Static Output Feedback controller design for Takagi-Sugeno systems - a Fuzzy Lyapunov LMI approach

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Abstract—**This paper deals with Takagi-Sugeno (T-S) systems stabilization based on a Static Output Feedback (SOF) non-PDC control law. To investigate SOF stabilization, the closed loop dynamics is written using a descriptor redundancy formulation. This approach allows avoiding appearance of crossing terms between the controller's and the T-S system's matrices. Thus, based on a Fuzzy Lyapunov candidate Function (FLF), a LMI based design methodology is provided first without then with considering a H-infinity criterion to attenuate external disturbances. Finally, an academic example illustrates the efficiency of the proposed approach.**

I. INTRODUCTION

Among control theory, static output feedback is of some interests for practical applications [1]. Indeed, a Static Output Feedback Controller (SOFC) is easy to implement since they only require available signals from the plant to be controlled. Moreover, unlike Dynamic Output Feedback Controllers (DOFC) or Observer Based Controllers (OBC), SOFC doesn't need any online differential equation solving and so reduces the online computational cost for practical applications [1]. Among nonlinear control theory, Takagi-Sugeno fuzzy systems [2] have shown their interests since they allow extending some of the linear control concepts to the non linear cases [3]. Indeed, a T-S fuzzy model is a collection of linear time invariant systems blended together with nonlinear membership functions. Therefore, convenient control for such systems has been proposed through the concept of Parallel Distributed Compensation (PDC) [3][4]. Then, PDC controllers design has been studied using a quadratic Lyapunov functions, see [3][4][5] and references therein. This approach remains conservative since it needs to find a common Lyapunov matrix for the whole set of linear matrix inequalities (LMI) constraints. Thus, many ways have been proposed to relax these conditions. For instance, relaxation schemes have been developed based on rewriting the closed-loop interconnection structure of the considered control plant [6][7]. Other works have considered piecewise Lyapunov functions [8] and, more accurately with the fuzzy aggregation of T-S models, through a non quadratic / fuzzy Lyapunov approach [9][10][11][12]. Regarding to output stabilization of T-S fuzzy models, many works have been done for OBC design [13][14][15][16][17][18], DOFC design in both the quadratic and the non quadratic case [19][20][21][22][23][24]. More recently, SOFC controller design has been also considered in the quadratic case [25][26][27] but remains conservative. In [26] and [27], the results are provided in terms of Bilinear Matrix Inequalities instead of LMI. In [28], a descriptor redundancy formulation has been employed to derive new fuzzy Lyapunov LMI stability conditions for state feedback PDC controllers. Based on this descriptor approach, LMI based fuzzy Lyapunov DOFC design has been proposed in [23][24]. In this study, in aid of the redundancy property, one proposes non quadratic strict LMI based SOFC design using a fuzzy Lyapunov approach. The paper is organized as follows. First, convenient notations and lemma will be described. Then, in section 3, the problem statement of SOFC for T-S fuzzy models is proposed. Afterward, fuzzy Lyapunov LMI based SOFC design is investigated and extended to T-S fuzzy models subject to external disturbances. Finally, a design example is proposed to show the efficiency of the proposed approaches.

This work was supported by the French Ministry of Research, the "Département des Ardennes" and the GIS 3SGS within the framework of the project COSMOS and the RCA within the CPER MOSYP.

