^{v_ih_ih_i} & -\tau_1^iI & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & & & \vdots \\ \vdots & \vdots & \ddots & -\tau_{i-1}^iI & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & -\tau_{i+1}^iI & \ddots & \vdots \\ \vdots & \vdots & & & \ddots & \ddots & 0 \\ \tilde{W}_i^{v_ih_ih_i} & 0 & \cdots & \cdots & \cdots & 0 & -\tau_{ni}I \end{array}\right) < 0 \quad (17)$$

with :

$$\psi_i^{v_ih_ih_i} = \left(\tilde{W}_i^{v_ih_ih_i}\right)^T\left(\tilde{G}_i^{v_ih_i}\right)^T + \tilde{G}_i^{v_ih_i}\tilde{W}_i^{v_ih_ih_i} - \tilde{E}\dot{\tilde{W}}_i^{v_ih_ih_i} + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_\alpha^i\tilde{F}_{i\alpha}^{h_i}\left(\tilde{F}_{i\alpha}^{h_i}\right)^T \quad (18)$$

Extending (18) with the matrices defined in (5) and (11), one obtains $\psi_i^{v_ih_ih_i}$ defined below. Note that (18) is not LMI. Thus, a solution is to set $W_{9i}^{h_ih_i} = 0$, $W_{10i}^{h_ih_i} = 0$ and $W_{12i}^{h_ih_i} = 0$ since they are free decision variables. Moreover, the non singularity of $W_{6i}^{h_ih_i}$, $W_{11i}^{v_ih_i}$ and $W_{16i}^{v_ih_i}$ have to be verified in simulation.

Let us now focus on the term $\dot{W}_{i1}^{vh}$ in $\psi_i^{v_ih_ih_i}$. One assumes that, for $i = 1,...,n$, $j = 1,...,l_i$, $s = 1,...,r_i$, ρ_i^j and ϖ_i^s are respectively the lower bound of $\dot{v}_i^j(z)$ and $\dot{h}_i^s(z)$.

$$\psi_i^{v_ih_ih_i} = \begin{bmatrix} \left(W_{5i}^{h_ih_i}\right)^T + W_{5i}^{h_ih_i} - \dot{W}_{1i}^{v_ih_i} & (*) & (*) & (*) \\ \begin{pmatrix}\left(W_{6i}^{h_ih_i}\right)^T + A_i^{h_i}W_{1i}^{v_ih_i} \\ -E_i^{v_i}W_{5i}^{h_ih_i} + B_i^{h_i}W_{13i}^{v_ih_i}\end{pmatrix} & \begin{pmatrix}-\left(W_{6i}^{h_ih_i}\right)^T\left(E_i^{v_i}\right)^T + \left(W_{14i}^{v_ih_i}\right)^T\left(B_i^{h_i}\right)^T \\ -E_i^{v_i}W_{6i}^{h_ih_i} + B_i^{h_i}W_{14i}^{v_ih_i} + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i\alpha}F_{i\alpha}^{h_i}\left(F_{i\alpha}^{h_i}\right)^T\end{pmatrix} & (*) & (*) \\ \begin{pmatrix}\left(W_{7i}^{h_ih_i}\right)^T + C_i^{h_i}W_{1i}^{v_ih_i} \\ -W_{9i}^{h_ih_i} + D_i^{h_i}W_{13i}^{v_ih_i}\end{pmatrix} & \begin{pmatrix}-\left(W_{7i}^{h_ih_i}\right)^T E_i^{v_i} + \left(W_{15i}^{v_ih_i}\right)^T\left(B_i^{h_i}\right)^T \\ -W_{10i}^{h_ih_i} + D_i^{h_i}W_{14i}^{v_ih_i}\end{pmatrix} & \begin{pmatrix}-\left(W_{11i}^{v_ih_i}\right)^T - W_{11i}^{v_ih_i} \\ +\left(W_{15i}^{v_ih_i}\right)^T\left(D_i^{h_i}\right)^T + D_i^{h_i}W_{15i}^{v_ih_i}\end{pmatrix} & (*) \\ \begin{pmatrix}\left(W_{8i}^{h_ih_i}\right)^T - W_{13i}^{v_ih_i} \\ +K_i^{v_ih_i}\left(W_{11i}^{v_ih_i}\right)^{-1}W_{9i}^{h_ih_i}\end{pmatrix} & \begin{pmatrix}-\left(W_{8i}^{h_ih_i}\right)^T\left(E_i^{v_i}\right)^T + \left(W_{16i}^{v_ih_i}\right)^T\left(B_i^{h_i}\right)^T \\ +K_i^{v_ih_i}\left(W_{11i}^{v_ih_i}\right)^{-1}W_{10i}^{h_ih_i} - W_{14i}^{v_ih_i}\end{pmatrix} & \begin{pmatrix}-\left(W_{12i}^{h_ih_i}\right)^T + \left(W_{16i}^{v_ih_i}\right)^T\left(D_i^{h_i}\right)^T \\ +K_i^{v_ih_i} - W_{15i}^{v_ih_i}\end{pmatrix} & \begin{pmatrix}\left(W_{12i}^{v_ih_i}\right)^T\left(W_{11i}^{v_ih_i}\right)^{-T}\left(K_i^{v_ih_i}\right)^T \\ +K_i^{h_iv_i}\left(W_{11i}^{v_ih_i}\right)^{-1}W_{12i}^{h_ih_i} \\ -\left(W_{16i}^{v_ih_i}\right)^T - W_{16i}^{v_ih_i}\end{pmatrix} \end{bmatrix}$$

