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***Abstract*—This paper deals with decentralized stabilization of nonlinear systems composed of interconnected Takagi-Sugeno fuzzy descriptors. To ensure the stability of the overall closed-loop system, a set of decentralized Parallel Distributed Compensations (PDC) controllers is employed. The stability conditions are then derived into Linear Matrix Inequalities (LMI) using a fuzzy Lyapunov function for less conservatism. Nevertheless, it contains decision parameters that are not available in practice. So the LMIs are casted into relaxed quadratic conditions using simple assumptions. Finally, a numerical example is proposed to illustrate the effectiveness of the suggested decentralized approach.**

I. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy systems are known as effective and suitable in modelling complex nonlinear systems. Indeed, T-S fuzzy models are composed of a set of linear time invariant systems blended together by nonlinear membership functions [1]. Hence, one of their major interests is that they make possible to extend some of the linear system methodology to the case of nonlinear systems. Stability conditions and stabilization of T-S fuzzy systems are often derived via the second Lyapunov paradigm and formulated using linear matrix inequality (LMI) tools, see e.g. [2][3][4] and references therein. Note that, to take advantages of the fuzzy structure of a considered T-S model to be stabilized, the most commonly used control law is based on the concept of parallel distributed compensation (PDC) [3]. Many works have dealt with synthesizing T-S fuzzy control law for different classes of nonlinear systems. For example, uncertain perturbed systems have been considered in [5] [6] [7] [8] or time delay systems in [9] [10]. In the past few years, enlarging the class of nonlinear systems to be treated with T-S fuzzy approaches, descriptors have been studied in [11] [12] [13] [14] [15]. Indeed, this kind of systems are able to model algebraic constrains in differential equations as included in singular systems [16] and, for instance, mechanical systems with time varying inertia [17]. Moreover, due to complex physical configuration and high dimension of large scale interconnected systems, new approaches for T-S fuzzy decentralized control law design have been developed [18] [19] [20] [21] [22]. These systems are composed of a set of interconnected T-S fuzzy subsystems. Nevertheless, stabilizing controller synthesis for the T-S interconnected descriptors has been seldom treated in the literature. In [23], a first approach was developed. However, this result still restricts on a particular class of interconnected descriptors. The objective of this paper is to provide LMI conditions for decentralized T-S fuzzy controller synthesis to stabilize a wide class of T-S decentralised descriptor.

The paper is organized as follows. First, the class of T-S fuzzy decentralised descriptors studied in this work is presented. Then, a fuzzy state feedback decentralized controller is proposed and, following the second Lyapunov methodology with a fuzzy Lyapunov function, LMI based design is provided. For more practical applicability, conditions leading to relaxed quadratic stabilization are then proposed. Finally, a simulation example, followed by a conclusion, is provided to illustrate the efficiency of the design approach.

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II. T-S DECENTRALIZED DESCRIPTORS

Consider the class of nonlinear interconnected system S composed of n T-S fuzzy descriptor subsystems S_i as follows, for $i = 1,...,n$:

$$\sum_{j=1}^{l_i} v_i^j(z_i) E_i^j \dot{x}_i(t) = \sum_{k=1}^{r_i} h_i^k(z_i) \left\{ A_i^k x_i(t) + B_i^k u_i(t) + \sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha}^k x_\alpha(t) \right\} \quad (1)$$

with $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $z_i(t) \in \mathbb{R}^{p_i}$ represent respectively the state, the input and the premise vectors associated to the i^{th} model. $x_\alpha(t) \in \mathbb{R}^{n_\alpha}$ denotes the state vector of the α^{th} model with $\alpha = 1,...,n$ and $\alpha \neq i$. l_i is the number of fuzzy rules associated to the left-hand side of the state model (1). So, for $j = 1,...,l_i$, $E_i^j \in \mathbb{R}^{n_i \times n_i}$ are constant matrices, if necessary singular, and $v_i^j(z_i) \geq 0$ are the left-hand side membership functions verifying the convexes sum propriety $\sum_{j=1}^{l_i} v_i^j(z_i) = 1$. In the same way, r_i is the number of fuzzy rules associated to the right-hand term in (1). Thus, for $k = 1,...,r_i$ $A_i^k \in \mathbb{R}^{n_i \times n_i}$, $B_i^k \in \mathbb{R}^{n_i \times m_i}$ and $F_{i\alpha}^k \in \mathbb{R}^{n_i \times n_\alpha}$ are constants matrixes and $h_i^k(z_i) \geq 0$ are positive membership functions associated to the right hand side fuzzy rules satisfying the convex sum proprieties $\sum_{k=1}^{r_i} h_i^k(z_i) = 1$. Note that $F_{i\alpha}^k$ is an interconnection matrix which express the influence of the α^{th} subsystem on the i^{th} one.