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II. NOTATIONS AND LEMMA

In the sequel, when there is no ambiguity, the time t in a time varying variable will be omitted for space convenience. As usual, in a matrix, $(*)$ indicates a symmetrical transpose quantity. Let us consider the scalar functions $h_i(z)$, the matrices Y_i and T_{ij} for $i \in \{1,\dots,r\}$ and $j \in \{1,\dots,l\}$ with appropriate dimensions, we will denote $Y_h = \sum_{i=1}^{r} h_i(z) Y_i$, $T_{hv} = \sum_{k=1}^{l}\sum_{i=1}^{r} v_k(z) h_i(z) T_{ik}$. Note that h will be identically used as subscript or superscript in order to lighten the notations. Also for more simplicity, we will use the subscript $\underline{h}$ to indicate a matrix depending on inverse summation structures as $Q_{h\underline{h}} = L_h (M_h)^{-1}$. Finally, one denotes :

$$\dot{X}_h = \frac{d\left(\sum_{i=1}^{r} h_i(z) X_i\right)}{dt} \text{ and } \overline{(X_h)^{-1}}^{\,\boldsymbol{\cdot}} = \frac{d\left(\left(\sum_{i=1}^{r} h_i(z) X_i\right)^{-1}\right)}{dt}.$$

Lemma 1 [7]: Consider the proposition: "For all combinations of $i, j = 1,2,...,r$ we have $\Omega_{ij} < 0$".

This proposition is equivalent to: "For all combinations of $i, j = 1,2,...,r$, we have $\Omega_{ii} < 0$ and for $1 \le i \ne j \le r$, we have $\frac{1}{r-1}\Omega_{ii} + \frac{1}{2}\left(\Omega_{ij} + \Omega_{ji}\right) < 0$".

III. PROBLEM STATEMENT

Let us consider the class of T-S fuzzy systems described by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left[A_i x(t) + B_i u(t) + F_i \varphi(t)\right] \\ y(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left[C_i x(t) + D_i u(t) + G_i \varphi(t)\right] \end{cases} \tag{1}$$

where r represents the number of fuzzy rules. $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$ and $\varphi(t) \in \mathbb{R}^{d \le n}$ represent respectively the state, the input, the output and the external disturbances vectors. $h_i\left(z(t)\right)$ are positive membership functions satisfying the convex sum proprieties $0 \le h_i\left(z(t)\right) \le 1$ and $\sum_{i=1}^{r} h_i\left(z(t)\right) = 1$. $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $C_i \in \mathbb{R}^{q \times n}$, $D_i \in \mathbb{R}^{q \times m}$, $F_i \in \mathbb{R}^{d \times n}$, $G_i \in \mathbb{R}^{d \times q}$ are real matrices.

Let us consider the following non PDC SOFC:

$$u(t) = \left(\sum_{i=1}^{r} h_i\left(z(t)\right) L_i\right)\left(\sum_{i=1}^{r} h_i\left(z(t)\right) W_5^i\right)^{-1} y(t) \tag{2}$$

where $L_1 \in \mathbb{R}^{m \times q}$, $W_5^i \in \mathbb{R}^{q \times q}$ are real matrices to be synthesized.

In [28], LMI based design for state feedback controller using the descriptor redundancy has been proposed. To take advantage of a descriptor redundancy formulation in the case of SOFC design, (1) and (2) can be easily rewritten with the above defined notations respectively as:

$$\begin{cases} \dot{x}(t) = A_h x(t) + B_h u(t) + F_h \varphi(t) \\ 0\dot{y}(t) = -y(t) + C_h x(t) + D_h u(t) + G_h \varphi(t) \end{cases} \tag{3}$$

and

$$0\dot{u}(t) = -u(t) + L_h\left(W_5^h\right)^{-1} y(t) \tag{4}$$

Note that, here the redundancy consists on introducing virtual dynamics in the output equations of (3) and in (4). Then, a descriptor formulation can be obtained considering the extended state vector $\tilde{x}(t) = \left[x^T(t) \quad y^T(t) \quad u^T(t)\right]^T$ and the closed loop dynamics can be expressed as:

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}_{h\underline{h}}\tilde{x}(t) + \tilde{F}_h \varphi(t) \tag{5}$$

with $\tilde{E} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\tilde{A}_{h\underline{h}} = \begin{bmatrix} A_h & 0 & B_h \\ C_h & -I & D_h \\ 0 & L_h\left(W_5^h\right)^{-1} & -I \end{bmatrix}$ and $\tilde{F}_h = \begin{bmatrix} F_h \\ G_h \\ 0 \end{bmatrix}$.

