H-infinity decentralized Static Output Feedback controller design for large scale Takagi-Sugeno systems

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Abstract—**This paper deals with large scale Takagi-Sugeno (T-S) systems stabilization based on a decentralized Static Output Feedback (SOF) non-PDC control scheme. To do so, the overall closed loop dynamics is written using a descriptor redundancy formulation. The latter allows avoiding crossing terms between the controller's and the system's matrices. Thus, based on a multiple Fuzzy Lyapunov candidate function and a H-infinity criterion, employed to minimize the effects of the interconnections between subsystems, a LMI based design methodology is proposed. Finally, an academic example illustrates the efficiency of the proposed approach.**

I. Introduction

Due to the growing complexity of dynamical systems many works have dealt with nonlinear control in the past few decades. Among these works, Takagi-Sugeno (T-S) fuzzy systems have shown their significance since they allow representing non linear systems by a set of linear ones blended together by nonlinear functions [1]. Therefore, such polytopic modeling of nonlinear systems is useful since it allows extending some of the linear control concepts to the nonlinear framework [2]. To do so, Parallel Distributed Compensation (PDC) controllers design has been firstly studied using quadratic Lyapunov functions leading to a set of Linear Matrix Inequalities (LMI) to be satisfied, see e.g. [2][3][4] and references therein. It is well-known that LMI based PDC controller design suffers from conservativeness, see [5] for an exhaustive review of conservatism sources. Therefore, non-PDC based controllers have been proposed for T-S system's stabilization through nonquadratic / Fuzzy Lyapunov based approaches [6][7][8][9].

Another lock of complex nonlinear systems analysis may be due, when this is the case, to their high dimension. Thus, recent works have dealt with fuzzy decentralized state feedback controllers design for large scale nonlinear systems [10][11][12][13]. Indeed, a set of interconnected Takagi-Sugeno systems can be considered to represent a large scale dynamics. In this context, most of the proposed results have been done in the quadratic framework and, to the best of authors' knowledge, only few recent results have been proposed using nonquadratic Lyapunov functions [14][15]. Independently to the case of large scale systems, many works have dealt with the synthesis of observer based output feedback fuzzy controllers [16][17][18][19][20], Dynamic Output Feedback Controllers (DOFC) [21][22][23][24] or Static Output Feedback Controllers (SOFC) [25][26][27]. Among them, SOFC are of some interests for practical applications [28]. Indeed, they don't need any online differential equation solving and so reduce the online computational cost. In most of the cases, output feedback controller design has been considered in the quadratic case and the results are often provided in terms of bilinear matrix inequalities instead of LMI. More recently, nonquadratic LMI conditions have been proposed using a descriptor redundancy approach for the synthesis of DOFC [29] and SOFC [30].

In this study, our aim is to provide a non quadratic decentralized SOFC design methodology for large scale / interconnected T-S fuzzy systems. As suggested in [15], a H_∞ criterion is employed to minimize the effects of the interconnections between subsystems. The next section will depict the studied class of large scale T-S fuzzy systems and the considered decentralized SOFC design problem. Then, in section III, with the aid of the descriptor redundancy property, fuzzy Lyapunov based LMI conditions will be derived. Finally, in section IV, a numerical example will illustrate the efficiency of the proposed approach.

II. Interconnected T-S Fuzzy Systems and Decentralized SOF Controllers

Let's consider the class of nonlinear systems S composed of n interconnected T-S fuzzy subsystems S_i described as follows:

for $i = 1,...,n$,

$$\begin{cases} \dot{x}_i(t) = \sum_{j=1}^{r_i} h_i^j(z_i(t)) \left(A_i^j x_i(t) + B_i^j u_i(t) + \sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} F_{i\alpha}^j x_\alpha(t) \right) \\ y_i(t) = \sum_{j=1}^{r_i} h_i^j(z_i(t)) \left(C_i^j x_i(t) + D_i^j u_i(t) \right) \end{cases} \quad (1)$$

This work was supported by the French Ministry of Research, the CNOUS, the French Embassy in Tunisia, the GIS 3SGS within the framework of the project COSMOS2 and the RCA within the CPER MOSYP.