To ensure the stabilization of the overall closed loop system S, a decentralised Parallel Distributed Compensation (PDC) approach is proposed. The basic idea is to synthesize a decentralized PDC controller composed of n local controller. Each i^{th} fuzzy local controller is able to ensure the stability of the subsystem S_i by considering the interconnections among the others subsystems of the whole system. For more convenience, the local PDC control law $u_i(t)$ shares the same fuzzy sets with the T-S descriptor model of the subsystem S_i. This decentralized PDC controller is given by, for $i = 1,...,n$:

$$u_i(t) = \left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j(z_i)h_i^s(z_i)K_i^{js}\right)\left(\sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j(z_i)h_i^s(z_i)X_{i1}^{js}\right)^{-1} x_i(t) \quad (2)$$

where K_i^{js} are the gain matrices to be synthesized and $X_{i1}^{js} > 0$ are some Lyapunov dependant gain matrices which are justified to obtain LMI conditions in the following (see proof of theorem 1).

Substituting (2) into (1), one obtains the overall closed-loop system S described as, for all $i = 1,...,n$:

$$\sum_{j=1}^{l_i} v_i^j(z_i)E_i^j \dot{x}_i(t)$$

$$= \sum_{j=1}^{l_i}\sum_{k=1}^{r_i}\sum_{s=1}^{r_i} v_i^j(z_i)h_i^k(z_i)h_i^s(z_i)\left\{\begin{aligned}&\left(A_i^k + B_i^k K_i^{js}\left(X_{i1}^{vh}\right)^{-1}\right)x_i(t)\\ &+\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n} F_{i\alpha}^k x_\alpha(t)\end{aligned}\right\} \quad (3)$$

with $X_{i1}^{vh} = \sum_{j=1}^{l_i}\sum_{s=1}^{r_i} v_i^j(z_i)h_i^s(z_i)X_{i1}^{js}$.

The goal is now to design the matrices K_i^{js} and $X_{i1}^{js} > 0$, for $i = 1,...,n$, $j = 1,...,l_i$ and $s = 1,...,r_i$, ensuring the stabilisation of the whole interconnected closed loop system (3).

Notations:

The following notations will be used in the sequel to clarify the mathematical expression.

$$E_i^v = \sum_{j=1}^{l_i} v_i^j(z_i)E_i^j, \; Y_i^{hh} = \sum_{j=1}^{l_i}\sum_{k=1}^{l_i} h_i^j(z_i)h_i^k(z_i)Y_i^{jk},$$

$$T_i^{vhh} = \sum_{j=1}^{l_i}\sum_{s=1}^{r_i}\sum_{k=1}^{r_i} v_i^j(z_i)h_i^s(z_i)h_i^k(z_i)T_i^{jsk} \; \ldots$$

We will also distinguish, for a regular quantity Γ_i^h of appropriate dimension, $\left(\Gamma_i^h\right)^{-1} = \left(\sum_{s=1}^{r_i} h_i^s(z_i)\Gamma_i^s\right)^{-1}$ and $\Gamma_i^{-h} = \sum_{s=1}^{r_i} h_i^s(z_i)\left(\Gamma_i^s\right)^{-1}$. As usual, a star (*) indicates a transpose quantity in a symmetric matrix. The time t will be omitted when there is no ambiguity.

Moreover, the following lemmas will be used in the sequel to run to LMI conditions.

Lemma 1 [24] **:**

Let us consider A and B two matrices of appropriate dimensions and a positive constant $\varepsilon > 0$:

$$A^T B + B^T A \leq \varepsilon AA + \varepsilon^{-1} BB \quad (4)$$

III. Fuzzy Lyapunov LMI controller design

The main purpose of this paper is to provide a design methodology for decentralized PDC fuzzy controller in order to stabilize nonlinear interconnected descriptors described by (1). The main result is summarized in the following theorem.

