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***Abstract*—Stabilization issue for discrete-time interconnected switched system is studied in this paper. A global large scale discrete-time system can be decomposed into a set of small interconnected switched subsystems. Thus, a decentralized switched state feedback controller is considered to stabilize the global large scale switched system. The stability conditions, obtained from a candidate multiple switched Lyapunov function, are proposed in term of Linear Matrix Inequality (LMI). A numerical example is given to illustrate the effectiveness of the proposed approach.**

***Keywords*—**Discrete-time large scale switched system, interconnected switched systems, decentralized switched controller, multiple switched Lyapunov function,

I. Introduction

MANY real systems such as chemical control systems, navigation systems, etc., encompass both continues and discrete dynamics. In recent years, stability analysis and control problem of hybrid systems and specially switched systems have attracted growing attention [1] [2] [3] [4]. Indeed, switched systems are a particular class of hybrid dynamic systems which are composed of a set of continuous-time or discrete-time dynamical subsystems and a rule that governs the switching among them [1] [3] [5] [6]. Therefore, switching rule, in this kind of systems, can be considered arbitrary [7] [8] or can be constraint by a dwell time [9] [10]. The main concern of several researches, interested by the stabilization issue of switched systems under arbitrary switching law, is to obtain less conservative stability conditions. At first, many approaches use a global Lyapunov function for all the subsystems [11] [12]. However, these quadratic based approaches are very conservative since they need to check the existence of a common Lyapunov matrix for a set of linear matrix inequalities (LMI) constraints. In order to reduce the conservatism, authors have proposed a multiple Lyapunov function [13] and reference therein.

In other hand, many researches are interested recently in studying the stability and the stabilization of smoothed large scale dynamical systems; see e.g. [14]. Nevertheless, few investigations can be found in the literature dealing with the problem of large scale discrete time switched systems stabilization. The lack of previous results in this field has motivated the present study. Thus, this paper focuses on a decentralized controller design for a class of discrete-time interconnected switched systems and it is organized as follows. First, the studied class of interconnected discrete time switched system is presented. Then, a set of decentralized switched control laws is proposed. The design procedure is obtained from a multiple switched Lyapunov function through LMI conditions. Finally, a simulation example is provided to illustrate the efficiency of the design approach.

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II. Interconnected Switched System

Consider the class of hybrid systems S composed of n interconnected switched discrete subsystems S_i given as follows:

For $i = 1,...,n$,

$$x_i(k+1) = \sum_{j=1}^{m_i} \xi_{ij}(k) \left[A_{ij} x_i(k) + B_{ij} u_i(k) + \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} F_{i\alpha j} x_\alpha(k) \right] \quad (1)$$

where $x_i(k) \in \mathbb{R}^{n_i}$, $u_i(k) \in \mathbb{R}^{m_i}$ represent respectively the state and the input vectors associated to the i^{th} model. $x_\alpha(k) \in \mathbb{R}^{n_\alpha}$ denotes the state vector of the α^{th} model with $\alpha = 1,...,n$ and $\alpha \neq i$. m_i is the number of modes of the i^{th} model. $A_{ij} \in \mathbb{R}^{n_i \times n_i}$, $B_{ij} \in \mathbb{R}^{n_i \times p_i}$ and $F_{i\alpha j} \in \mathbb{R}^{n_i \times n_\alpha}$ are constant matrices and $F_{i\alpha j}$ are matrices representing the interconnections expressing the influence of the α^{th} subsystem on the i^{th} one. $\xi_{ij}(k)$ are the switching rules, considered unknown but assumed to be real time available.

These are defined such that the i^{th} subsystem is active in the l^{th} mode as follow:

$$\begin{cases} \xi_{ij}(k)=1 & \text{if } j=l \\ \xi_{ij}(k)=0 & \text{if } j\neq l \end{cases} \tag{2}$$

In order to ensure the stabilization of the overall closed loop system S, a decentralized switched state feedback control law is proposed. The basic idea is to synthesize a global controller composed of n local switched controller assuming that each local controller is able to ensure the stability of the subsystem S_i regarding to the interconnections among the others subsystems. Therefore, the set of decentralized controllers is proposed as:

For all $i=1,\dots,n$:

$$u_i(k)=\sum_{j=1}^{m_i}\xi_{ij}(k)K_{ij}x_i(k) \tag{3}$$

where K_{ij} are the gain matrices to be synthesized.