Therefore, (1) is stabilized via the control law (2) if (5) is stable. Thus, the goal is now to provide LMI stability conditions allowing to find the matrices L_h and W_5^h ensuring the stability of (5).

Remark 1: Unlike previous studies on static output feedback [26][27] where the stability conditions are not strictly LMI, rewriting the closed-loop system (5) by the use of descriptor redundancy allows to avoid appearance of crossing terms between the state space matrices and the controller's ones. Therefore, the benefit of this descriptor formulation will be emphasized in the following section since it makes easy the LMI formulation of non quadratic stability conditions.

IV. SOFC LMI CONDITIONS

The main result is presented in this section. Let us first focus on the non quadratic stabilization of uncertain T-S systems (1) but without external disturbances ($\varphi(t) = 0$).

Theorem 1: The T-S fuzzy model (1) (with $\varphi(t) = 0$) is globally asymptotically stable via the non PDC static output feedback control law (2) if there exist, for $i, j = 1,...,r$, the matrices $W_1^j = W_1^{jT} > 0$, $W_5^j > 0$, W_7^j, W_8^j, W_9^j and L_i such that the following LMI conditions are satisfied for $i = 1,2,...,r$, $1 \le i \ne j \le r$ and $k = 1,2,...,r-1$:

$$\Gamma_{ii} < 0 \tag{6}$$

$$\frac{1}{r-1}\Gamma_{ii} + \frac{1}{2}\left(\Gamma_{ij} + \Gamma_{ji}\right) < 0 \tag{7}$$

$$W_1^k - W_1^r \ge 0 \tag{8}$$

where $\Gamma_{ij} = \begin{bmatrix} \Psi_{ij}^{(1,1)} & (*) & (*) \\ \Psi_{ij}^{(2,1)} & \Psi_{ij}^{(2,2)} & (*) \\ W_9^{jT} B_i^T - W_7^j & W_9^{jT} D_i^T + L_i - W_8^j & \Psi_j^{(3,3)} \end{bmatrix}$,

$\Psi_{ij}^{(1,1)} = A_i W_1^j + W_1^j A_i^T + B_i W_7^j + W_7^{jT} B_i^T - \sum_{k=1}^{r-1} \phi_k \left(W_1^k - W_1^r \right)$,

$\Psi_{ij}^{(2,1)} = W_8^{jT} B_i^T + C_i W_1^j + D_i W_7^j$,

$\Psi_{ij}^{(2,2)} = -W_5^j - W_5^{jT} + D_i W_8^j + W_8^{jT} D_i^T$,

$\Psi_j^{(3,3)} = -W_9^j - W_9^{jT}$ and where the scalars ϕ_k are defined as the lower bound of $\dot{h}_k(z)$ for all $k = 1,2,...,r$.

Proof: Let us consider the non quadratic candidate fuzzy Lyapunov function given by:

$$v(x) = \tilde{x}^T \tilde{E} \left(\tilde{W}_h \right)^{-1} \tilde{x} \tag{9}$$

The closed-loop system (5) is stable if:

$$\dot{v}(x) = \dot{\tilde{x}}^T \tilde{E} \left(\tilde{W}_h \right)^{-1} \tilde{x} + \tilde{x}^T \tilde{E} \left(\tilde{W}_h \right)^{-1} \dot{\tilde{x}} + \tilde{x}^T \tilde{E} \overset{\bullet}{\overline{\left(\tilde{W}_h \right)^{-1}}} \tilde{x} < 0 \tag{10}$$

Classically for descriptor systems, from (10) one needs:

$$\tilde{E} \left(\tilde{W}_h \right)^{-1} = \left(\tilde{W}_h \right)^{-T} \tilde{E} > 0 \tag{11}$$