Theorem 1: Assume that, for $i = 1,...,n$, $j = 1,...,l_i$ and $s = 1,...,r_i$, $\dot{h}_i^s\left(z(t)\right) \geq \varpi_i^s$ and $\dot{v}_i^j\left(z(t)\right) \geq \lambda_i^j$. The closed loop system S composed of n Takagi-Sugeno interconnected descriptor systems S_i described in (1) is stabilized by the *n* PDC decentralized control laws described in (2) if there exist, for all combination of $\{ i = 1,...,n$, $j = 1,...,l_i$, $k = 1,...,r_i$, $s = 1,...,r_i \}$, the matrices $X_{i1}^{js} = \left(X_{i1}^{js}\right)^T > 0$, X_{i3}^{ks}, X_{i4}^{ks}, and K_i^{js} and the positive scalars $\rho_{1i}^k > 0$, $\rho_{2i}^k > 0$, …, $\rho_{ni}^k > 0$, excepted ρ_{ii}^k which do not exist, such that the following LMIs are satisfied:

$$\left(\begin{array}{c|cccc} \Omega_i^{jks} & & (*) & & \\ \hline X_i^{jks} & -\rho_{1i}^k I & 0 & \cdots & 0 \\ \vdots & 0 & -\rho_{2i}^k I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ X_i^{jks} & 0 & \cdots & 0 & -\rho_{ni}^k I \end{array}\right) < 0 \quad (5)$$

with $\tilde{X}_i^{jks}=\begin{bmatrix} X_{i1}^{js} & 0 \\ X_{i3}^{ks} & X_{i4}^{ks}\end{bmatrix}$,

$$\Omega_i^{jks}=\begin{pmatrix} \left(X_{i3}^{ks}\right)^T+X_{i3}^{ks}-(n-1)\Phi_{i1}^{js} & (*) \\ \begin{pmatrix}\left(X_{i4}^{ks}\right)^T+A_i^k X_{i1}^{js} \\ +B_i^s K_i^{jk}-E_i^j X_{i3}^{ks}\end{pmatrix} & \begin{pmatrix} -\left(X_{i4}^{ks}\right)^T\left(E_i^j\right)^T-E_i^j X_{i4}^{ks} \\ +\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\rho_{i\alpha}^k F_{i\alpha}^k\left(F_{i\alpha}^k\right)^T\end{pmatrix}\end{pmatrix},$$

$\Phi_{js}=\sum_{s=1}^{r_i}\varpi_i^s X_{i1}^{js}+\sum_{s=1}^{l}\lambda_i^j \dot{X}_{i1}^{js}$ and removing the column and the row corresponding to ρ_{ii}^k in each right bottom corner of the LMIs defined by (5).

Proof : Let, for $i=1,\ldots,n$, $\tilde{x}_i=\begin{bmatrix}x_i & \dot{x}_i\end{bmatrix}^T$ be extended state vectors. The overall closed loop T-S fuzzy decentralised descriptor system described in (3) can be rewritten, with the proposed notations, as:

$$\text{For all } i=1,\ldots,n,\ \tilde{E}\dot{\tilde{x}}_i=\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\left(\tilde{A}_i^{vhh}\tilde{x}_i+\tilde{F}_{i\alpha}^h\tilde{x}_\alpha\right) \quad (6)$$

with $\tilde{E}=\begin{bmatrix} I & 0 \\ 0 & 0\end{bmatrix}$, $\tilde{A}_i^{vhh}=\frac{1}{n-1}\begin{bmatrix} 0 & I \\ A_i^h+B_i^h K_i^{vh}\left(X_{i1}^{vh}\right)^{-1} & -E_i^v\end{bmatrix}$ and $\tilde{F}_{i\alpha}{}^h=\begin{bmatrix} 0 & 0 \\ F_{i\alpha}{}^h & 0\end{bmatrix}$.

