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Dalel Jabri*,, Kevin Guelton*, Noureddine Manamanni***

**CReSTIC, Université de Reims Champagne Ardenne, Moulin de la Housse, 51000 Reims, France*
e-mail: {kevin.guelton ; noureddine.manamanni ; dalel.jabri}@univ-reims.fr, Tel: +33 3 26 91 83 86;
***Unité de Recherche MACS, Université de Gabès, Route Médenine, 6029 Gabès, Tunisia*

Abstract: This paper proposes, for nonlinear systems composed of interconnected Takagi-Sugeno fuzzy descriptors, a nonlinear H∞ based controller design. A set of decentralized non-Parallel-Distributed-Compensations (non-PDC) control law is employed to ensure the stability of the overall closed loop system and to achieve the H∞ performance in order to minimize the interconnection effects between subsystems. Sufficient conditions are derived, based on the Lyapunov theory. These ones are written into Linear Matrix Inequalities (LMI). Finally, a numerical example is given to illustrate the efficiency of the proposed approach.

Keywords: Interconnected Takagi-Sugeno fuzzy descriptors, non-quadratic Lyapunov function, H∞ controller.

1. INTRODUCTION

Among nonlinear control theory, Takagi-Sugeno (T-S) fuzzy systems have shown their significance in resolution of both control and modelling problems (Takagi et al., 1985). Their interest is that they allow extending some of the linear control concepts to the nonlinear cases. Within T-S fuzzy stabilization, the most commonly used controllers are based on the concept of parallel distributed compensation (PDC) (Wang et al., 1996). The main idea of such controllers is to associate compensators to each rule of the fuzzy system. More recently, a wider class of nonlinear T-S fuzzy systems, called descriptor systems, have been studied (Taniguchi et al, 2000) and some applications have been proposed in robotics fields (Guelton et al., 2008), (Schulte et al., 2009). Only few of the existing approaches deal with stability and stabilisation of T-S fuzzy descriptors (Taniguchi et al. 2000), (Bouarar et al., 2007). These studies are based on the quadratic Lyapunov function. Nevertheless, these quadratic based approaches are very conservative since they need to check the existence of a common Lyapunov matrix for a set of linear matrix inequalities (LMI) constraints. There exist many ways to relax these conditions. For instance, relaxation schemes have been proposed (Tuan et al., 2001), (Liu et al., 2003). Moreover, another way is to use another type of Lyapunov function. Owing to that, less conservative stability conditions based on piecewise Lyapunov functions have been proposed (Johansen et al., 1999) but they do not provide major improvement when the considered T-S model is derived from the sector nonlinearity approach (Tanaka et al., 2001). On other hand, more relaxed conditions have been derived using a non quadratic-fuzzy Lyapunov function (Feng, 2006), (Guerra et al., 2004), (Rhee et al., 2006), (Tanaka et al., 2007), (Bouarar et al., 2008).

In this paper, we are interested in stabilizing a set of non linear interconnected descriptors. Indeed, these are useful to deal with, for instance, large scale systems for which classical approaches failed to ensure the overall stability. One other interest should be to propose a distributed controller design for networked systems. Therefore, with the growing interest for large scale systems, many different studies have been proposed to stabilize T-S fuzzy interconnected systems (Akar et al., 2000), (Lin et al., 2006), (Taniguchi et al., 2007), (Tseng et al., 2001) (Wang et al., 2005). However, the decentralized control of T-S interconnected descriptors has been seldom treated in the literature. Indeed, T-S interconnected descriptor has been firstly studied by (Wang et al., 2001). In the latter, the proposed conditions are based on a quadratic Lyapunov function, and the left hand side of the considered descriptors is a LTI model ($E\dot{x}(t)=......$ instead of the general case $\sum_{k=1}^{li} v_k E^k \dot{x}(t)=......$). In order to relax and to extend the previous work, a first non quadratic approach has been proposed in a preliminary work (Jabri et al., 2009). Nevertheless, in the latter, performances criterion had not been taking into account to optimize the closed loop dynamics. Thus, to minimize the effects of the interconnections between subsystems, we propose a methodology, based on a H_∞ criterion, to design a set of decentralized T-S fuzzy controllers ensuring the closed-loop stability of the whole set of interconnected descriptors.