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where $x_i(t) \in \mathbb{R}^{n_i}$ is the i^{th} state vector, $u_i(t) \in \mathbb{R}^{m_i}$ is the i^{th} control signal, $z_i(t) \in \mathbb{R}^{p_i}$ is the i^{th} premise vector, $x_\alpha(t) \in \mathbb{R}^{n_\alpha}$ is the state vector of the α^{th} model with $\alpha = 1,...,n$ and $\alpha \neq i$, r_i is the S_i's number of fuzzy rules. $A_i^j \in \mathbb{R}^{n_i \times n_i}$, $B_i^j \in \mathbb{R}^{n_i \times m_i}$, $C_i^j \in \mathbb{R}^{q_i \times n_i}$ and $D_i^j \in \mathbb{R}^{q_i \times m_i}$ are constant matrices constituting the i^{th} T-S fuzzy subsystem and $F_{i\alpha}^j \in \mathbb{R}^{n_i \times n_\alpha}$ denotes the influence of the α^{th} subsystem on the i^{th} one. $h_i^j(z_i(t)) \geq 0$ are the S_i's membership functions verifying the convex sums propriety $\sum_{k=1}^{r_i} h_i^k(z_i(t)) = 1$.

To ensure the stabilization of the overall system S, a decentralized SOF controller, based on the well-known non-Parallel Distributed Compensation (non-PDC) scheme, is proposed. The basic idea is to synthesize a decentralized controller composed of n local SOF controllers. Each i^{th} local SOF controller must be able to guarantee the stability of the subsystem S_i while considering interconnections among the others subsystems. For more convenience, the local non-PDC SOF controller shares the same fuzzy sets with the subsystem S_i. This set of decentralized non-PDC SOF controllers is given by,

For $i = 1,...,n$:

$$u_i(t) = \left(\sum_{j=1}^{r_i} h_i^j(z_i(t)) K_i^j\right)\left(\sum_{j=1}^{r_i} h_i^j(z_i(t)) X_{5i}^j\right)^{-1} y_i(t) \qquad (2)$$

where K_i^j are non-PDC gain matrices and $X_{5i}^j > 0$ are Lyapunov dependant gain matrices to be synthesized.

Notations:

For space convenience, in the sequel, the time t as well as the premises $z_i(t)$ will be omitted when there is no ambiguity. Moreover, to clarify the mathematical expression, the notations $Y_i^{h_i} = \sum_{j=1}^{r_i} h_i^j Y_i^j$ and $Y_i^{h_i h_i} = \sum_{j=1}^{r_i}\sum_{k=1}^{r_i} h_i^j(z(t)) h_i^k(z(t)) Y_i^{jk}$ will be used and, as usual, a star (*) in a matrix indicates a transpose quantity.

To take advantage of a descriptor redundancy formulation [29][30] in the case of decentralized SOFC design, (1) and (2) can be easily rewritten with the above defined notations respectively as:

For $i = 1,...,n$:

$$\begin{cases} \dot{x}_i(t) = A_i^{h_i} x_i(t) + B_i^{h_i} u_i(t) + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} F_{i\alpha}^{h_i} x_\alpha(t) \\ 0 = -y_i(t) + C_i^{h_i} x_i(t) + D_i^{h_i} u_i(t) \end{cases} \qquad (3)$$

and

For $i = 1,...,n$:

$$0 = -u_i(t) + \left(K_i^{h_i}\right)\left(X_{5i}^{h_i}\right)^{-1} y_i(t) \qquad (4)$$

Therefore, by considering the extended state vector $\tilde{x}_i = \begin{bmatrix} x_i^T & y_i^T & u_i^T \end{bmatrix}^T$, and substituting (2) into (1), the overall closed-loop system S_{cl} can be described by the following descriptor:

For all $i = 1,...,n$,

$$\tilde{E}_i \dot{\tilde{x}}_i = \tilde{G}_i^{h_i} \tilde{x}_i + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \tilde{F}_{i\alpha}^{h_i} x_\alpha \qquad (5)$$

with $\tilde{E}_i = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\tilde{G}_i^{h_i} = \begin{bmatrix} A_i^{h_i} & 0 & B_i^{h_i} \\ C_i^{h_i} & -I & D_i^{h_i} \\ 0 & K_i^{h_i}\left(X_{5i}^{h_i}\right)^{-1} & -I \end{bmatrix}$ and $\tilde{F}_{i\alpha}^{h_i} = \begin{bmatrix} F_{i\alpha}^{h_i} \\ 0 \\ 0 \end{bmatrix}$.

The objective is now to propose a convenient decentralized SOF controller design methodology which ensure the stability of (5). Note that each subsystem i is influenced by the other subsystems $\alpha = 1,...,n$, $\alpha \neq i$. In this study, one proposes the design of the controller (2) based on the minimization of a H_∞ performance related to attenuate exotic influences to each considered state x_i, $i = 1,...,n$.