From the convex sum propriety, one can write [28]:

$$-\dot{W}_{i1}^{v_i h_i} \le -\Phi_{i1}^{h_i v_i} \quad (19)$$

with

$$\Phi_{i1}^{h_i v_i} = \sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j h_i^s \left(\sum_{\xi=1}^{r_i-1} \varpi_i^s \left(W_{i1}^{j\xi} - W_{i1}^{jr_i} \right) + \sum_{\zeta=1}^{l_i-1} \rho_i^j \left(W_{i1}^{\zeta s} - W_{i1}^{l_i s} \right) \right),$$

$W_{i1}^{j\xi} - W_{i1}^{jr_i} > 0$ and $W_{i1}^{\zeta s} - W_{i1}^{l_i s} > 0$.

Therefore, from (19), if (7), (8) and (9) hold, (17) and so (12) are verified. That ends the proof. ■

Remark 1: For $i = 1,...,n$, $j = 1,...,l_i$, $k = 1,...,r_i$, $s = 1,...,r_i$, $v_i^j(z)$ and $h_i^s(z)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach [21] or, for instance, when membership functions are chosen with a smoothed shape.

Remark 2: In theorem 1 the lower bounds of the membership functions derivatives have to be a priori known. Nevertheless, as suggested in [5], these bounds can be arbitrarily fixed (negatives) in advance and then post verified in simulation. Therefore, a trial and error procedure, acceptable since it is made offline, gives a fine approximation of these bounds but leads to a local solution depending on the chosen initial state for the simulation. Another way to run to strict LMIs is to set W_{1i} common inside each LMIs (9), but, in this case, the price to pay is leaving the proposed multiple non quadratic framework to a more conservative quadratic one. Let us point out that the goal of this paper is not to give a solution to this point. Moreover, a recent study has dealt with this point [29]. Nevertheless, the latter remains to complex LMI conditions in the simplest case of standard T-S model local stability analysis. Therefore, it will necessitate further strong research effort before being suitable for the class of systems depicted by (1).

IV. Numerical Example

In this section, an academic example is provided to illustrate the efficiency of the proposed decentralized SOF controller design methodology. Let us consider the following set of T-S descriptors S composed of two subsystems S_1 and S_2 described respectively by:

$$\begin{cases} \sum_{j=1}^{2} v_1^j(y_1) E_1^j \dot{x}_1 = \sum_{k=1}^{2} h_1^k(y_1)\left(A_1^k x_1 + B_1^k u_1 + F_{12}^k x_2 \right) \\ y_1 = \sum_{k=1}^{2} h_1^k(y_1)\left(C_1^k x_1 + D_1^k u_1 \right) \end{cases} \quad (20)$$

with $E_1^1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $E_1^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $A_1^1 = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$, $A_1^2 = \begin{bmatrix} -3 & 0 \\ 1 & -3 \end{bmatrix}$, $B_1^1 = \begin{bmatrix} 1.2 \\ 1 \end{bmatrix}$, $B_1^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$,