This paper is organized as follows: First, the studied class of T-S fuzzy decentralized descriptors will be described. Then, a fuzzy state feedback decentralized controller is developed. Next, the problem position followed by the main result in terms of LMI is proposed. Finally, a simulation example is given to illustrate the efficiency of the design approach.

2. T-S DECENTRALIZED DESCRIPTORS

Let's consider the class of nonlinear interconnected system S composed of n T-S fuzzy descriptor subsystems S_i described as follows:

for $i = 1,...,n$,

$$\begin{aligned}&\sum_{j=1}^{l_i} v_i^j\left(z_i\left(t\right)\right)E_i^j\dot{x}_i\left(t\right)\\&=\sum_{k=1}^{r_i} h_i^k\left(z_i\left(t\right)\right)\left(A_i^k x_i\left(t\right)+B_i^k u_i\left(t\right)+\sum_{\substack{\alpha=1\\\alpha\neq i}}^{n} F_{i\alpha}^k x_\alpha\left(t\right)\right)\end{aligned} \tag{1}$$

where $x_i\left(t\right)\in\mathbb{R}^{n_i}$ is the i^{th} state vector, $u_i\left(t\right)\in\mathbb{R}^{m_i}$ is the i^{th} control signal, $z_i\left(t\right)\in\mathbb{R}^{p_i}$ is the i^{th} premise vector and $x_\alpha\left(t\right)\in\mathbb{R}^{n_\alpha}$ is the state vector of the α^{th} model with $\alpha=1,...,n$ and $\alpha\neq i$. l_i and r_i are respectively the number of fuzzy rules in the left-hand and right-hand side of the state equation (1). $E_i^j\in\mathbb{R}^{n_i\times n_i}$, $A_i^k\in\mathbb{R}^{n_i\times n_i}$, $B_i^k\in\mathbb{R}^{n_i\times m_i}$ are constant matrices constituting the i^{th} T-S fuzzy subsystem and $F_{i\alpha}^k\in\mathbb{R}^{n_i\times n_\alpha}$ denotes the influence of the α^{th} subsystem on the i^{th} one. $h_i^k\left(z_i\left(t\right)\right)\geq 0$ and $v_i^j\left(z_i\left(t\right)\right)\geq 0$ are respectively the right-hand and the left-hand side membership functions verifying the convex sums propriety $\sum_{j=1}^{l_i} v_i^j\left(z_i\left(t\right)\right)=1$ and $\sum_{k=1}^{r_i} h_i^k\left(z_i\left(t\right)\right)=1$. Note that, (1) may represent singular systems. In that case, one assumes that, in the sequel, (1) is regular and impulse free (Dai, 1989).

To ensure the stabilization of the overall closed-loop system S, a decentralized non-Parallel Distributed Compensation (non-PDC) approach is proposed. The basic idea is to synthesize a decentralized controller composed of n local controller. Each i^{th} local fuzzy controller is able to guarantee the stability of the subsystem S_i while considering interconnections among the others subsystems. For more convenience, the local non-PDC control law $u_i\left(t\right)$ shares the same fuzzy sets with the T-S descriptor model of the subsystem S_i. This set of decentralized non-PDC controllers is given by,

For $i = 1,...,n$:

$$\begin{aligned}u_i\left(t\right)=&\left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j\left(z_i\left(t\right)\right)h_i^s\left(z_i\left(t\right)\right)K_i^{js}\right)\\&\times\left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j\left(z_i\left(t\right)\right)h_i^s\left(z_i\left(t\right)\right)X_{i1}^{js}\right)^{-1}x_i\left(t\right)\end{aligned} \tag{2}$$

where K_i^{js} are non-PDC gain matrices and $X_{i1}^{js}>0$ are Lyapunov dependant gain matrices to be synthesized.
For space convenience, in the sequel, the time t as well as the premises $z_i\left(t\right)$ will be omitted when there is no ambiguity.
Combining (2) and (1), the overall closed-loop system S can be described by:

For all $i = 1,...,n$,

$$\sum_{j=1}^{l_i} v_i^j E_i^j\dot{x}_i=\sum_{j=1}^{l_i}\sum_{k=1}^{r_i}\sum_{s=1}^{r_i} v_i^j h_i^k h_i^s G_i^{jks}\left(v_i^j,h_i^s\right) \tag{3}$$

with

$$G_i^{jks}\left(v_i^j,h_i^s\right)=\left(A_i^k+B_i^k K_i^{js}\left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j h_i^s X_{i1}^{js}\right)^{-1}\right)x_i+\sum_{\substack{\alpha=1\\\alpha\neq i}}^{n} F_{i\alpha}^k x_\alpha$$

The objective is now to propose a convenient controller design methodology which ensure the stability of (3). Note that each subsystem i is influenced by the other subsystems $\alpha=1,...,n$, $\alpha\neq i$. In this study, one proposes the design of the controller (2) based on the minimization of a H_∞ performance related to attenuate exotic influences to each considered state x_i , $i=1,...,n$.

Notations:
To clarify the mathematical expression, the following notations will be used in the sequel: $E_i^v=\sum_{j=1}^{l_i} v_i^j E_i^j$, $Y_i^{hh}=\sum_{j=1}^{r_i}\sum_{k=1}^{r_i} h_i^j h_i^k Y_i^{jk}$, $T_i^{vhh}=\sum_{j=1}^{l_i}\sum_{s=1}^{r_i}\sum_{k=1}^{r_i} v_i^j h_i^s h_i^k T_i^{jsk}$, …
A star (*) in a matrix indicates a transpose quantity.

The following lemma will be used in the next section to relax the LMI conditions.

Lemma 1: (Tuan et al., 2001)
The following propositions are equivalent:

- *For all* $i=1,...,q$ *and* $j=1,...,q$, $\Gamma_{ij}<0$
- *For all* $i=1,...,q$ *and* $j=1,...,q$,

$$\begin{cases}\Gamma_{ii}<0,\\ \dfrac{1}{q-1}\Gamma_{ii}+\dfrac{1}{2}\left(\Gamma_{ij}+\Gamma_{ji}\right)<0,\ j\neq i\end{cases}$$

3. FUZZY LYAPUNOV LMI CONTROLLER DESIGN

In this section, the main purpose is to design a decentralized controller ensuring the stability of the whole set of interconnected T-S descriptors and minimizing the effect of the interactions between subsystems using the following H_∞ criterion:

$$\int_{t_0}^{t_f} x_i^T x_i dt < \rho_i^2 \int_{t_0}^{t_f} \varphi_i^T \varphi_i dt \tag{4}$$

with $\varphi_i(z_i) = \sum_{k=1}^{r_i} \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} h_i^k F_{i\alpha}^{\ k} x_\alpha$ and the $H\infty$ performances ρ_i.

To provide sufficient conditions for the existence of a decentralized controller minimizing the H_∞ criterion (4) one proposes the following theorem.

Theorem 1: *Assume that, for* $i = 1,...,n$, $j = 1,...,l_i$ *and* $s = 1,...,r_i$, $\dot{h}_i^s(z(t)) \geq \varpi_i^s$ *and* $\dot{v}_i^j(z(t)) \geq \lambda_i^j$. *The closed-loop system* S *composed of n T-S interconnected descriptors* S_i *(1) is stabilized by the network of n non-PDC decentralized control laws (2) and guarantees the* H_∞ *performances* ρ_i *if there exist, for all combination of {*$i = 1,...,n$, $j = 1,...,l_i$, $k = 1,...,r_i$, $s = 1,...,r_i$*}, the matrices* $X_{i1}^{js} = (X_{i1}^{js})^T > 0$, X_{i3}^{ks}, X_{i4}^{ks}, *and* K_i^{js}, *such that the following LMIs are satisfied:*

Minimize ρ_i *such that:*

- $\Gamma_{i\alpha}^{jkk} < 0 \tag{5}$
- $\frac{1}{r_i - 1}\Gamma_{i\alpha}^{jkk} + \frac{1}{2}\left(\Gamma_{i\alpha}^{jsk} + \Gamma_{i\alpha}^{jks}\right) < 0 \tag{6}$

with

$$\Gamma_{i\alpha}^{jks} = \begin{bmatrix} \Gamma_{i(1,1)}^{jks} & & & (*) \\ \Gamma_{i(2,1)}^{jks} & \Gamma_{i(2,2)}^{jks} & & \\ 0 & (n-1)\left(F_{i\alpha}^k\right)^T & \Gamma_{i\alpha(3,3)}^k & \\ \left(X_{i1}^{js}\right)^T & 0 & 0 & -I \end{bmatrix}$$