III. Fuzzy Lyapunov Design of Decentralized SOF Controllers

In this section, the main purpose is to design a decentralized controller ensuring the stability of the whole set of interconnected closed-loop T-S fuzzy systems (5) and minimizing the effect of the interactions between subsystems using the following H_∞ criterion:

$$\int_{t_0}^{t_f} x_i^T x_i dt < \rho_i^2 \int_{t_0}^{t_f} \left(\sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \tilde{F}_{i\alpha}^{h_i} x_\alpha\right)^T \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \tilde{F}_{i\alpha}^{h_i} x_\alpha dt \qquad (6)$$

where ρ_i are the $H\infty$ performances level.

Many relaxation schemes have been proposed to relax LMI conditions for T-S fuzzy models analysis [5]. Among them, the following one, proposed by Tuan et al., will be used in the sequel since it leads to a good compromise between conservatism and computational cost.

Lemma 1 [31] **:**

The inequality $\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\Gamma_{ij} < 0$ *is verified if, for all* $i=1,...,r$, $j=1,...,r$, $j \neq i$, $\Gamma_{ii}<0$ *and* $\frac{1}{r-1}\Gamma_{ii}+\frac{1}{2}\left(\Gamma_{ij}+\Gamma_{ji}\right)<0$.

The following theorem provides sufficient conditions for the existence of a decentralized controller minimizing the H_∞ criterion (6).

Theorem 1: *Assume that, for* $i=1,...,n$, $s=1,...,r_i$, $\dot{h}_i^s\left(z(t)\right)\geq\theta_i^s$. *The system* S *composed of n interconnected T-S fuzzy systems* S_i *(1) is stabilized by the network of n non-PDC decentralized control laws (2), guaranteeing the* H_∞ *performances* $\left[\rho_i=\sqrt{\lambda_i}\right]_{i=1}^{n}$, *if there exist, for all combination of* $i=1,...,n$, $\alpha=1,...,n$, $\alpha\neq i$, $j=1,...,r_i$, $k=1,...,r_i$, *the matrices* $X_{i1}^k=\left(X_{i1}^k\right)^T>0$, $X_{i5}^k>0$, X_{i7}^k, X_{i8}^k, X_{i9}^k, K_i^j *and the scalars* $\lambda_i>0$ *such that the following LMIs are satisfied:*

Maximize λ_i *such that :*

- $\Gamma_{i\alpha}^{jj}<0$ (7)

- $\frac{1}{r-1}\Gamma_{i\alpha}^{jj}+\frac{1}{2}\left(\Gamma_{i\alpha}^{jk}+\Gamma_{i\alpha}^{kj}\right)<0$, $j\neq i$ (8)

- $X_{i1}^s-X_{i1}^{r_i}>0$, $s=1,...,r_i-1$ (9)

with $\Gamma_{i\alpha}^{jk}$ *defined below and* $\eta=\frac{1}{(n-1)}$.

Proof:

First of all, from (6) and since $X^TY^T+YX\leq X^TX+Y^TY$, one can write:

$$\int_{t_0}^{t_f}\tilde{x}_i^T\tilde{E}_i\tilde{x}_i dt$$

$$<\rho_i^2\int_{t_0}^{t_f}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left((n-1)\left(x_\alpha^T\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}x_\alpha\right)+\sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\left(x_\beta^T\left(\tilde{F}_{i\beta}^{h_i}\right)^T\tilde{F}_{i\beta}^{h_i}x_\beta\right)\right)dt \tag{10}$$

Note that $\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\Psi_{i\alpha}+\sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\Psi_{i\beta}\right)=(n-1)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Psi_{i\alpha}$. Thus, (10) can be rewritten as:

$$\int_{t_0}^{t_f}\tilde{x}_i^T\tilde{E}_i\tilde{x}_i dt<(2n-3)\rho_i^2\int_{t_0}^{t_f}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(x_\alpha^T\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}x_\alpha\right)dt \tag{11}$$

Now, let the candidate multiple fuzzy Lyapunov function be:

$$V(t)=\sum_{i=1}^{n}V_i\left(x_i(t)\right)\geq 0 \tag{12}$$

with $V_i\left(x_i(t)\right)=\tilde{x}_i^T(t)\tilde{E}\left(\tilde{X}_i^{h_i}\right)^{-1}\tilde{x}_i(t)$.