Let us consider $\tilde{W}_h = \begin{bmatrix} W_1^h & W_2^h & W_3^h \\ W_4^h & W_5^h & W_6^h \\ W_7^h & W_8^h & W_9^h \end{bmatrix}$. Multiplying (11), left by $\tilde{W}_h^T$ and right by $\tilde{W}_h$, one has $\tilde{W}_h^T \tilde{E} = \tilde{E} \tilde{W}_h > 0$ which leads to $W_1^h = W_1^{hT} > 0$, $W_2^h = 0$ and $W_3^h = 0$. Considering (5), (10) is obviously satisfied if:

$$\tilde{A}_{h\underline{h}}^T \left(\tilde{W}_h \right)^{-1} + \left(\tilde{W}_h \right)^{-T} \tilde{A}_{h\underline{h}} + \tilde{E} \overset{\bullet}{\overline{\left(\tilde{W}_h \right)^{-1}}} < 0 \tag{12}$$

Multiplying left by $\tilde{W}_h^T$ and right by $\tilde{W}_h$ and since $\tilde{W}_h^T \tilde{E} = \tilde{E} \tilde{W}_h > 0$, (12) yields:

$$\tilde{W}_h^T \tilde{A}_{h\underline{h}}^T + \tilde{A}_{h\underline{h}} \tilde{W}_h + \tilde{E} \tilde{W}_h \overset{\bullet}{\overline{\left(\tilde{W}_h \right)^{-1}}} \tilde{W}_h < 0 \tag{13}$$

It is well-known that $\tilde{W}_h \overset{\bullet}{\overline{\left(\tilde{W}_h \right)^{-1}}} \tilde{W}_h = -\dot{\tilde{W}}_h$, see e.g. [23]. Thus (13) can be rewritten as:

$$\tilde{W}_h^T \tilde{A}_{h\underline{h}}^T + \tilde{A}_{h\underline{h}} \tilde{W}_h - \tilde{E} \dot{\tilde{W}}_h < 0 \tag{14}$$

Extending (14), it yields

$$\begin{bmatrix} \Phi_{hh}^{(1,1)} & (*) & (*) \\ \Phi_{hh}^{(2,1)} & \Phi_{hh}^{(2,2)} & (*) \\ \Phi_{hh\underline{h}}^{(3,1)} & \Phi_{hh}^{(3,2)} & \Phi_{hh\underline{h}}^{(3,3)} \end{bmatrix} < 0 \tag{15}$$

with $\Phi_{hh}^{(1,1)} = A_h W_1^h + W_1^h A_h^T + B_h W_7^h + W_7^{hT} B_h^T - \dot{W}_1^h$,

$\Phi_{hh}^{(2,1)} = W_8^{hT} B_h^T + C_h W_1^h - W_4^h + D_h W_7^h$,

$\Phi_{hh}^{(2,2)} = -W_5^h - W_5^{hT} + D_h W_8^h + W_8^{hT} D_h^T$,

$\Phi_{hh\underline{h}}^{(3,1)} = W_9^{hT} B_h^T + L_h \left(W_5^h \right)^{-1} W_4^h - W_7^h$,

$\Phi_{hh}^{(3,2)} = -W_6^{hT} + W_9^{hT} D_h^T + L_h - W_8^h$

and $\Phi_{hh\underline{h}}^{(3,3)} = L_h \left(W_5^h \right)^{-1} W_6^h + W_6^{hT} \left(W_5^h \right)^{-T} L_h^T - W_9^h - W_9^{hT}$