$\Gamma_{i(1,1)}^{jks} = X_{i3}^{ks} + \left(X_{i3}^{ks}\right)^T - \sum_{s=1}^{r_i} \varpi_i^s X_{i1}^{js} - \sum_{s=1}^{l_i} \lambda_i^j X_{i1}^{js}$,

$\Gamma_{i(2,1)}^{jks} = A_i^k X_{i1}^{js} + B_i^k K_i^{js} - E_i^j X_{i3}^{ks} + \left(X_{i4}^{ks}\right)^T$,

$\Gamma_{i(2,2)}^{jks} = -E_i^j X_{i4}^{ks} - \left(E_i^j\right)^T \left(X_{i4}^{ks}\right)^T$

and $\Gamma_{i\alpha(3,3)}^k = -\rho_i^2 (n-1)(2n-3)\left(F_{i\alpha}^k\right)^T F_{i\alpha}^k$.

Proof: Let, for $i = 1,...,n$, $\tilde{x}_i = \begin{bmatrix} x_i^T & \dot{x}_i^T \end{bmatrix}^T$, be an extended state vector. The $H\infty$ criterion (4) can be rewritten as:

$$\int_{t_0}^{t_f} \tilde{x}_i^T \tilde{Q}_i \tilde{x}_i dt$$
$$< \rho_i^2 \int_{t_0}^{t_f} \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \left((n-1)\left(\tilde{x}_\alpha^T \left(\tilde{F}_{i\alpha}^h\right)^T \tilde{F}_{i\alpha}^h \tilde{x}_\alpha\right) + \sum_{\substack{\beta=1 \\ \beta \neq i \\ \beta \neq \alpha}}^{n} \left(\tilde{x}_\beta^T \left(\tilde{F}_{i\beta}^h\right)^T \tilde{F}_{i\beta}^h \tilde{x}_\beta\right) \right) dt \tag{7}$$

with $\tilde{Q}_i = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$.

The overall closed-loop T-S fuzzy decentralized descriptor (3) can be expressed as:

$$\text{For all } i = 1,...,n,\ \tilde{E}\dot{\tilde{x}}_i = \sum_{\substack{\alpha=1 \\ \alpha \neq i}}^{n} \left(\tilde{A}_i^{vhh} \tilde{x}_i + \tilde{F}_{i\alpha}^h \tilde{x}_\alpha \right) \tag{8}$$

with $\tilde{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{A}_i^{vhh} = \frac{1}{n-1}\begin{bmatrix} 0 & I \\ A_i^h + B_i^h K_i^{vh} \left(X_{i1}^{vh}\right)^{-1} & -E_i^v \end{bmatrix}$

and $\tilde{F}_{i\alpha}^{\ h} = \begin{bmatrix} 0 \\ F_{i\alpha}^{\ h} \end{bmatrix}$.

Let the candidate multiple fuzzy Lyapunov function be:

$$V(t) = \sum_{i=1}^{n} V_i(x_i(t)) \geq 0 \tag{9}$$

with $V_i(x_i(t)) = \tilde{x}_i^T(t) \tilde{E} \left(\tilde{X}_i^{vhh}\right)^{-1} \tilde{x}_i(t)$.

As usual, (9) needs $\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1} = \left(\tilde{X}_i^{vhh}\right)^{-T} \tilde{E} \geq 0$. Leading to condition the Lyapunov matrix such that $\tilde{X}_i^{vhh} = \begin{bmatrix} X_{i1}^{vh} & 0 \\ X_{i3}^{hh} & X_{i4}^{hh} \end{bmatrix}$ and $X_{i1}^{vh} = \left(X_{i1}^{vh}\right)^T > 0$.