$C_1^1 = \begin{bmatrix} 0.01 & -0.01 \end{bmatrix}$, $C_1^2 = \begin{bmatrix} 0.05 & 0.01 \end{bmatrix}$, $D_1^1 = 0.04$,

$D_1^2 = 0.06$, $F_{12}^1 = \begin{bmatrix} 0 & 0 \\ 0.3 & 0.01 \end{bmatrix}$, $F_{12}^2 = \begin{bmatrix} 0 & 0 \\ 0.02 & 0.01 \end{bmatrix}$,

$v_1^1(y_1) = \cos^2 y_1$, $v_1^2(y_1) = \sin^2 y_1$, $h_1^1(y_1) = \frac{1}{2}(1 + \sin y_1)$,

$h_1^2(y_1) = \frac{1}{2}(1 - \sin y_1)$.

$$\begin{cases} \sum_{j=1}^{2} v_2^j(y_2) E_2^j \dot{x}_2 = \sum_{k=1}^{2} h_2^k(y_2)\left(A_2^k x_2 + B_2^k u_2 + F_{21}^k x_1 \right) \\ y_2 = \sum_{k=1}^{2} h_2^k(y_2)\left(C_2^k x_2 + D_2^k u_2 \right) \end{cases} \quad (21)$$

with $E_2^1 = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}$, $E_2^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_2^1 = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}$,

$A_2^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B_2^1 = \begin{bmatrix} 0.47 \\ 1 \end{bmatrix}$, $B_2^2 = \begin{bmatrix} 0.47 \\ 0.8 \end{bmatrix}$,

$C_2^1 = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}$, $C_2^2 = \begin{bmatrix} 0.01 & 0.02 \end{bmatrix}$, $D_2^1 = 0.03$,

$D_2^2 = 0.02$, $F_{21}^1 = \begin{bmatrix} 0 & 0 \\ 0.03 & 0.01 \end{bmatrix}$, $F_{21}^2 = \begin{bmatrix} 0 & 0 \\ 0.05 & 0.01 \end{bmatrix}$,

$v_2^1(y_2) = \frac{1}{2}(1 + \sin y_2)$, $v_2^2(y_2) = \frac{1}{2}(1 - \sin y_2)$,

$h_2^1(y_2) = \cos^2 y_2$, $h_2^2(y_2) = \sin^2 y_2$.

Assume that the lower bound of the derivative membership functions are $\varpi_1^1 = \varpi_2^1 = \rho_1^1 = \rho_2^1 = -2$ (post verified in simulation). The Matlab LMI toolbox is used to solve the LMI conditions provided in theorem 1 leading to the synthesis of a set of two decentralized SOF controllers (2) with:

$K_1^{11} = \text{-}0.022$, $K_1^{12} = \text{-}0.0206$, $K_1^{21} = \text{-}0.0038$,
$K_1^{22} = 0.0046$, $K_2^{11} = \text{-}0.0191$, $K_2^{12} = \text{-}0.0188$,
$K_2^{21} = \text{-}0.0175$, $K_2^{22} = \text{-}0.0171$, $W_{111}^{11} = 0.6395$,
$W_{111}^{12} = 0.6396$, $W_{111}^{21} = 0.6398$, $W_{111}^{22} = 0.6398$,
$W_{112}^{11} = 0.6317$, $W_{112}^{12} = 0.6317$, $W_{112}^{21} = 0.6317$,
$W_{112}^{22} = 0.6317$, $\tau_2^1 = 2.7118$ and $\tau_1^2 = 3.091$

The closed-loop subsystem dynamics, the output signals, the membership function derivatives evolution and the control signals are presented in Fig.1 for initial states $x_1(0) = \begin{bmatrix} 3 & -3 \end{bmatrix}^T$ and $x_2(0) = \begin{bmatrix} 2 & -5 \end{bmatrix}^T$. As it is shown, the overall system is stabilized by the synthesized set of decentralized SOF controllers.