Let us recall that, due to the nature of the candidate Lyapunov function (9), $W_4^h, W_5^h, ..., W_9^h$ are slack decision matrices which are free of choice. At a first glance on (15), in order to run to LMI conditions, a solution should be to choose, for instance $W_4^h = W_5^h = W_6^h$. Nevertheless, in that case, the problem remains more restrictive regarding to the considered class of T-S fuzzy systems since $W_4^h \in \mathbb{R}^{q \times n}$, $W_5^h \in \mathbb{R}^{q \times q}$ and $W_6^h \in \mathbb{R}^{q \times m}$. Indeed, with the latter solution, one has to consider T-S fuzzy systems where the input, output and the state vector have to be casted into the same dimension. Therefore, for the sake of generality, the choice $W_5^h > 0$, $W_4^h = 0$ and $W_6^h = 0$ appears as a convenient solution. Thus, (15) becomes:

$$\begin{bmatrix} \Phi_{hh}^{(1,1)} & (*) & (*) \\ W_8^{hT} B_h^T + C_h W_1^h + D_h W_7^h & \Phi_{hh}^{(2,2)} & (*) \\ W_9^{hT} B_h^T - W_7^h & \Psi_{hh}^{(3,2)} & \Psi_{hh}^{(3,3)} \end{bmatrix} < 0 \tag{16}$$

with $\Psi_{hh}^{(3,2)} = W_9^{hT} D_h^T + L_h - W_8^h$ and $\Psi_{hh}^{(3,3)} = -W_9^h - W_9^{hT}$.

Let us now focus on the term $\Phi_{hh}^{(1,1)}$ which contains $\dot{W}_1^h = \sum_{k=1}^{r} \dot{h}_k(z) W_1^k$. From the convex property of the membership functions $h_k(z)$ one has $\sum_{k=1}^{r} h_k(z) = 1$, so $\dot{h}_r(z) = -\sum_{k=1}^{r-1} \dot{h}_k(z)$. Therefore, in order to reduce the number of membership function derivates to be taking into account, one can write:

$$\dot{W}_1^h = \sum_{k=1}^{r-1} \dot{h}_k(z) \left(W_1^k - W_1^r \right) \tag{17}$$

Let us consider for $k=1,...,r-1$, ϕ_k the lower bounds of $\dot{h}_k(z)$. One can write $\dot{W}_1^h \geq \sum_{k=1}^{r-1}\phi_k\left(W_1^k - W_1^r\right)$ with $W_1^k - W_1^r \geq 0$ for $k=1,...,r-1$. Thus, after applying lemma 1, (16) holds if the conditions (6), (7) and yield. That ends the proof. ■

V. H_∞ PERFORMANCES

This section aims at extending the previous results to the case of T-S fuzzy systems with external disturbances. Hence, considering $\varphi(t)\neq 0$ and using a H_∞ criterion, The objective is now to stabilize (1) such that the influence of the external disturbance $\varphi(t)$ on the output behavior is minimized. Let us consider the following H_∞ criterion [3]:

$$\int_0^\infty \left(y^T(t)y(t)-\lambda^2\varphi^T(t)\varphi(t)\right)dt \leq 0 \tag{18}$$

Recall that $\tilde{x}(t)=\begin{bmatrix}x^T(t) & y^T(t) & u^T(t)\end{bmatrix}^T$, thus (18) can be rewritten as:

$$\int_0^\infty \left(\tilde{x}^T(t)\tilde{Q}\tilde{x}(t)-\lambda^2\varphi^T(t)\varphi(t)\right)dt \leq 0 \tag{19}$$

with $\tilde{Q}=\begin{bmatrix}0&0&0\\0&I&0\\0&0&0\end{bmatrix}$.

In that case, the stability of the closed loop-system (5) is guaranteed under the constraint (19) if the LMI conditions summarized in the following theorem hold.