Substituting (3) into (1), one obtains the overall closed-loop system S described as,

For all $i=1,\dots,n$:

$$x_i(k+1)=\sum_{j=1}^{m_i}\xi_{ij}(k)\left[\left(A_{ij}+B_{ij}K_{ij}\right)x_i(k)+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}F_{i\alpha j}x_\alpha(k)\right] \tag{4}$$

Now, the goal is to find the matrices K_{ij} for $i=1,\dots,n$, $j=1,\dots,m_i$, in order to guarantee the stability of the whole interconnected switched closed loop system (4).

Note that the following lemma will be useful in the sequel.

Lemma 1 [15]

Let us consider A and B two matrices with appropriate dimensions, the following inequality holds:

$$A^TB+B^TA\leq A^TA+B^TB \tag{5}$$

As usual a star $(*)$ indicates a transpose quantity in a matrix.

I. Decentralized Controller Design For Interconnected Switched Systems

In this section, we are interested in designing a decentralized switched controller able to stabilize the close loop interconnected switched system (4). The main result is given in the following theorem.

Theorem 1:

Assuming that the active mode is denoted by l and the upcoming one by h. The closed loop system S composed of n interconnected switched systems S_i described in (1) is stabilized by the set of n decentralized switched state feedback control laws described in (3) if there exist, for all combinations of $i=1,\dots,n$, $j=1,\dots,m_i$, $l=1,\dots,m_i$, $\alpha=1,\dots,n$ and $\alpha\neq i$, the matrices $X_{ij}=X_{ij}^T>0$ and Y_{ij} such that the following LMIs are satisfied:

$$\begin{bmatrix} -X_{il} & (*) & (*) & \cdots & (*) \\ A_{il}X_{il}+B_{il}Y_{il} & -\frac{1}{n}X_{ih} & 0 & \cdots & 0 \\ F_{1il}X_{il} & 0 & -\frac{1}{2(n-1)}X_{1h} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ F_{nil}X_{il} & 0 & \cdots & 0 & -\frac{1}{2(n-1)}X_{nh} \end{bmatrix}<0 \tag{6}$$

Then the controllers' gain matrices are obtained from the following change of variable $K_{il}=Y_{il}X_{il}^{-1}$.

Proof: Let us consider the following candidate switched Lyapunov function:

For $i=1,\dots,n$,

$$v_i\left(k,x_i(k)\right)=\sum_{j=1}^{m_i}\xi_{ij}(k)\left(x_i^T(k)P_{ij}x_i(k)\right) \tag{7}$$

The interconnected closed-loop switched system (4) is stable if :

For $i=1,\dots,n$,

$$\Delta v_i=v_i\left(k+1,x_i(k+1)\right)-v_i\left(k,x_i(k)\right)<0 \tag{8}$$

This can be rewritten as:

For $i=1,\dots,n$,

$$\Delta v_i=\sum_{j=1}^{m_i}\xi_{ij}(k+1)\left(x_i^T(k+1)P_{ij}x_i(k+1)\right) -\sum_{j=1}^{m_i}\xi_{ij}(k)\left(x_i^T(k)P_{ij}x_i(k)\right)<0 \tag{9}$$

Recall that the active mode is denoted by l and the upcoming one by h. It means that:

$$\begin{cases} \xi_{ij}(k+1)=1 & \text{if } j=h \\ \xi_{ij}(k+1)=0 & \text{if } j\neq h \end{cases} \tag{10}$$

and
$$\begin{cases} \xi_{ij}(k)=1 & \text{if } j=l \\ \xi_{ij}(k)=0 & \text{if } j\neq l \end{cases} \tag{11}$$

Therefore, the inequality (9) can be rewritten as follow:

For $i=1,\dots,n$,

$$\Delta v_i = G_{il}^T P_{ih} G_{il} - x_i^T(k) P_{il} x_i(k) < 0 \tag{12}$$

with $G_{il} = (A_{il} + B_{il}K_{il})x_i(k) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha l} x_\alpha(k)$.