$$\Gamma_{i\alpha}^{jk}=\begin{bmatrix}\eta\begin{pmatrix}\left(X_{1i}^k\right)^T\left(A_i^j\right)^T+A_i^jX_{1i}^k\\ +\left(X_{7i}^k\right)^T\left(B_i^j\right)^T+B_i^jX_{7i}^k\\ -\sum_{s=1}^{r_i-1}\theta_i^s\left(X_{i1}^s-X_{i1}^{r_i}\right)\end{pmatrix} & (*) & (*) & (*) & (*)\\ \eta\begin{pmatrix}\left(X_{8i}^k\right)^T\left(B_i^j\right)^T\\ +C_i^jX_{1i}^k+D_i^jX_{7i}^k\end{pmatrix} & \eta\begin{pmatrix}-\left(X_{5i}^k\right)^T-X_{5i}^k\\ +\left(X_{8i}^k\right)^T\left(D_i^j\right)^T+D_i^jX_{8i}^k\end{pmatrix} & (*) & 0 & 0\\ \eta\left(\left(X_{9i}^k\right)^T\left(B_i^j\right)^T-X_{7i}^k\right) & \eta\left(\left(X_{9i}^k\right)^T\left(D_i^j\right)^T+K_i^j-X_{8i}^k\right) & -\eta\left(\left(X_{9i}^k\right)^T+X_{9i}^k\right) & 0 & 0\\ \left(F_{i\alpha}^j\right)^T & 0 & 0 & -(2n-3)\lambda_i\left(F_{i\alpha}^j\right)^TF_{i\alpha}^j & 0\\ X_{1i}^k & 0 & 0 & 0 & -(n-1)I\end{bmatrix}$$

As usual, (12) needs $\tilde{E}\left(\tilde{X}_i^{h_i}\right)^{-1}=\left(\tilde{X}_i^{h_i}\right)^{-T}\tilde{E}\geq 0$. This obviously leads to condition the Lyapunov matrix such that $\tilde{X}_i^{h_i}=\begin{bmatrix} X_{1i}^{h_i} & 0 & 0 \\ X_{4i}^{h_i} & X_{5i}^{h_i} & X_{6i}^{h_i} \\ X_{7i}^{h_i} & X_{8i}^{h_i} & X_{9i}^{h_i}\end{bmatrix}$ with $X_{1i}^{h_i}=\left(X_{1i}^{h_i}\right)^T>0$.

From (12), the closed loop system (5) is stable under the criterion (11) if:

$$\sum_{i=1}^{n}\left(\dot{V}_i(t)+\tilde{x}_i^T\tilde{E}_i\tilde{x}_i-(2n-3)\rho_i^2\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}x_\alpha^T\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}x_\alpha\right)<0 \quad (13)$$

That is to say if:

$$\sum_{i=1}^{n}\left(\dot{\tilde{x}}_i^T\tilde{E}_i\left(\tilde{X}_i^{h_i}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\tilde{E}_i\left(\tilde{X}_i^{h_i}\right)^{-1}\dot{\tilde{x}}_i+\tilde{x}_i^T\tilde{E}_i\overset{\bullet}{\overline{\left(\tilde{X}_i^{h_i}\right)^{-1}}}\tilde{x}_i \right. \\ \left. +\tilde{x}_i^T\tilde{E}_i\tilde{x}_i-(2n-3)\rho_i^2\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}x_\alpha^T\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}x_\alpha\right)<0 \quad (14)$$

This can be rewritten as:

$$\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{bmatrix}\tilde{x}_i\\ x_\alpha\end{bmatrix}^T\begin{bmatrix}\Omega_i^{h_ih_i} & (*)\\ \left(\tilde{F}_{i\alpha}^{h_i}\right)^T\left(\tilde{X}_i^{h_i}\right)^{-1} & -(2n-3)\rho_i^2\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}\end{bmatrix}\begin{bmatrix}\tilde{x}_i\\ x_\alpha\end{bmatrix}<0 \quad (15)$$

where

$$\Omega_i^{h_ih_i}=\frac{1}{(n-1)}\left(\left(\tilde{G}_i^{h_i}\right)^T\left(\tilde{X}_i^{h_i}\right)^{-1}+\left(\tilde{X}_i^{h_i}\right)^{-T}\tilde{G}_i^{h_i}+\tilde{Q}_i+\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{h_i}\right)^{-1}}}\right)$$

Which is verified if:

For all $i=1,...,n$, $\alpha=1,...,n$, $\alpha\neq i$,

$$\begin{bmatrix}\Omega_i^{h_ih_i} & (*)\\ \left(\tilde{F}_{i\alpha}^{h_i}\right)^T\left(\tilde{X}_i^{h_i}\right)^{-1} & -(2n-3)\rho_i^2\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}\end{bmatrix}<0 \quad (16)$$