From (9), the closed loop system (3) is stable under the $H\infty$ criterion (7) if:

$$\sum_{i=1}^{n} \left(\dot{V}_i(t) + \tilde{x}_i^T \tilde{Q}_i \tilde{x}_i - \rho_i^2 \varphi_i(z_i)^T \varphi_i(z_i) \right) < 0 \tag{10}$$

That is to say if:

$$\sum_{i=1}^{n}\begin{pmatrix} \dot{\tilde{x}}_i^T\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i + \tilde{x}_i^T\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}\dot{\tilde{x}}_i \\ +\tilde{x}_i^T\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\tilde{x}_i + \tilde{x}_i^{\,T}\tilde{Q}_i\tilde{x}_i \\ -\rho_i^{\,2}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left((n-1)\tilde{x}_\alpha^{\,T}\left(\tilde{F}_{i\alpha}^h\right)^T\tilde{F}_{i\alpha}^h\tilde{x}_\alpha + \sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\tilde{x}_\beta^{\,T}\left(\tilde{F}_{i\beta}^h\right)^T\tilde{F}_{i\beta}^h\tilde{x}_\beta\right)\end{pmatrix}<0 \tag{11}$$

This can be rewritten as:

$$\sum_{i=1}^{n}\begin{pmatrix} \tilde{x}_i^T\begin{pmatrix}(n-1)\left(\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\right)\\ +\tilde{Q}_i+\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\end{pmatrix}\tilde{x}_i \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tilde{x}_\alpha^T\left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{F}_{i\alpha}^h\tilde{x}_\alpha\right) \\ -\rho_i^{\,2}\begin{pmatrix}(n-1)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tilde{x}_\alpha^T\left(\tilde{F}_{i\alpha}^h\right)^T\tilde{F}_{i\alpha}^h\tilde{x}_\alpha \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\tilde{x}_\beta^T\left(\tilde{F}_{i\beta}^h\right)^T\tilde{F}_{i\beta}^h\tilde{x}_\beta\end{pmatrix}\end{pmatrix}<0 \tag{12}$$

Note that $\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\Psi_{i\alpha}+\sum_{\substack{\beta=1\\ \beta\neq i\\ \beta\neq\alpha}}^{n}\Psi_{i\beta}\right)=(n-1)\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\Psi_{i\alpha}$. Thus, (12) can be rewritten as:

$$\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{pmatrix}\tilde{x}_i^T\begin{pmatrix}\left(\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\right)\\ +\frac{1}{(n-1)}\left(\tilde{Q}_i+\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\right)\end{pmatrix}\tilde{x}_i \\ +\begin{pmatrix}\tilde{x}_\alpha^T\left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{F}_{i\alpha}^h\tilde{x}_\alpha\\ -\rho_i^2(2n-3)\tilde{x}_\alpha^T\left(\tilde{F}_{i\alpha}^h\right)^T\tilde{F}_{i\alpha}^h\tilde{x}_\alpha\end{pmatrix}\end{pmatrix}<0 \tag{13}$$

i.e.:

$$\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{bmatrix}\tilde{x}_i\\ \tilde{x}_\alpha\end{bmatrix}^T\begin{bmatrix}\Omega_{i(1,1)}^{vhh} & (*)\\ \left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1} & \Omega_{i(2,2)}^{h}\end{bmatrix}\begin{bmatrix}\tilde{x}_i\\ \tilde{x}_\alpha\end{bmatrix}<0 \tag{14}$$

with $\Omega_{i(1,1)}^{vhh}=\begin{pmatrix}\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\\ +\frac{1}{(n-1)}\left(\tilde{Q}_i+\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\right)\end{pmatrix}$

and $\Omega_{i(2,2)}^{h}=-\rho_i^2(2n-3)\left(\tilde{F}_{i\alpha}^h\right)^T\tilde{F}_{i\alpha}^h$.

Then, (14) is verified if:

For all $i=1,\dots,n$, $\alpha=1,\dots,n$, $\alpha\neq i$,

$$\begin{bmatrix}\Omega_{i(1,1)}^{vhh} & (*)\\ \left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1} & \Omega_{i(2,2)}^{h}\end{bmatrix}<0 \tag{15}$$

Multiplying (15) left and right respectively by $\begin{bmatrix}\left(\tilde{X}_i^{vhh}\right)^T & 0\\ 0 & I\end{bmatrix}$ and its transpose, it yields:

For all $i=1,\dots,n$, $\alpha=1,\dots,n$, $\alpha\neq i$,

$$\begin{bmatrix}\Lambda_{i(1,1)}^{vhh} & (*)\\ \left(\tilde{F}_{i\alpha}^h\right)^T & \Omega_{i(2,2)}^{h}\end{bmatrix}<0 \tag{16}$$

with $\Lambda_{i(1,1)}^{vhh}=\begin{pmatrix}\left(\tilde{X}_i^{vhh}\right)^T\left(\tilde{A}_i^{vhh}\right)^T+\tilde{A}_i^{vhh}\tilde{X}_i^{vhh}\\ +\frac{1}{(n-1)}\left(\tilde{X}_i^{vhh}\right)^T\left(\tilde{Q}_i+\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\right)\tilde{X}_i^{vhh}\end{pmatrix}$.