Inequality (12) can be rearranged as

For $i=1,\dots,n$,

$$\begin{pmatrix} x_i^T(k)\left(\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) - P_{il}\right)x_i(k) \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih}\left(A_{il}+B_{il}K_{il}\right)x_i(k) \\ +x_i^T(k)\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha l} x_\alpha(k) \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih}\sum_{\substack{\beta=1\\ \beta\neq i}}^{n} F_{i\beta l} x_\beta(k) \end{pmatrix} < 0 \tag{13}$$

Let us focus on the term $\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih}\sum_{\substack{\beta=1\\ \beta\neq i}}^{n} F_{i\beta l} x_\beta(k)$ in (13). It is equivalent to

$$\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih}\sum_{\substack{\beta=1\\ \beta\neq i}}^{n} F_{i\beta l} x_\beta(k) = \begin{pmatrix} \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih} F_{i\alpha l} x_\alpha(k) \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n} \begin{pmatrix} x_\alpha^T(k) F_{i\alpha l}^T P_{ih} F_{i\beta l} x_\beta(k) \\ +x_\beta^T(k) F_{i\beta l}^T P_{ih} F_{i\alpha l} x_\alpha(k) \end{pmatrix} \end{pmatrix} \tag{14}$$

Applying lemma 1 and since $\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\Psi_{i\alpha} + \sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\Psi_{i\beta}\right) = (n-1)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Psi_{i\alpha}$, (13) yields:

For $i=1,\dots,n$,

$$x_i^T(k)\left(\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) - P_{il}\right)x_i(k) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{pmatrix} x_\alpha^T(k) F_{i\alpha l}^T P_{ih}\left(A_{il}+B_{il}K_{il}\right)x_i(k) \\ +x_i^T(k)\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih} F_{i\alpha l} x_\alpha(k) \\ +(2n-3)\, x_\alpha^T(k) F_{i\alpha l}^T P_{ih} F_{i\alpha l} x_\alpha(k) \end{pmatrix} < 0 \tag{15}$$

Then, using lemma 1, (15) is satisfied if:

For $i=1,\dots,n$,

$$x_i^T(k)\left(n\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) - P_{il}\right)x_i(k) + 2(n-1)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} x_\alpha^T(k) F_{i\alpha l}^T P_{ih} F_{i\alpha l} x_\alpha(k) < 0 \tag{16}$$

Let us now define $a_{i\alpha} = 2(n-1)$ if $\alpha\neq i$ and $a_{i\alpha}=0$ if $\alpha = i$; (16) can be rewritten as

For $i=1,\dots,n$,

$$x_i^T(k)\left(n\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) - P_{il}\right)x_i(k) + \sum_{\alpha=1}^{n} x_\alpha^T(k)\, a_{i\alpha} F_{i\alpha l}^T P_{ih} F_{i\alpha l} x_\alpha(k) < 0 \tag{17}$$

witch is equivalent

For $i=1,\dots,n$,

$$x_i^T(k)\sum_{\alpha=1}^{n}\begin{pmatrix} \left(\dfrac{n}{n-1}\right)\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) \\ -\dfrac{1}{(n-1)}P_{il} + a_{\alpha i} F_{\alpha il}^T P_{\alpha h} F_{\alpha il} \end{pmatrix} x_i(k) < 0 \tag{18}$$

which is obviously satisfied, $\forall x_i(k)$, if:

For $i=1,\dots,n$,

$$n\left(A_{il}^T + K_{il}^T B_{il}^T\right)P_{ih}\left(A_{il}+B_{il}K_{il}\right) - P_{il} + \sum_{\alpha=1}^{n} a_{\alpha i} F_{\alpha il}^T P_{\alpha h} F_{\alpha il} < 0 \tag{19}$$

Then, applying the Schur complement, one obtains:

For all $i=1,\dots,n$:

$$\begin{bmatrix} -P_{il} & (*) & (*) & \cdots & (*) \\ (A_{il}+B_{il}K_{il}) & -\frac{1}{n}P_{ih}^{-1} & 0 & \cdots & 0 \\ F_{1il} & 0 & -\frac{1}{a_{1i}}P_{1h}^{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ F_{nil} & 0 & \cdots & 0 & -\frac{1}{a_{ni}}P_{nh}^{-1} \end{bmatrix} < 0 \quad (20)$$

Note that, in (20), the columns and the rows corresponding to a_{ii} don't exist. Multiplying the left and the right (20) by $diag\left[P_{il}^{-1} \quad I \quad \cdots \quad I\right]$, one obtains:

For all $i = 1,...,n$:

$$\begin{bmatrix} -P_{il}^{-1} & (*) & (*) & \cdots & (*) \\ (A_{il}+B_{il}K_{il})P_{il}^{-1} & -\frac{1}{n}P_{ih}^{-1} & 0 & \cdots & 0 \\ F_{1il}P_{il}^{-1} & 0 & -\frac{1}{a_{1i}}P_{1h}^{-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ F_{nil}P_{il}^{-1} & 0 & \cdots & 0 & -\frac{1}{a_{ni}}P_{nh}^{-1} \end{bmatrix} < 0 \quad (21)$$

Now, with the changes of variables $X_{ij} = P_{ij}^{-1}$ and $Y_{il} = K_{il}P_{il}^{-1}$, the proof is completed. ■

Remark 1: the intrinsic stability analysis of a set of autonomous (unforced) interconnected switched systems (1) , with $u_i(k) = 0$, can be easily done from theorem 1 by considering $Y_{ij} = 0$ in LMIs (6).

II. Numerical Example

In order to show the efficiency of the proposed switched decentralized controller design approach, let us consider the hybrid system S composed of two interconnected discrete-time switched subsystems S_1 and S_2:

Subsystem 1

$$S_1 : x_1(k+1) = \sum_{j=1}^{2} \xi_{1j}(k)\left[A_{1j}x_1(k) + B_{1j}u_1(k) + F_{12j}x_2(k)\right] \quad (22)$$

with $A_{11} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, $B_{11} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $F_{121} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ defining mode 1 and $A_{12} = \begin{bmatrix} -1 & 0 \\ 3 & 1 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $F_{122} = \begin{bmatrix} 0.1 & 0 \\ 2 & 0.1 \end{bmatrix}$ defining mode 2.

Subsystem 2

$$S_2 : x_2(k+1) = \sum_{j=1}^{2} \xi_{2j}(k)\left[A_{2j}x_2(k) + B_{2j}u_2(k) + F_{21j}x_1(k)\right] \quad (23)$$

with $A_{21} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, $B_{21} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $F_{211} = \begin{bmatrix} 1 & 0.2 \\ 1 & 0 \end{bmatrix}$ defining mode 1 and $A_{22} = \begin{bmatrix} 2 & 1 \\ 0 & 0.3 \end{bmatrix}$, $B_{22} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $F_{212} = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}$ defining mode 2.

For simulation purpose, let us assume that the subsystems switch according to $S_{11} = 0.9x_{11} + x_{12}$, $S_{12} = -0.2x_{11} + 9x_{12}$, $S_{21} = -x_{21} + x_{22}$ and $S_{22} = x_{21} - 2x_{22}$.

Following remark 1, the intrinsic stability of the overall unforced interconnected switched systems S has been investigate and no solution has been found from LMIs (6). Therefore, one can expect the system to be unstable. This point is confirmed in simulation. Figure 1 presents the states dynamics of the unforced overall system which show an unstable behaviour with initial states $x_1(0) = \begin{bmatrix} 5 & 2 \end{bmatrix}^T$ and $x_2(0) = \begin{bmatrix} -6 & -3 \end{bmatrix}^T$.