Theorem 2: The T-S fuzzy model (1) is globally asymptotically stable via the non PDC static output feedback control law (2) and guarantees the attenuation level $\lambda=\sqrt{\eta}$ if there exist, for $i,j=1,...,r$, the matrices $W_1^j=W_1^{jT}>0$, $W_5^j>0$, W_7^j, W_8^j, W_9^j and L_i such that the following LMI conditions are satisfied for $i=1,2,...,r$, $1\leq i\neq j\leq r$ and $k=1,2,...,r-1$:

Minimize $\eta>0$ such that:

$$\Theta_{ii}<0 \tag{20}$$

$$\frac{1}{r-1}\Theta_{ii}+\frac{1}{2}\left(\Theta_{ij}+\Theta_{ji}\right)<0 \tag{21}$$

$$W_1^i - W_1^r \geq 0 \tag{22}$$

where $\Theta_{ij}=\left[\begin{array}{ccc|cc} & & & 0 & (*)\\ & \Gamma_{ij} & & (*) & (*)\\ & & & 0 & 0\\ \hline 0 & W_5^j & 0 & -I & 0\\ F_i^T & G_i^T & 0 & 0 & -\eta I\end{array}\right]$ and with the matrices Γ_{ij} defined in theorem 1.

Proof: The stability of the closed-loop system (5) is guarantee, under the constraint (19), if:

$$\dot{v}(x)+\tilde{x}^T\tilde{Q}\tilde{x}-\lambda^2\varphi^T\varphi<0 \tag{23}$$

That is to say if:

$$\begin{gathered}\tilde{x}^T\left(\tilde{A}_{h\underline{h}}{}^T\tilde{W}_h^{-1}+\tilde{W}_h^{-T}\tilde{A}_{h\underline{h}}+\tilde{E}\dot{\overline{\tilde{W}_h^{-1}}}+\tilde{Q}\right)\tilde{x}\\ +\varphi^T\tilde{F}_h{}^TW_h^{-1}\tilde{x}+\tilde{x}^T\tilde{W}_h^{-T}\tilde{F}_h\varphi-\lambda^2\varphi^T\varphi<0\end{gathered} \tag{24}$$

which is obviously satisfied if:

$$\begin{bmatrix}\tilde{A}_{h\underline{h}}{}^T\tilde{W}_h^{-1}+\tilde{W}_h^{-T}\tilde{A}_{h\underline{h}}+\tilde{E}\dot{\overline{\tilde{W}_h^{-1}}}+\tilde{Q} & (*)\\ \tilde{F}_h{}^TW_h^{-1} & -\lambda^2I\end{bmatrix}<0 \tag{25}$$

Multiplying left by $\begin{bmatrix}W_h^T & 0\\0 & I\end{bmatrix}$ and right by $\begin{bmatrix}W_h & 0\\0 & I\end{bmatrix}$, one has:

$$\begin{bmatrix}W_h{}^T\tilde{A}_{h\underline{h}}{}^T+\tilde{A}_{h\underline{h}}W_h+\tilde{E}W_h\dot{\overline{\tilde{W}_h^{-1}}}W_h+W_h{}^T\tilde{Q}W_h & (*)\\ \tilde{F}_h{}^T & -\lambda^2I\end{bmatrix}<0 \tag{26}$$

Following the same way as for the proof of theorem 1, (26) is satisfied if (22) holds as well as:

$$\begin{bmatrix}\Gamma_{hh}+W_h{}^T\tilde{Q}W_h & (*)\\ \tilde{F}_h{}^T & -\lambda^2I\end{bmatrix}<0 \tag{27}$$

Note that $W_h{}^T\tilde{Q}W_h=\begin{bmatrix}0&0&0\\0&W_5^{hT}W_5^h&0\\0&0&0\end{bmatrix}$, using the Schur complement and lemma 2, (20) and (21) yield. That ends the proof. ■

Remark 2: The LMI conditions proposed in theorems 1 and 2 are depending on the lower bounds of $\dot{h}_k(z)$ for $k=1,...,r-1$. Even if it is often pointed out as a criticism to fuzzy Lyapunov approach since these parameters may be difficult to choose, a way to obtain these bound has been proposed in [9] in some special cases. Moreover, let us recall that this approach remains one of the least conservative in terms of LMI based design. In [28][29] and [24], a fuzzy Lyapunov candidate function has been reduced leading to relaxed quadratic stability. Indeed, some elements in the Lyapunov matrix can be set common in order to make the LMI free of membership function's lower bounds. In the present study, this remains on setting W_1 common matrices in the previous theorems. Note finally that, obviously, the 'price' to pay for more practical applicability is an increase of the conservatism.