Multiplying (16) left and right respectively by $\begin{bmatrix}\left(\tilde{X}_i^{h_i}\right)^T & 0\\ 0 & I\end{bmatrix}$ and $\begin{bmatrix}\tilde{X}_i^{h_i} & 0\\ 0 & I\end{bmatrix}$, and since $-\overset{\bullet}{\overline{\left(\tilde{X}_i^{h_i}\right)^{-1}}}=\left(\tilde{X}_i^{h_i}\right)^{-1}\dot{\tilde{X}}_i^{h_i}\left(\tilde{X}_i^{h_i}\right)^{-1}$, see e.g. [14], it yields:

For all $i=1,...,n$, $\alpha=1,...,n$, $\alpha\neq i$,

$$\begin{bmatrix}\bar{\Omega}_{i\alpha}^{h_ih_i} & (*)\\ \left(\tilde{F}_{i\alpha}^{h_i}\right)^T & -(2n-3)\rho_i^2\left(\tilde{F}_{i\alpha}^{h_i}\right)^T\tilde{F}_{i\alpha}^{h_i}\end{bmatrix}<0 \quad (17)$$

with

$$\bar{\Omega}_i^{h_ih_i}=\frac{1}{(n-1)}\left(\left(\tilde{X}_i^{h_i}\right)^T\left(\tilde{G}_i^{h_i}\right)^T+\tilde{G}_i^{h_i}\tilde{X}_i^{h_i}+\left(\tilde{X}_i^{h_i}\right)^T\tilde{Q}_i\tilde{X}_i^{h_i}-\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{h_i}\right)}}\right)$$

Now, extending (17) with the matrices defined in (5), one has:

$$\bar{\Omega}_i^{h_ih_i}=\frac{1}{(n-1)}\begin{bmatrix}\bar{\Omega}_{i(1,1)}^{h_ih_i} & (*) & (*)\\ \bar{\Omega}_{i(2,1)}^{h_ih_i} & \bar{\Omega}_{i(2,2)}^{h_ih_i} & (*)\\ \bar{\Omega}_{i(3,1)}^{h_ih_i} & \bar{\Omega}_{i(3,2)}^{h_ih_i} & \bar{\Omega}_{i(3,3)}^{h_ih_i}\end{bmatrix} \quad (18)$$

with:

$$\bar{\Omega}_{i(1,1)}^{h_ih_i}=\left(\left(X_{1i}^{h_i}\right)^T\left(A_i^{h_i}\right)^T+A_i^{h_i}X_{1i}^{h_i}+\left(X_{7i}^{h_i}\right)^T\left(B_i^{h_i}\right)^T+B_i^{h_i}X_{7i}^{h_i}+\left(X_{1i}^{h_i}\right)^TX_{1i}^{h_i}-\dot{X}_{1i}^{h_i}\right.,$$

$$\bar{\Omega}_{i(2,1)}^{h_ih_i}=\left(X_{8i}^{h_i}\right)^T\left(B_i^{h_i}\right)^T+C_i^{h_i}X_{1i}^{h_i}-X_{4i}^{h_i}+D_i^{h_i}X_{7i}^{h_i},$$

$$\bar{\Omega}_{i(3,1)}^{h_ih_i}=\left(X_{9i}^{h_i}\right)^T\left(B_i^{h_i}\right)^T+K_i^{h_i}\left(X_{\beta i}^{h_i}\right)^{-1}X_{4i}^{h_i}-X_{7i}^{h_i},$$

$$\bar{\Omega}_{i(2,2)}^{h_ih_i}=-\left(X_{5i}^{h_i}\right)^T-X_{5i}^{h_i}+\left(X_{8i}^{h_i}\right)^T\left(D_i^{h_i}\right)^T+D_i^{h_i}X_{8i}^{h_i},$$

$$\bar{\Omega}_{i(3,2)}^{h_ih_i}=-\left(X_{6i}^{h_i}\right)^T+\left(X_{9i}^{h_i}\right)^T\left(D_i^{h_i}\right)^T+K_i^{h_i}\left(X_{\beta i}^{h_i}\right)^{-1}X_{5i}^{h_i}-X_{8i}^{h_i},$$

$$\bar{\Omega}_{i(3,3)}^{h_ih_i}=\left(X_{6i}^{h_i}\right)^T\left(X_{\beta i}^{h_i}\right)^{-T}\left(K_i^{h_i}\right)^T+K_i^{h_i}\left(X_{\beta i}^{h_i}\right)^{-1}X_{6i}^{h_i}-\left(X_{9i}^{h_i}\right)^T-X_{9i}^{h_i}$$