Recall that $-\dot{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}=\left(\tilde{X}_i^{vhh}\right)^{-1}\dot{\tilde{X}}_i^{vhh}\left(\tilde{X}_i^{vhh}\right)^{-1}$, see e.g. (Jabri et al. 2009). $\Lambda_{i(1,1)}^{vhh}$ can be rewritten as:

$$\Lambda_{i(1,1)}^{vhh}=\begin{pmatrix}\left(\tilde{X}_i^{vhh}\right)^T\left(\tilde{A}_i^{vhh}\right)^T+\tilde{A}_i^{vhh}\tilde{X}_i^{vhh}\\ +\frac{1}{(n-1)}\left(\left(\tilde{X}_i^{vhh}\right)^T\tilde{Q}_i\tilde{X}_i^{vhh}-\tilde{E}\dot{\overline{\left(\tilde{X}_i^{vhh}\right)}}\right)\end{pmatrix} \tag{17}$$

Now, extending (16) with (17) and the matrices defined in (8) and after applying the Schur complement, one obtains:

For all $i=1,\dots,n$, $\alpha=1,\dots,n$, $\alpha\neq i$,

$$\Gamma_{i\alpha}^{vhh}=\begin{bmatrix}\Gamma_{i(1,1)}^{vhh} & & & (*)\\ \Gamma_{i(2,1)}^{vhh} & \Gamma_{i(2,2)}^{vhh} & & \\ 0 & (n-1)\left(\tilde{F}_{i\alpha}^h\right)^T & \Gamma_{i\alpha(3,3)}^{h} & \\ \left(X_{i1}^{vh}\right)^T & 0 & 0 & -I\end{bmatrix} \tag{18}$$

with $\Gamma_{i(1,1)}^{vhh}=X_{i3}^{hh}+\left(X_{i3}^{hh}\right)^T-\dot{\overline{\left(X_{1i}^{vh}\right)}}$,

$\Gamma_{i(2,1)}^{vhh}=A_i^hX_{i1}^{vh}+B_i^hK_i^{vh}-E_i^vX_{i3}^{hh}+\left(X_{i4}^{hh}\right)^T$,

$\Gamma_{i(2,2)}^{vhh}=-E_i^vX_{i4}^{hh}-\left(E_i^v\right)^T\left(X_{i4}^{hh}\right)^T$

and $\Gamma_{i\alpha(3,3)}^{h}=-\rho_i^2(n-1)(2n-3)\left(\tilde{F}_{i\alpha}^h\right)^T\tilde{F}_{i\alpha}^h$.

Let us now focus on the term $-\dot{\overline{\left(X_{1i}^{vh}\right)}}$ in (18). From the convex sum propriety, one can write:

$$\overbrace{\left(X_{1i}^{vh}\right)}^{\bullet}=\sum_{j=1}^{l_i}\sum_{s=1}^{r_i}v_i^j h_i^s\left(\sum_{s=1}^{r_i}\dot{h}_i^s X_{i1}^{js}+\sum_{s=1}^{l_i}\dot{v}_i^j X_{i1}^{js}\right) \quad (19)$$

Let, for $j=1,...,l_i$, $s=1,...,r_i$, λ_i^j and ϖ_i^s be respectively the lower bounds of $\dot{v}_i^j(z)$ and $\dot{h}_i^s(z)$. One can write:

$-\overbrace{\left(X_{1i}^{vh}\right)}^{\bullet}\leq-\Phi_{hv}$ with:

$$\Phi_{hv}=\sum_{j=1}^{l_i}\sum_{s=1}^{r_i}v_i^j h_i^s\left\{\sum_{s=1}^{r_i}\varpi_i^s X_{i1}^{js}+\sum_{s=1}^{l_i}\lambda_i^j X_{i1}^{js}\right\} \quad (20)$$

(18) is obviously satisfied, following lemma 1, if the LMI conditions (5) and (6) holds. And the proof is completed ■

Remark 1: For $j=1,...,l_i$, $k=1,...,r_i$, $s=1,...,r_i$, $v_i^j(z)$ and $h_i^s(z)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach (Tanaka et al., 2001) or, for instance, when membership functions are chosen with a smoothed shape (Gaussian...).