Let us consider the following candidate multiple fuzzy Lyapunov function:

$$V(t)=\sum_{i=1}^{n}V_i\left(x_i(t)\right)\ \geq 0 \quad (7)$$

with

$$V_i\left(x_i(t)\right)=\tilde{x}_i^T(t)\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i(t)\geq 0 \quad (8)$$

and, for $i=1,\ldots,n$, $j=1,\ldots,l_i$, $k=1,\ldots,r_i$, $s=1,\ldots,r_i$:

$$\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}=\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{E}\geq 0 \quad (9)$$

The symmetric condition (9) is verified with the Lyapunov matrix given by $\tilde{X}_i^{vhh}=\begin{bmatrix} X_{i1}^{vh} & 0 \\ X_{i3}^{hh} & X_{i4}^{hh}\end{bmatrix}$ and $X_{i1}^{vh}=\left(X_{i1}^{vh}\right)^T>0$.

The T-S fuzzy decentralized closed-loop descriptor (3) is stable if :

$$\dot{V}=\sum_{i=1}^{n}\left(\dot{\tilde{x}}_i^T\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\tilde{E}\left(\tilde{X}_i^{vhh}\right)^{-1}\dot{\tilde{x}}_i+\tilde{x}_i^T\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\tilde{x}_i\right)<0 \quad (10)$$

Substituting (6) into (10), we get:

$$\dot{V}=\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{pmatrix}\tilde{x}_i{}^T\left(\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\right)\tilde{x}_i \\ +\tilde{x}_\alpha{}^T\left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}\tilde{x}_i+\tilde{x}_i^T\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{F}_{i\alpha}^h\tilde{x}_\alpha \\ +\tilde{x}_i{}^T\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\tilde{x}_i\end{pmatrix}<0 \quad (11)$$

Using (4), (11) is verified if:

$$\sum_{i=1}^{n}\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\begin{pmatrix}\tilde{x}_i{}^T\begin{pmatrix}\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh} \\ +\tau_{i\alpha}^{-h}\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{F}_{i\alpha}^h\left(\tilde{F}_{i\alpha}^h\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1} \\ +\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\end{pmatrix}\tilde{x}_i \\ +\tau_{i\alpha}^h\tilde{x}_\alpha{}^T\tilde{x}_\alpha\end{pmatrix}<0 \quad (12)$$

Notes that, with $\tau_{ii}^h=0$, $\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i\alpha}^h\tilde{x}_\alpha{}^T\tilde{x}_\alpha=\sum_{p=1}^{n}\tau_{ip}^h\tilde{x}_p{}^T\tilde{x}_p$, (12) is verified if:

$$\sum_{i=1}^{n}\tilde{x}_i{}^T\begin{pmatrix}(n-1)\left(\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\right) \\ +\left(\tilde{X}_i^{vhh}\right)^{-T}\left(\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i\alpha}^{-h}\tilde{F}_{i\alpha}^h\left(\tilde{F}_{i\alpha}^h\right)^T\right)\left(\tilde{X}_i^{vhh}\right)^{-1} \\ +(n-1)\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\end{pmatrix}\tilde{x}_i+\sum_{p=1}^{n}\tilde{x}_p{}^T\left(\sum_{i=1}^{n}\tau_{ip}^h\right)\tilde{x}_p<0 \quad (13)$$

The inequality (13) can be rearranged as:

$$\sum_{i=1}^{n}\tilde{x}_i{}^T\begin{pmatrix}(n-1)\left(\left(\tilde{A}_i^{vhh}\right)^T\left(\tilde{X}_i^{vhh}\right)^{-1}+\left(\tilde{X}_i^{vhh}\right)^{-T}\tilde{A}_i^{vhh}\right) \\ +\left(\tilde{X}_i^{vhh}\right)^{-T}\left(\sum_{\substack{\alpha=1\\ \alpha\neq i}}^{n}\tau_{i\alpha}^{-h}\tilde{F}_{i\alpha}^h\left(\tilde{F}_{i\alpha}^h\right)^T\right)\left(\tilde{X}_i^{vhh}\right)^{-1} \\ +\left(\sum_{p=1}^{n}\tau_{pi}^h I\right)+(n-1)\tilde{E}\overset{\bullet}{\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}}\end{pmatrix}\tilde{x}_i<0 \quad (14)$$