From the convex sum propriety, one has $\overset{\bullet}{\overline{\left(X_{1i}^{h_i}\right)}}=\sum_{s=1}^{r_i-1}h_i^s\left(X_{i1}^s-X_{i1}^{r_i}\right)$. Let, for $s=1,...,r_i$, θ_i^s be the lower bounds of $\dot{h}_i^s(z)$, one can write [6]:

$$-\overset{\bullet}{\overline{\left(X_{1i}^{h_i}\right)}}\leq-\sum_{s=1}^{r_i-1}\theta_i^s\left(X_{i1}^s-X_{i1}^{r_i}\right) \quad (19)$$

Now, from (18), for LMI purpose, one chooses $X_{\beta i}^{h_i}=X_{5i}^{h_i}$, $X_{4i}^{h_i}=0$ and $X_{6i}^{h_i}=0$. Then, considering (19) and applying the Schur complement, (17) is satisfied if:

For all $i=1,...,n$, $\alpha=1,...,n$, $\alpha\neq i$,

$$\Gamma_{i\alpha}^{h_ih_i}=\left[\begin{array}{ccc|cc}\Psi_i^{h_ih_i} & & & (*) & (*)\\ \hline \left(F_{i\alpha}^{h_i}\right)^T & 0 & 0 & -(2n-3)\rho_i^2\left(F_{i\alpha}^{h_i}\right)^TF_{i\alpha}^{h_i} & 0\\ X_{1i}^{h_i} & 0 & 0 & 0 & -(n-1)I\end{array}\right]<0 \quad (20)$$

with $\Psi_i^{h_ih_i}=\frac{1}{(n-1)}\begin{bmatrix}\Psi_{i(1,1)}^{h_ih_i} & (*) & (*)\\ \Psi_{i(2,1)}^{h_ih_i} & \Psi_{i(2,2)}^{h_ih_i} & (*)\\ \Psi_{i(3,1)}^{h_ih_i} & \Psi_{i(3,2)}^{h_ih_i} & \Psi_{i(3,3)}^{h_ih_i}\end{bmatrix}$,

$$\Psi_{i(1,1)}^{h_ih_i}=\left(X_{1i}^{h_i}\right)^T\left(A_i^{h_i}\right)^T+A_i^{h_i}X_{1i}^{h_i}+\left(X_{7i}^{h_i}\right)^T\left(B_i^{h_i}\right)^T+B_i^{h_i}X_{7i}^{h_i}-\Phi_i,$$

$$\Psi_{i(2,1)}^{h_ih_i}=\left(X_{8i}^{h_i}\right)^T\left(B_i^{h_i}\right)^T+C_i^{h_i}X_{1i}^{h_i}+D_i^{h_i}X_{7i}^{h_i},$$

$$\Psi_{i(2,2)}^{h_i h_i} = -\left(X_{5i}^{h_i}\right)^T - X_{5i}^{h_i} + \left(X_{8i}^{h_i}\right)^T \left(D_i^{h_i}\right)^T + D_i^{h_i} X_{8i}^{h_i},$$
$$\Psi_{i(3,1)}^{h_i h_i} = \left(X_{9i}^{h_i}\right)^T \left(B_i^{h_i}\right)^T - X_{7i}^{h_i},$$
$$\Psi_{i(3,2)}^{h_i h_i} = \left(X_{9i}^{h_i}\right)^T \left(D_i^{h_i}\right)^T + K_i^{h_i} - X_{8i}^{h_i}, \quad \Psi_{i(3,3)}^{h_i} = -\left(X_{9i}^{h_i}\right)^T - X_{9i}^{h_i}$$
and $\Phi_i = \sum_{s=1}^{r_i - 1} \theta_i^s \left(X_{i1}^s - X_{i1}^{r_i}\right)$.

Inequality (20) is obviously satisfied if (8) holds. The proof is completed ■

Remark 1: For $i = 1,...,n$, $j = 1,...,r_i$, $h_i^k(z)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach [2] or, for instance, when membership functions are chosen with a smoothed shape (Gaussian...).

Remark 2: The LMI conditions proposed in theorems 1 are depending on the lower bounds of the membership functions derivatives $\theta_i^k \le \dot{h}_i^k(z)$ for $i = 1,...,n$ and $k = 1,...,r_i - 1$. Therefore these bounds have to be priory known for the conditions (7) and (8) to be LMI. Note that it is often pointed out as a criticism to fuzzy Lyapunov approach since these parameters may be difficult to choose in advance. Nevertheless, as suggested in [30], these bounds can be arbitrarily fixed (negatives) in advance and then post verified in simulation. Therefore a trial and error procedure, acceptable since it is made offline, gives a fine approximation of these bounds but inevitably leads to a local solution depending on the chosen initial state for the simulation. Note finally that another way to run to strict LMIs is to set X_{i1} common inside each subsystems $i = 1,...,n$, but, in this case, the price to pay is leaving the proposed multiple non quadratic framework to a multiple quadratic one.