4. NUMERICAL EXAMPLE

In order to illustrate the approach developed above, let us consider the following set of T-S descriptors S composed of two subsystems S_1 and S_2 described by:

$$S_1:\left|\begin{array}{l}\sum_{j=1}^{2}v_1^j\left(x_1(t)\right)E_1^j\dot{x}_1(t)\\ \quad=\sum_{k=1}^{2}h_1^k\left(x_1(t)\right)\left\{A_1^k x_1(t)+B_1^k u_1(t)+F_{12}^k x_2(t)\right\}\end{array}\right. \quad (21)$$

with $E_1^1=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}$, $E_1^2=\begin{bmatrix}1 & 0.5\\ -1 & 1\end{bmatrix}$, $A_1^1=\begin{bmatrix}0 & 1\\ 0.839 & -0.73\end{bmatrix}$, $A_1^2=\begin{bmatrix}-0.73 & 1\\ 0.839 & -0.73\end{bmatrix}$, $B_1^1=\begin{bmatrix}0.47\\ 1.263\end{bmatrix}$, $B_1^2=\begin{bmatrix}1\\ 1.263\end{bmatrix}$, $F_{12}^1=\begin{bmatrix}0 & 0\\ 0.1 & 0\end{bmatrix}$, $F_{12}^2=\begin{bmatrix}0 & 0\\ 0.2 & 0\end{bmatrix}$, $v_1^1\left(x_1(t)\right)=\left(\frac{1-\cos(x_{11})}{2}\right)$, $v_1^2\left(x_1(t)\right)=\left(\frac{1+\cos(x_{11})}{2}\right)$, $h_1^1\left(x_1(t)\right)=\sin^2\left(x_{11}(t)\right)$, $h_1^2\left(x_1(t)\right)=\cos^2\left(x_{11}(t)\right)$.

and

$$S_2:\left|\begin{array}{l}\sum_{j=1}^{2}v_2^j\left(x_2(t)\right)E_2^j\dot{x}_2(t)\\ \quad=\sum_{k=1}^{2}h_2^k\left(x_2(t)\right)\left\{A_2^k x_2(t)+B_2^k u_2(t)+F_{21}^k x_1(t)\right\}\end{array}\right. \quad (22)$$

with $E_2^1=\begin{bmatrix}1 & 0.2\\ 0 & 1\end{bmatrix}$, $E_1^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $A_2^1=\begin{bmatrix}-1 & 1\\ 0.839 & -0.931\end{bmatrix}$, $A_2^2=\begin{bmatrix}-1 & 1\\ 0 & 0.931\end{bmatrix}$, $B_2^1=\begin{bmatrix}0.47\\ 0.4\end{bmatrix}$, $B_2^2=\begin{bmatrix}0.47\\ 0.8\end{bmatrix}$, $F_{21}^1=\begin{bmatrix}0 & 0\\ 0.3 & 0\end{bmatrix}$, $F_{21}^2=\begin{bmatrix}0 & 0\\ 0.5 & 0\end{bmatrix}$, $v_2^1\left(x_2(t)\right)=\cos^2\left(x_{21}(t)\right)$, $v_2^2\left(x_2(t)\right)=\sin^2\left(x_{21}(t)\right)$, $h_2^1\left(x_2(t)\right)=\sin^2\left(x_{21}(t)\right)$, $h_2^2\left(x_2(t)\right)=\cos^2\left(x_{21}(t)\right)$.

Remark: The stability of (21) and (22) cannot be established with the LMI conditions proposed in (Jabri et al., 2009).