So, the inequality (14) is verified if, for all $i = 1,...,n$:

$$(n-1)\left(\left(\tilde{A}_i^{vhh}\right)^T \left(\tilde{X}_i^{vhh}\right)^{-1} + \left(\tilde{X}_i^{vhh}\right)^{-T} \tilde{A}_i^{vhh} + \tilde{E}\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} \right) + \left(\tilde{X}_i^{vhh}\right)^{-T} \left(\sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \tau_{i\alpha}^{-h} \tilde{F}_{i\alpha}^{h} \left(\tilde{F}_{i\alpha}^{h}\right)^T \right) \left(\tilde{X}_i^{vhh}\right)^{-1} + \left(\sum_{p=1}^{n} \left(\tau_{pi}^{h}\right)^{-1} I \right) < 0 \tag{15}$$

Left-multiply and right-multiply the inequality (15) respectively by $\left(\tilde{X}_i^{vhh}\right)^T$ and $\tilde{X}_i^{vhh}$, it yields, for $i = 1,...,n$:

$$(n-1)\left(\left(\tilde{X}_i^{vhh}\right)^T \left(\tilde{A}_i^{vhh}\right)^T + \tilde{A}_i^{vhh}\tilde{X}_i^{vhh} + \tilde{E}\left(\tilde{X}_i^{vhh}\right)^T \overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} \tilde{X}_i^{vhh} \right) + \sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \tau_{i\alpha}^{-h} \tilde{F}_{i\alpha}^{h} \left(\tilde{F}_{i\alpha}^{h}\right)^T + \left(\tilde{X}_i^{vhh}\right)^T \left(\sum_{p=1}^{n} \tau_{pi}^{h} \right) \tilde{X}_i^{vhh} < 0 \tag{16}$$

Note that :

$$\begin{aligned} -\overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} &= \frac{d}{dt}\left\{ \left(\tilde{X}_i^{vhh}\right)^{-1} \tilde{X}_i^{vhh} \right\} \left(\tilde{X}_i^{vhh}\right)^{-1} - \overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} \\ &= \overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} \tilde{X}_i^{vhh} \left(\tilde{X}_i^{vhh}\right)^{-1} + \left(\tilde{X}_i^{vhh}\right)^{-1} \dot{\tilde{X}}_i^{vhh} \left(\tilde{X}_i^{vhh}\right)^{-1} - \overline{\left(\tilde{X}_i^{vhh}\right)^{-1}}^{\cdot} \\ &= \left(\tilde{X}_i^{vhh}\right)^{-1} \dot{\tilde{X}}_i^{vhh} \left(\tilde{X}_i^{vhh}\right)^{-1} \end{aligned} \tag{17}$$

Thus, (16) can be rewritten as:

$$(n-1)\left(\left(\tilde{X}_i^{vhh}\right)^T \left(\tilde{A}_i^{vhh}\right)^T + \tilde{A}_i^{vhh}\tilde{X}_i^{vhh} - \tilde{E}\dot{\tilde{X}}_i^{vhh} \right) + \sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \tau_{i\alpha}^{-h} \tilde{F}_{i\alpha}^{h} \left(\tilde{F}_{i\alpha}^{h}\right)^T + \left(\tilde{X}_i^{vhh}\right)^T \left(\sum_{p=1}^{n} \tau_{pi}^{h} \right) \tilde{X}_i^{vhh} < 0 \tag{18}$$

Recall that $\tau_{ii}^{h} = 0$, applying the Schur complement, (18) can be rewritten as, for all $i = 1,...,n$:

$$\left(\begin{array}{c|cccc} \psi_i^{hhv} & & (*) & & \\ \hline \tilde{X}_i^{vhh} & -\tau_{i\alpha}^{-h} I & 0 & \cdots & 0 \\ \vdots & 0 & -\tau_{i\alpha}^{-h} I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \tilde{X}_i^{vhh} & 0 & \cdots & 0 & -\tau_{i\alpha}^{-h} I \end{array} \right) < 0 \tag{19}$$

with

$$\psi_i^{hhv} = (n-1)\left(\left(\tilde{X}_i^{vhh}\right)^T \left(\tilde{A}_i^{vhh}\right)^T + \tilde{A}_i^{vhh}\tilde{X}_i^{vhh} \right) - (n-1)\tilde{E}\dot{\tilde{X}}_i^{vhh} + \sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \tau_{i\alpha}^{-h} \tilde{F}_{i\alpha}^{h} \left(\tilde{F}_{i\alpha}^{h}\right)^T < 0 \tag{20}$$

and removing the column and the row corresponding to τ_{ii}^{h} in each right bottom corner of the LMIs defined by(19).