IV. EXAMPLE AND SIMULATION

In order to illustrate the approach developed above, let us consider the following set of interconnected T-S systems S composed of two subsystems S_1 and S_2 described respectively by:

$$\begin{cases} \dot{x}_1(t) = \sum_{j=1}^{2} h_1^j\left(x_1(t)\right)\left(A_1^j x_1(t) + B_1^j u_1(t) + F_{12}^j x_2(t)\right) \\ y_1(t) = \sum_{j=1}^{2} h_1^j\left(z_1(t)\right)\left(C_1^j x_1(t) + D_1^j u_1(t)\right) \end{cases} \quad (21)$$

with $A_1^1 = \begin{bmatrix} -6 & 6 & 0 \\ 0.5 & -3 & 1 \\ 0 & 0.2 & -1 \end{bmatrix}$, $A_1^2 = \begin{bmatrix} -1 & 0.1 & 0 \\ -0.2 & -2 & 0 \\ 0.3 & 0 & -1 \end{bmatrix}$,

$B_1^1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B_1^2 = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$, $C_1^1 = \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}$,

$C_1^2 = \begin{bmatrix} -0.5 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}$, $D_1^1 = \begin{bmatrix} 0.5 & 0.1 \\ 0.3 & -0.1 \end{bmatrix}$,

$D_1^2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}$, $F_{12}^1 = \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.4 \\ 0.01 & 0.1 \end{bmatrix}$, $F_{12}^2 = \begin{bmatrix} 0.04 & 0.02 \\ 0.01 & 0.01 \\ 0.03 & 0.02 \end{bmatrix}$,

$h_1^1\left(x_1(t)\right) = \sin^2\left(x_{11}(t)\right)$, $h_1^2\left(x_1(t)\right) = \cos^2\left(x_{11}(t)\right)$.

and

$$\begin{cases} \dot{x}_2(t) = \sum_{k=1}^{2} h_2^k\left(x_2(t)\right)\left(A_2^k x_2(t) + B_2^k u_2(t) + F_{21}^k x_1(t)\right) \\ y_2(t) = \sum_{j=1}^{2} h_2^j\left(z_2(t)\right)\left(C_2^j x_2(t) + D_2^j u_2(t)\right) \end{cases} \quad (22)$$

with $A_2^1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $A_2^2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$, $B_2^1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $B_2^2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$,

$C_2^1 = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix}$, $C_2^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1.01 \end{bmatrix}$, $D_2^1 = \begin{bmatrix} 0.01 & 0.5 \end{bmatrix}$,

$D_2^2 = \begin{bmatrix} 0.01 & 0.2 \end{bmatrix}$, $F_{21}^1 = \begin{bmatrix} 0.01 & 0.01 & 0.01 \\ 0.02 & 0.01 & 0.1 \end{bmatrix}$,

$F_{21}^2 = \begin{bmatrix} 0.02 & 0.01 & 0.02 \\ 0.03 & 1 & 0.01 \end{bmatrix}$, $h_2^1\left(x_2(t)\right) = \sin^2\left(x_{21}(t)\right)$,

$h_2^2\left(x_2(t)\right) = \cos^2\left(x_{21}(t)\right)$.

A set of decentralized non-PDC SOFC (2) can be synthesized using theorem 1. Assuming that $\theta_1^1 = -1$ and $\theta_2^1 = -1.5$, the Matlab LMI toolbox is used to solve the LMI conditions (7), (8) and (9). The result leads to the following gain matrices:

$K_1^1 = \begin{bmatrix} -0.44 & -0.09 \\ 0.19 & -0.02 \end{bmatrix}$, $K_1^2 = \begin{bmatrix} -0.22 & -0.04 \\ 0.15 & -0.02 \end{bmatrix}$,

$K_2^1 = \begin{bmatrix} -0.28 & -0.17 \end{bmatrix}$, $K_2^2 = \begin{bmatrix} -0.1 & -0.47 \end{bmatrix}$,

$X_{51}^1 = \begin{bmatrix} 0.46 & 0.39 \\ -0.44 & 0.54 \end{bmatrix}$, $X_{51}^2 = \begin{bmatrix} 0.54 & -0.04 \\ 0.02 & 0.54 \end{bmatrix}$,

$X_{52}^1 = \begin{bmatrix} 0.58 & 3.5 \\ -3.74 & 0.68 \end{bmatrix}$, $X_{52}^2 = \begin{bmatrix} 0.59 & 1.58 \\ -1.7 & 0.55 \end{bmatrix}$

and H_∞ performances given by the scalars $\rho_1 = 12.92$ and $\rho_2 = 5.39$.