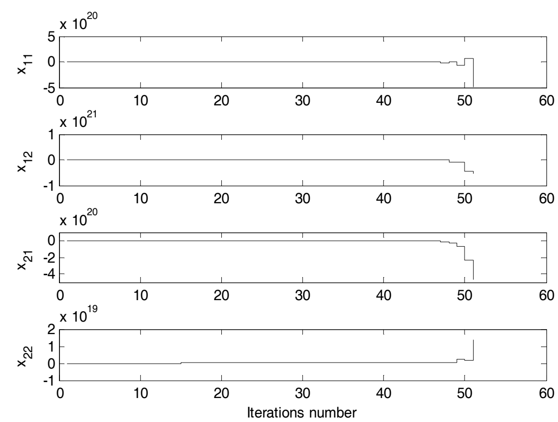

Fig.1. States dynamics of the unforced discrete-time interconnected switched system.

Then, using the Matlab LMI toolbox, a solution of theorem 1 is obtained and leads to the synthesis of two local switched controllers of the form (3) given by the following gain matrices:

$$K_{11} = 10^{8}\left[-0.77 \quad -1.53\right],\; K_{12} = 10^{8}\left[0.26 \quad -1.13\right],$$
$$K_{21} = 10^{-13}\left[0.21 \quad -0.11\right],\; K_{22} = 10^{-11}\left[0.02 \quad -0.11\right]$$
$$X_{11} = 10^{8}\begin{bmatrix} 0.82 & -0.92 \\ -0.92 & 5.46 \end{bmatrix},\; X_{12} = 10^{8}\begin{bmatrix} 0.68 & -0.81 \\ -0.81 & 5.75 \end{bmatrix},$$
$$X_{21} = 10^{-13}\begin{bmatrix} 0.12 & -0.04 \\ -0.04 & 0.07 \end{bmatrix} \text{ and } X_{22} = 10^{-11}\begin{bmatrix} 0.05 & -0.19 \\ -0.19 & 0.74 \end{bmatrix}.$$

The close-loop subsystem dynamics are shown in Figure 2 for initial states $x_1(0) = \left[5 \quad 2\right]^T$ and $x_2(0) = \left[-6 \quad -3\right]^T$. Figure 3 shows the control signals as well as the switching modes' evolution. As expected, the synthesized decentralized switched controller stabilizes the overall discrete-time switched system S.

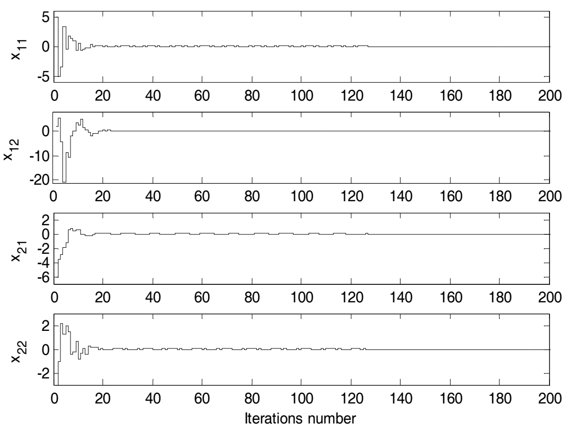

Fig.2. States dynamics of the overall closed loop interconnected switched system.

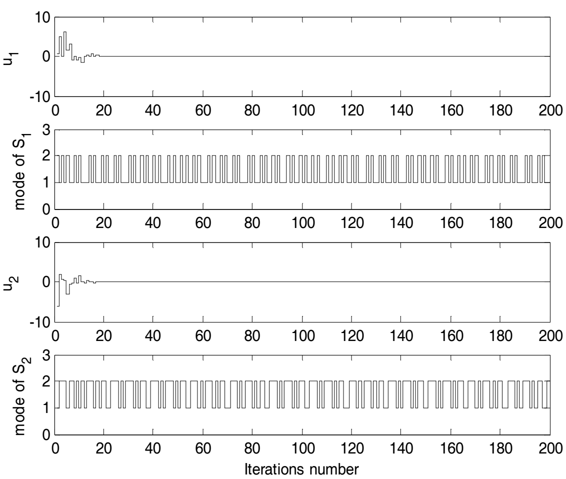

Fig. 3. Control signals and switching modes evolution.