(20) can be rewritten in its extended form, with the matrices defined in (6) and the constraints issued from (9), as:

$$\psi_i^{hhv} = \begin{pmatrix} \left(X_{i3}^{hh}\right)^T + X_{i3}^{hh} - (n-1)\dot{X}_{i1}^{vh} & (*) \\ \begin{pmatrix} \left(X_{i4}^{hh}\right)^T + A_i^h X_{i1}^{vh} \\ +B_i^h K_i^{vh} - E_i^v X_{i3}^{hh} \end{pmatrix} & \begin{pmatrix} -\left(X_{i4}^{hh}\right)^T E_i^{vT} - E_i^v X_{i4}^{hh} \\ +\sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \tau_{i\alpha}^{-h} F_{i\alpha}^{h} F_{i\alpha}^{hT} \end{pmatrix} \end{pmatrix} \tag{21}$$

Let us now focus on the term $-(n-1)\dot{X}_{i1}^{vh}$ in (21). From the convex sum propriety, one can write:

$$\dot{X}_{i1}^{vh} = \sum_{j=1}^{l_i} \sum_{s=1}^{r_i} v_i^j h_i^s \left(\sum_{s=1}^{r_i} \dot{h}_i^s X_{i1}^{js} + \sum_{s=1}^{l} \dot{v}_i^j \dot{X}_{i1}^{js} \right) \tag{22}$$

Let us assume that, for $j = 1,...,l_i$, $s = 1,...,r_i$, λ_i^j and ϖ_i^s are respectively the lower bound of $\dot{v}_i^j(z)$ and $\dot{h}_i^s(z)$. One can write $-\dot{X}_{hv}^1 \leq -\Phi_{hv}$ with:

$$\Phi_{hv} = \sum_{j=1}^{l_i} \sum_{s=1}^{r_i} v_i^j h_i^s \left\{ \sum_{s=1}^{r_i} \varpi_i^s X_{i1}^{js} + \sum_{s=1}^{l} \lambda_i^j \dot{X}_{i1}^{js} \right\} \tag{23}$$

Thus, considering (23), the inequality (19) can be bounded and leads to the LMI conditions proposed in theorem 1 with each $\rho_{i\alpha}^k = \left(\tau_{i\alpha}^k\right)^{-1}$. That ends the proof. ■

Remark 1: For $j = 1,...,l_i$, $k = 1,...,r_i$, $s = 1,...,r_i$, $\dot{v}_i^j(z)$ and $\dot{h}_i^s(z)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach [2] or, for instance, when membership functions are chosen with a smoothed shape (Gaussian...).

Remark 2: The LMI conditions proposed in theorems 1 depend on the lower bounds of $\dot{v}_i^j(z)$ and $\dot{h}_i^s(z)$ for $j = 1,...,l_i$, $s = 1,...,r_i$. It is often pointed out as a criticism to fuzzy Lyapunov approach since these parameters may be difficult to choose in practice. Nevertheless, let us recall that this approach remains one of the least conservative in terms of LMI based design. In [25], a fuzzy Lyapunov candidate function has been reduced to lead to relaxed quadratic stability conditions in the case of non decentralized descriptor systems. Indeed, some elements in the Lyapunov matrix can be set common in order to make the LMI free of membership function's lower bounds. For decentralized T-S descriptors, the result is summarized in the following corollary. Note finally that, obviously, the 'price' to pay for more practical applicability is an increase of the conservatism.