The close-loop subsystem dynamics and the output signals are given respectively in Fig.1 and Fig.2 for initial states $x_1(0) = [1 \quad 0.5 \quad -1]^T$ and $x_2(0) = [2 \quad -2]^T$. As it is shown, the overall closed-loop dynamics is stable.

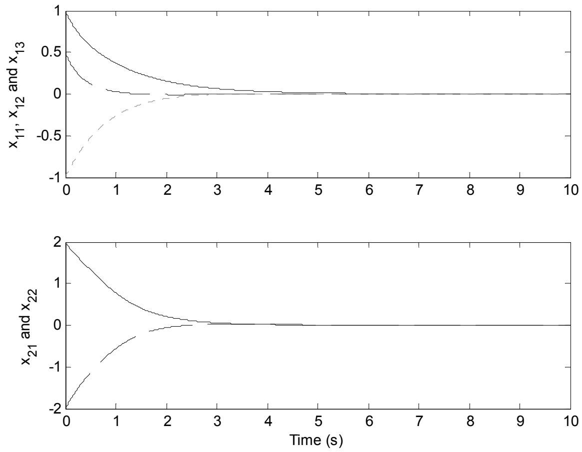

Fig.1. State's dynamics,
(x_{11} , x_{21}) solid lines, (x_{12} , x_{13} , x_{22})dashed lines.

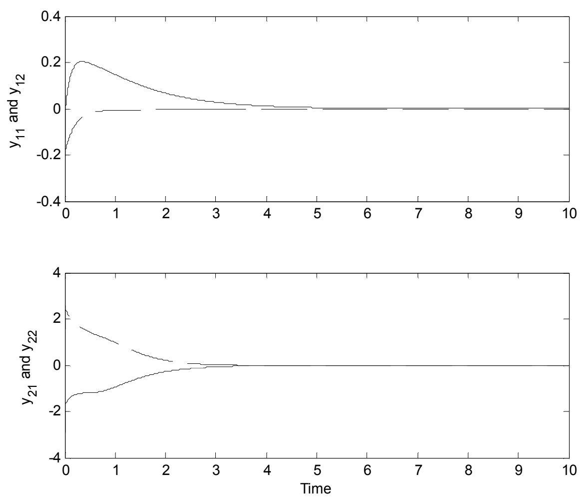

Fig.2: Output signals,
(y_{11} , y_{21}) dashed lines, (y_{12} , y_{22}) solid lines.

Fig. 3 provides the time evolution of the membership functions derivatives $\dot{h}_1^1(z(t))$ and $\dot{h}_2^1(z(t))$. This shows that the assumed lower bounds are verified in simulation for the considered initial conditions.

V. Conclusion

In this paper, the problem of decentralized static output feedback stabilization of large scale / interconnected Takagi-Sugeno models has been considered. A set of decentralized non PDC static output feedback control law has been proposed and its design has been involved through a multiple fuzzy Lyapunov approach. Thanks to the descriptor redundancy, LMI conditions have been easily obtained. This approach leads to less conservative results and is valuable for minimizing H_∞ performances of interaction effects between interconnected subsystems. Finally, an academic example has illustrated the efficiency of the proposed approach.

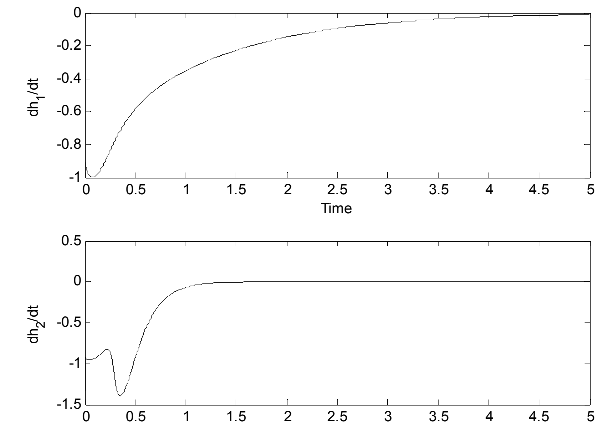

Fig. 3: Evolution of membership functions derivatives.

Acknowledgment

The authors would like to thanks Mr Amédé Sandt-Alizé for his valuable comments within this study.

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