A set of decentralized H_∞ controllers can be synthesized using theorem 1. The Matlab LMI toolbox is used to solve the LMI conditions provided in (5) and (6), the solution leads to the design of two decentralized non-PDC controllers with the following gain matrices:

$K_1^{11}=\begin{bmatrix}-1.5968 & -1.8869\end{bmatrix}$, $K_1^{12}=\begin{bmatrix}-1.0805 & -1.4521\end{bmatrix}$, $K_1^{21}=\begin{bmatrix}-1.3858 & -1.5140\end{bmatrix}$, $K_1^{22}=\begin{bmatrix}-1.0176 & -1.5563\end{bmatrix}$, $K_2^{11}=\begin{bmatrix}1.0781 & -9.6242\end{bmatrix}$, $K_2^{12}=\begin{bmatrix}1.3314 & -6.2205\end{bmatrix}$, $K_2^{21}=\begin{bmatrix}1.5534 & -6.2765\end{bmatrix}$, $K_2^{22}=\begin{bmatrix}1.2592 & -4.4276\end{bmatrix}$, $X_{11}^{11}=\begin{bmatrix}3.6662 & 0.8928\\ 0.8928 & 4.2713\end{bmatrix}$, $X_{11}^{12}=\begin{bmatrix}2.0172 & 0.1673\\ 0.1673 & 2.8931\end{bmatrix}$, $X_{11}^{21}=\begin{bmatrix}2.3189 & 0.3671\\ 0.3671 & 3.1360\end{bmatrix}$, $X_{11}^{22}=\begin{bmatrix}1.0466 & -0.1627\\ -0.1627 & 1.8607\end{bmatrix}$, $X_{21}^{11}=\begin{bmatrix}3.4822 & 0.4084\\ 0.4084 & 5.2171\end{bmatrix}$, $X_{21}^{12}=\begin{bmatrix}2.2474 & -0.1658\\ -0.1658 & 3.1969\end{bmatrix}$, $X_{21}^{21}=\begin{bmatrix}2.5471 & 0.0923\\ 0.0923 & 3.3726\end{bmatrix}$, $X_{21}^{22}=\begin{bmatrix}1.3482 & -0.3622\\ -0.3622 & 1.8596\end{bmatrix}$

and H_∞ performances given by the scalars $\rho_1=3.7176$ and $\rho_2=4.4442$.

The close-loop subsystem dynamics and the control laws are given in Fig.1, for initial states $x_1(0)=\begin{bmatrix}-1 & 1\end{bmatrix}^T$ and $x_2(0)=\begin{bmatrix}-1 & 1\end{bmatrix}^T$. As we show, the controller stabilized the overall system.

5. CONCLUSIONS

In this paper, a decentralized fuzzy H_∞ based Takagi-Sugeno fuzzy controller design has been provided for n interconnected nonlinear T-S descriptors. The H_∞ criterion has been considered to minimize the fuzzy structure interconnection effects between subsystems. The use of a multiple fuzzy Lyapunov function and a network of n non-PDC control laws lead to less conservativeness in the LMI conditions rather than the classical quadratic case. Finally, a numeric example has been given to illustrate the efficiency and the applicability of the proposed fuzzy approach.

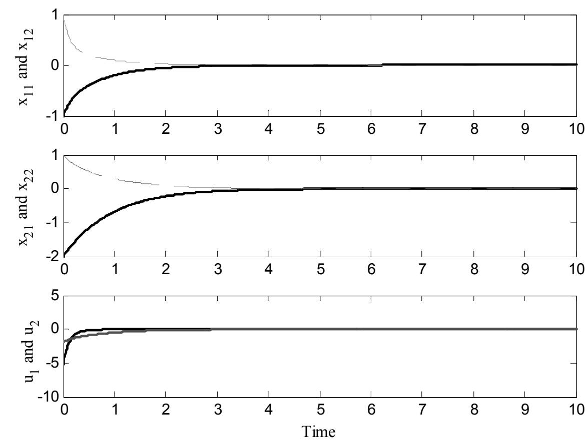

Fig.1. State's dynamics of the closed-loop interconnected descriptors and control signals, (x_{11} , x_{21} , u_1) dotted line, (x_{12} , x_{22} , u_2) solid line.

ACKNOWLEDGEMENT

The authors would like to thank the FEDER and the Champagne Ardennes Region within the CPER MOSYP and the GIS3SGS within the project COSMOS for their support. Many thanks also to Prof. Grant T. Aury for his comments.

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