Corollary 1: The closed loop system S composed of n Takagi-Sugeno interconnected descriptor systems S_i described in (1) is stabilized by the PDC decentralized control law described in (2) if there exist, for all combination of $\{ i=1,...,n$, $j=1,...,l_i$, $k=1,...,r_i$, $s=1,...,r_i$ $\}$, the matrices $X_{i1}=X_{i1}^T>0$, X_{i3}^{ks}, X_{i4}^{ks}, and K_i^{js} and the positive scalars $\rho_{1i}^k>0$, $\rho_{2i}^k>0$, …, $\rho_{ni}^k>0$, excepted ρ_{ii}^k which does not exist, such that the following LMI conditions are verified:

$$\begin{pmatrix} \Omega_i^{jks} & & (*) & & \\ X_i^{jks} & -\rho_{1i}^k I & 0 & \cdots & 0 \\ \vdots & 0 & -\rho_{2i}^k I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ X_i^{jks} & 0 & \cdots & 0 & -\rho_{ni}^k I \end{pmatrix} < 0 \tag{24}$$

with $\Omega_i^{jks} = \begin{pmatrix} \left(X_{i3}^{ks}\right)^T + X_{i3}^{ks} & (*) \\ \begin{pmatrix} \left(X_{i4}^{ks}\right)^T + A_i^k X_{i1} \\ +B_i^s K_i^{jk} - E_i^j X_{i3}^{ks} \end{pmatrix} & \begin{pmatrix} -\left(X_{i4}^{ks}\right)^T \left(E_i^j\right)^T - E_i^j X_{i4}^{ks} \\ +\sum_{\substack{\alpha=1 \\ \alpha\neq i}}^{n} \rho_{i\alpha}^k F_{i\alpha}^k \left(F_{i\alpha}^k\right)^T \end{pmatrix} \end{pmatrix}$,

and $\tilde{X}_i^{ks} = \begin{bmatrix} X_{i1} & 0 \\ X_{i3}^{ks} & X_{i4}^{ks} \end{bmatrix}$ and removing the column and the row corresponding to ρ_{ii}^k in each right bottom corner of the LMIs defined by (24).

Proof: straightforward from theorem 1 with X_{i1} common for all $j=1,...,l_i$, $s=1,...,r_i$, in the LMI conditions (5) and in the decentralized control law (2). ■

IV. Numerical Example

In order to show the effectiveness of the proposed approach, let us consider the non linear decentralized descriptor S composed of two subsystems S_1 and S_2 described by:

$$S_1: \left| \begin{array}{l} E_1\left(x_1(t)\right)\dot{x}_1(t) \\ = A_1\left(x_1(t)\right)x_1(t) + B_1\left(x_1(t)\right)u_1(t) + F_{12}\left(x_1(t)\right)x_2(t) \end{array} \right. \tag{25}$$

with $x_1 = \begin{bmatrix} x_{11} & x_{12} \end{bmatrix}^T$, $E_1(x_1) = \begin{bmatrix} 1 & \cos^2(x_{11}) \\ -1 & 1 \end{bmatrix}$,

$A_1(x_1) = \begin{bmatrix} -0.73\sin^2(x_{11}) & 1 \\ 0.839 & -0.73 \end{bmatrix}$,

$B_1(x_1) = \begin{bmatrix} 0.47 + 0.53\sin^2(x_{11}) \\ 1.263 \end{bmatrix}$,

$F_{12}(x_1) \begin{bmatrix} 0 & 0 \\ 0.1 + 0.1\sin^2(x_{11}) & 0 \end{bmatrix}$,

and

$$S_2: \left| \begin{array}{l} E_2\left(x_2(t)\right)\dot{x}_2(t) \\ = A_2\left(x_2(t)\right)x_2(t) + B_2\left(x_2(t)\right)u_2(t) + F_{21}\left(x_2(t)\right)x_1(t) \end{array} \right. \tag{26}$$

with $x_2 = \begin{bmatrix} x_{21} & x_{22} \end{bmatrix}^T$, $E_2(x_2) = \begin{bmatrix} 1 & 0.2\cos^2(x_{21}) \\ 0 & 1 \end{bmatrix}$,

$A_2(x_2) = \begin{bmatrix} -1 & 1 \\ 0.839 - 0.839\sin^2(x_{21}) & 0.931 \end{bmatrix}$,

$B_2(x_2) = \begin{bmatrix} 0.47 \\ 0.4 + 0.4\sin^2(x_{21}) \end{bmatrix}$,

$F_{21}(x_2) = \begin{bmatrix} 0 & 0 \\ 0.3 + 0.2\sin^2(x_{21}) & 0 \end{bmatrix}$.