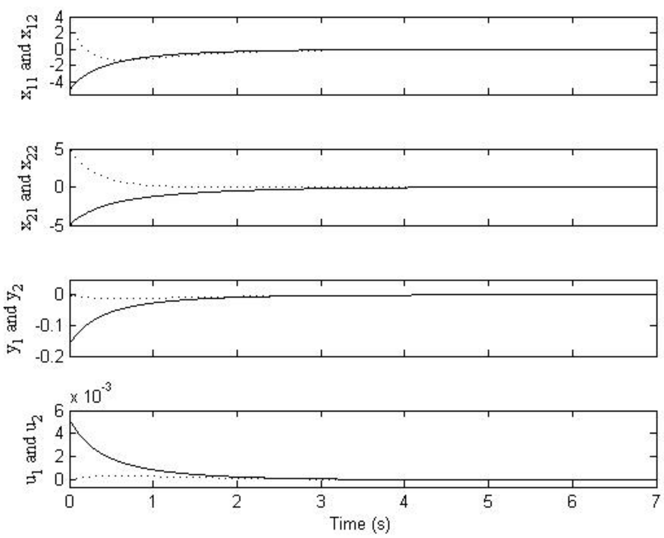

Fig.1. Simulated signals of the stabilized interconnected descriptors.

V. CONCLUSION

In this paper, the design of a decentralized non-PDC static output feedback controller has been proposed for stabilizing a network of n interconnected T-S descriptors. Based on a multiple fuzzy Lyapunov function, LMI based non quadratic stability conditions have been obtained thanks to the descriptor redundancy. Indeed, this property avoids appearance of crossing terms in the closed-loop dynamics formulation. Finally, to show the efficiency of the proposed control approach, a numerical example has been provided.

REFERENCES

[1] V.L. Syrmos, C.T. Abdallah, P. Dorato, K. Grigoriadis, “Static output feedback: a survey”, *Automatica* 33 (2) (1997) 125–137.

[2] M. Chadli, D. Maquin, J. Ragot “Static Output Feedback for Takagi-Sugeno systems: An Lmi Approach”, *Proceedings of the 10th Mediterranean Conference on Control and Automation - MED2002 Lisbon, Portugal,* July 9-12, 2002.

[3] Y. Chang, S-S. Chen, S-F. Su, T-T. Lee, “Static output feedback stabilization for nonlinear interval time-delay systems via fuzzy control approach”, *Fuzzy Sets and Systems* 148 (2004) 395–410, 2004.

[4] D. Huang, S. K. Nguang, “Static output feedback controller design for fuzzy systems: An ILMI approach”, *Information Sciences* 177 (2007) 3005–3015, 2007.

[5] T. Bouarar, K. Guelton, N. Manamanni, “Static output feedback controller design for Takagi-Sugeno Systems – A fuzzy Lyapunov LMI approach”, In Proc. Of the joint *48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, CDC-CCC’09, pp. 4150-4155, Shanghai, P.R. China, December 16-18, 2009.

[6] T. Takagi, M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” *IEEE Trans. Syst., Man, Cybern.*, Vol. 15, no. 1, 1985, pp. 116-132.

[7] K. Tanaka, H.O. Wang, “Fuzzy control systems design and analysis. A linear matrix inequality approach”, *Wiley*, New York, 2001.

[8] Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V., Linear Matrix Inequalities in System and Control Theory. Philadelphia, PA: *SIAM*, 1994.

[9] H.O. Wang, K. Tanaka, and M.F. Griffin, “An approach to fuzzy control of nonlinear systems: stability and the design issues,” *IEEE Trans. Fuzzy Systems.,* vol. 4, no. 1, pp. 14-23, 1996.

[10] J. Yoneyama, M. Nishikawa, H. Katayama, A. Ichikawa, “Output stabilization of Takagi–Sugeno fuzzy systems”, *Fuzzy Sets and Systems* 111 (2000) 253–266.

[11] J. Yoneyama, M. Nishikawa, H. Katayama, A. Ichikawa, “Design of output feedback controllers for Takagi–Sugeno fuzzy systems”, *Fuzzy Sets and Systems* 121 (2001) 127–148, 2001.

[12] B. Mansouri, N. Manamanni, K. Guelton, A. Kruszewski, T.M. Guerra “Output feedback LMI tracking control conditions with H∞ criterion for uncertain and disturbed T-S models”, *Information Sciences*, Vol 179, no. 4, pp. 446-457, 2009.

[13] J. Li, H. O. Wang, D. Niemann and K. Tanaka, “Dynamic parallel distributed compensation for Takagi-Sugeno fuzzy systems: An LMI approach”, *Information Sciences*, Vol. 123, no. 3-4, pp. 201-221, 2000.