III. Conclusion

In this paper, a class of discrete-time large scale systems is considered. These are classically represented by a set of smaller interconnected discrete-time switched systems. Thus, a set of decentralized discrete-time switched control laws is proposed to stabilize the considered class of hybrid systems. Based on the Lyapunov theory, a multiple Lyapunov function is used to derive LMI conditions allowing the design of such decentralized controllers. Finally, to show the efficiency of the proposed control approach, a numerical example has been provided.

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References

[1] Branicky M-S, "Stability of hybrid systems: state of the art", in *Proc. 36th Conf. Decision and Control,* San Diego, USA, Dec. 10-12, 1997, pp. 120–125.

[2] Branicky, M. S, "Stability of Switched and Hybrid Systems", in *Proc. 33th Conf. Decision and Control*, Lake Buena Vista, USA, Dec. 14-16, 1994, vol. 4, pp. 3498–3503.

[3] Sun Z., "Switched stability of discrete-time linear hybrid systems", *in Proc. IEEE Int. Conf. on Control and Automation*, May. 30-June 1, 2007, Guangzhou, China, pp. 2777-2779.

[4] A. Benzaouia, E. De Santis, P. Caravani and N. Daraoui Constrained Control of Switching Systems: A Positive Invariance Approach. *Int. Journal of Control*, Sep. 2007, vol. 80, n° 9, pp.1379-1387.

[5] Decarlo R.A, Branicky M.S, Pettersson S, Lennartson P.J, Antsaklis E, "Perspectives and results on the stability and stabilizability of hybrid systems", *Proc. of the IEEE*, Jul. 2000, vol. 88, n°7, pp. 1069-1082.

[6] Lin H., Antsaklis P., "Stability and Stabilizability of Switched Linear Systems: A Survey of Recent Results", *IEEE Transactions on Automatic Control*, Feb. 2009, vol. 54, n° 2, pp. 308-322.

[7] Daafouz J, Riedinger P, Iung C, "Stability analysis and control synthesis for switched systems: a switched Lyapunov function approach", *IEEE Transactions on Automatic Control*, Nov. 2002, vol. 47, n°11, pp. 1883 - 1887.

[8] Mansouri. B, Manamanni. N, Guelton. K, Djemai M., "Robust pole placement controller design in LMI region for uncertain and disturbed switched systems", *Nonlinear Analysis: Hybrid Systems*, Nov. 2008, vol. 2, n° 4, pp. 1136-1143.

[9] Hespanha J. P., Morse A. S. "Stability of switched systems with average dwell-time". in *Proc. 38th IEEE. Conf. Decision and Control, Phoenix, Arizona*, USA, Dec. 07-10, 1999, vol. 3, pp. 2655-2660.

[10] Chiou J.S, "Stability analysis for a class of switched large-scale time-delay systems via time-switched method", *IEE Proc. Control Theory Appl.*, Nov. 2006, vol. 153, n° 6, pp. 684-688.

[11] Sun H., Zhao J., "Control Lyapunov Functions for Switched Control Systems", in *Proc. American Control Conference*, Arlington, USA, June 25-27, 2001, vol. 3, pp. 1890-1891.

[12] Wang Z., Xie L., Xie, G., Hao F., "Linear matrix inequality approach to quadratic stabilisation of switched systems", *IEE Proc. Control Theory Appl.*, May 23, 2004, vol. 151, n°3, pp. 289–294.

[13] Zhai G., Matsune I., Imae J., Kobayashi T., "A Note on Multiple Lyapunov Functions and Stability Condition for Switched and Hybrid Systems", in *Proc. 16th IEEE International Conference on Control Applications*, Singapore, May. 2009, vol. 5, n° 5, pp. 1189-1199.

[14] Jabri D., Guelton K., Manamanni N., Abdelkrim M.N., "Fuzzy Lyapunov Decentralized Control of Takagi-Sugeno Interconnected Descriptors", in *Proc. Symposium on Computational Intelligence in Control and Automation*, Nashville, USA, Mar.30-Apr 2, 2009, pp. 30-40.

[15] Zhou K., Khargonedkar P., "Robust Stabilization of linear systems with norm-bounded time-varying uncertainty", *Sys. Control Letters*, 1988, vol. 10, pp. 17-20.