VI. EXAMPLE AND SIMULATION

In this example, both the relaxed quadratic (remark 2) and the non quadratic (theorem 2) design of a SOFC is considered for the following T-S fuzzy model:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))\left[A_i x(t) + B_i u(t) + F_i \varphi(t)\right] \\ y(t) = \sum_{i=1}^{2} h_i(z(t))\left[C_i x(t) + D_i u(t) + G_i \varphi(t)\right] \end{cases} \quad (28)$$

with $A_1 = \begin{bmatrix} -5 & -4 \\ -1 & -2 \end{bmatrix}$, $A_2 = \begin{bmatrix} -2 & -4 \\ 10 & -2 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, $C_1 = \begin{bmatrix} 2 & -10 \\ 5 & -1 \end{bmatrix}$, $C_2 = \begin{bmatrix} -3 & 20 \\ -7 & -2 \end{bmatrix}$, $D_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $D_2 = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}$, $F_1 = F_2 = \begin{bmatrix} 0 \\ -0.25 \end{bmatrix}$, $G_1 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$, $G_2 = \begin{bmatrix} 0.35 \\ 0.5 \end{bmatrix}$, $z(t) \equiv x_1(t)$ and $h_1(z(t)) = \cos^2(x_1(t)) = 1 - h_2(z(t))$.

Case 1 (relaxed quadratic result):

In this case, following remark 2, the result is obtained by considering W_1 common in theorem 2. Therefore, the fuzzy Lyapunov function remains quadratic but the knowledge of the lower bound ϕ_1 of $\dot{h}_1(z(t))$ is not required. This provides the SOFC gain matrices given by:

$W_1 = \begin{bmatrix} 0.0310 & -0.0033 \\ -0.0033 & 0.0477 \end{bmatrix}$, $W_5^1 = \begin{bmatrix} 1.1163 & 0.0817 \\ 0.0812 & 1.0576 \end{bmatrix}$, $W_5^2 = \begin{bmatrix} 1.0004 & -0.0002 \\ -0.0002 & 1.0003 \end{bmatrix}$, $L_1 = \begin{bmatrix} 0.0465 & 0.0254 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -0.1983 & 0.1196 \end{bmatrix}$.

This solution ensures a H_∞ performance given by the minimal attenuation level $\lambda = 1.182$.

Case 2 (non quadratic result):

To improve the conservatism of LMI conditions, non quadratic results can also be considered (i.e., for $i = 1, 2..., r$, non common W_1^i matrices in theorems 1 and 2). Nevertheless, in that case, the lower bound ϕ_1 of $\dot{h}_1(z)$, which is difficult in practice, is required. For the sake of generality, one proposes to study the influence of these bounds on the conservatism of the proposed LMI conditions. Thus, the attenuation level has been computed from theorem 2 for several values of ϕ_1.

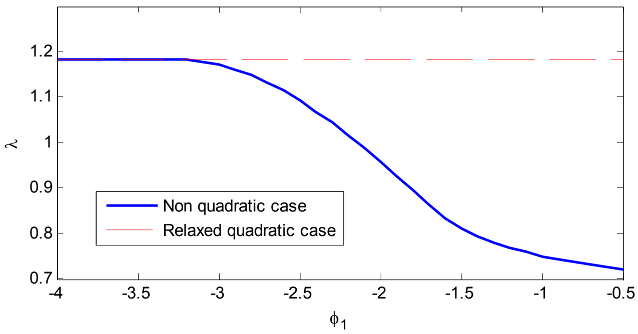

Fig. 1: Attenuation level λ for several values of ϕ_1.