[14] W. Assawinchaichote, S.K. Nguang and P. Shi, “output feedback control design for uncertain singularly perturbed systems: an LMI approach”, *Automatica*, Vol. 40, no. 12, pp. 2147-2152, 2004.

[15] S.K. Nguang, P. Shi, “Robust H∞ output feedback control design for fuzzy dynamic systems with quadratic stability constraints: An LMI approach”, *Information Sciences*, Vol. 176, no. 15, pp. 2161-2191, 2006.

[16] M. Zerar, K. Guelton, N. Manamanni, “Linear fractional transformation based H-infinity output stabilization for Takagi-Sugeno fuzzy models”, *Mediterranean Journal of Measurement and Control*, Vol. 4, no. 3, pp. 111-121, 2008.

[17] K. Guelton, T. Bouarar, N. Manamanni, “Robust dynamic output feedback fuzzy Lyapunov stabilization of Takagi-Sugeno systems - A descriptor redundancy approach”, Elsevier, *Fuzzy Sets and Systems*, 160(19):2796-2811, octobre 2009.

[18] A. Sala, “On the conservativeness of fuzzy and fuzzy-polynomial control of nonlinear systems”, *Annual Reviews in Control*, Vol. 33, no 1, pp. 48-58, 2009.

[19] T.M. Guerra, L. Vermeiren., “LMI based relaxed non quadratic stabilizations for non-linear systems in the Takagi-Sugeno’s form”, *Automatica*, Vol. 40, no. 5, pp. 823-829, 2004.

[20] K. Tanaka, T. Hori, and H.O. Wang, “A multiple Lyapunov function approach to stabilization of fuzzy control systems,” *IEEE Trans. Fuzzy Syst.,* vol. 11, no. 4, pp. 582-589, 2003.

[21] K. Tanaka, H. Ohtake, H.O. Wang, “A Descriptor System Approach to Fuzzy Control System Design via Fuzzy Lyapunov Functions”, *IEEE Trans Fuzzy Systems*, vol. 15, no. 3, pp.333- 341, 2007.

[22] W.W. Lin, W.J. Wang, S.H. Yang, Y.J. Chen, “Stabilization for Large-Scale Fuzzy Systems by Decentralized Fuzzy Control”, *IEEE International Conference on Fuzzy Systems*, 795 – 799, 2006.

[23] W-W. Lin, W-J. Wang, S-H. Yang, “A Novel Stabilization Criterion for Large-Scale T–S Fuzzy Systems”, *IEEE Transactions on Systems, Man, and Cybernetics—Part B: Cybernetics*, vol. 37, n°4, 1074 – 1079, 2007.

[24] W.J. Wang, W. Lin, “Decentralized PDC for Large Scale TS Fuzzy Systems”, *IEEE Transactions on Fuzzy Systems*, Vol. 8, No. 4, 2005.

[25] Y. Wang, Q.L. Zhang, “Robust Fuzzy Decentralized Control for Nonlinear Interconnected Descriptor Systems’, *10th IEEE conf of Fuzzy Systems*, vol. 3, 1392 - 1395 2001.

[26] D. Jabri, K. Guelton, N. Manamanni, M.N. Abdelkrim, “Fuzzy Lyapunov decentralized control of Takagi-Sugeno interconnected descriptors”, *In Proc. of CICA’2009, IEEE Symposium on Computational Intelligence in Control and Automation*, Nashville, Tennessee, USA, March 30 -April 2 2009, pp. 34 - 40.

[27] Zhou K., Khargonedkar P., “Robust Stabilization of linear systems with norm-bounded time-varying uncertainty”, *Sys. Control Letters*, 1988, vol. 10, pp. 17-20.

[28] T. Bouarar, K. Guelton, N. Manamanni, P. Billaudel, “Stabilization of uncertain Takagi-Sugeno descriptors: a fuzzy Lyapunov approach”, *16th Mediterranean Conference on Control and Automation (MED'08)*. IEEE, Ajaccio, Corsica, France, June 2008.

[29] T.M. Guerra, M. Bernal, A. Jaadari, K. Guelton, “Stabilisation non quadratique locale pour des modèles continus de type Takagi-Sugeno”, In Proc. Sixième Conférence Internationale Francophone d’Automatique, Nancy, June 2-4 2010.