From Fig. 1, it is shown that, to stabilize (28), the non quadratic case is of some interest for $\phi_1 \geq -3$. Therefore, as example, the following matrices are obtained for $\phi_1 = -1$ from theorem 2 with a minimum attenuation level $\lambda = 0.7504$:

$W_1^1 = \begin{bmatrix} 0.0691 & -0.0174 \\ -0.0174 & 0.0604 \end{bmatrix}$, $W_1^2 = \begin{bmatrix} 0.0227 & 0.0028 \\ 0.0028 & 0.0513 \end{bmatrix}$, $W_5^1 = \begin{bmatrix} 1.0018 & 0.0039 \\ 0.0025 & 1.0078 \end{bmatrix}$, $W_5^2 = \begin{bmatrix} 1.0022 & -0.0017 \\ 0.0007 & 1.0005 \end{bmatrix}$, $L_1 = \begin{bmatrix} 0.0543 & 0.0513 \end{bmatrix}$ and $L_2 = \begin{bmatrix} -0.2596 & 0.0948 \end{bmatrix}$.

The closed-loop dynamics has been simulated in the non quadratic cases with the initial values $x_1(0) = 1$, $x_2(0) = 1$ under the external disturbance $\varphi(t) = \sin(5t)$. Fig. 2 shows respectively the behavior of the state signals $x_1(t)$, $x_2(t)$, the control signal $u(t)$ and the derivative of the membership function $\dot{h}_1(z(t))$. Note that, in that case, one has to assume that the lower bound derivative is always lower than $\phi_1 = -1$. This hypothesis is verified a posteriori in simulation since, for this example, one has $\forall t, \min\left(\dot{h}_1(z(t))\right) \approx -0.9911$.

VII. CONCLUSION

In this paper, the problem of static output feedback stabilization Takagi-Sugeno models has been considered. A non PDC static output feedback control law has been proposed and its design has been involved through a fuzzy Lyapunov approach. Thanks to the descriptor redundancy, strict LMI conditions have been easily obtained. This approach leads to less conservative result and is valuable for disturbed T-S fuzzy models using a H_∞ criterion. Finally, an academic example has illustrated the efficiency of the proposed approach.

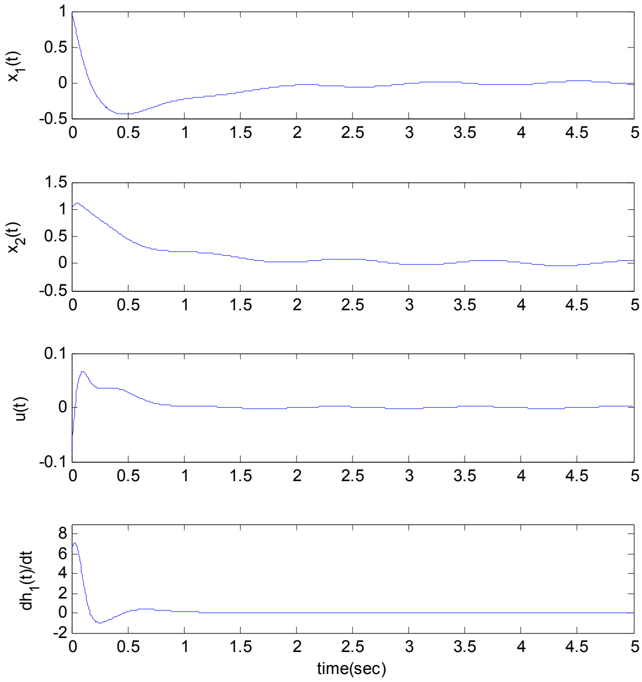

Fig. 2: Evolution of the states, control signal and membership function derivative.

ACKNOWLEDGMENT

The authors would like to thanks Mr Elie Maille for his valuable comments within this study.

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