Since S_1 and S_2 contain bounded nonlinearities, an exact fuzzy modelling of the non linear system S can be obtained via the sector non linearity approach [2]. This can be achieved by the following T-S fuzzy descriptors:

$$S_1: \left| \begin{array}{l} \sum_{j=1}^{2} v_1^j\left(x_1(t)\right)E_1^j\dot{x}_1(t) \\ = \sum_{k=1}^{2} h_1^k\left(x_1(t)\right)\left\{A_1^k x_1(t) + B_1^k u_1(t) + F_{12}^k x_2(t)\right\} \end{array} \right. \tag{27}$$

with $E_1^1 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $E_1^2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, $A_1^1 = \begin{bmatrix} 0 & 1 \\ 0.839 & -0.73 \end{bmatrix}$,

$A_1^2 = \begin{bmatrix} -0.73 & 1 \\ 0.839 & -0.73 \end{bmatrix}$, $B_1^1 = \begin{bmatrix} 0.47 \\ 1.263 \end{bmatrix}$, $B_1^2 = \begin{bmatrix} 1 \\ 1.263 \end{bmatrix}$,

$F_{12}^1 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}$, $F_{12}^2 = \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix}$ and the membership functions $v_1^1\left(x_1(t)\right) = \cos^2\left(x_{11}(t)\right)$, $v_1^2\left(x_1(t)\right) = \sin^2\left(x_{11}(t)\right)$, $h_1^1\left(x_1(t)\right) = \sin^2\left(x_{11}(t)\right)$ and $h_1^2\left(x_1(t)\right) = \cos^2\left(x_{11}(t)\right)$.

and

$$S_2: \left| \begin{array}{l} \sum_{j=1}^{2} v_2^j\left(x_2(t)\right)E_2^j\dot{x}_2(t) \\ = \sum_{k=1}^{2} h_2^k\left(x_2(t)\right)\left\{A_2^k x_2(t) + B_2^k u_2(t) + F_{21}^k x_1(t)\right\} \end{array} \right. \tag{28}$$

with $E_2^1 = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$, $E_1^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A_2^1 = \begin{bmatrix} -1 & 1 \\ 0.839 & -0.931 \end{bmatrix}$,

$A_2^2 = \begin{bmatrix} -1 & 1 \\ 0 & 0.931 \end{bmatrix}$, $B_2^1 = \begin{bmatrix} 0.47 \\ 0.4 \end{bmatrix}$, $B_2^2 = \begin{bmatrix} 0.47 \\ 0.8 \end{bmatrix}$,

$F_{21}^1 = \begin{bmatrix} 0 & 0 \\ 0.3 & 0 \end{bmatrix}$, $F_{21}^2 = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$, and the membership functions $v_2^1\left(x_2(t)\right) = \cos^2\left(x_{21}(t)\right)$,

$v_2^2\left(x_2(t)\right) = \sin^2\left(x_{21}(t)\right)$, $h_2^1\left(x_2(t)\right) = \sin^2\left(x_{21}(t)\right)$,

$h_2^2\left(x_2(t)\right) = \cos^2\left(x_{21}(t)\right)$.

The Matlab LMI toolbox is used to solve the conditions provided in corollary 1. This solution leads to the synthesis of a fuzzy controller composed of two PDC decentralized control of the form (2) given by the matrices:

$K_1^{11} = [\text{-5.3775} \quad \text{-8.0881}]$, $K_1^{12} = [\text{-1.9931} \quad \text{-9.9280}]$,

$K_1^{21} = [\text{-3.4877} \quad \text{-11.3587}]$, $K_1^{22} = [\text{-1.2986} \quad \text{-13.3339}]$,

$K_2^{11} = [\text{-9.7975} \quad \text{-26.6441}]$, $K_2^{12} = [\text{-0.2273} \quad \text{-32.3822}]$,

$K_2^{21} = [\text{-7.3477} \quad \text{-26.1544}]$, $K_2^{22} = [1.9413 \quad \text{-31.8932}]$,

$X_{11} = \begin{bmatrix} 9.0663 & -3.9758 \\ -3.9758 & 3.1366 \end{bmatrix}$ and $X_{21} = \begin{bmatrix} 9.7584 & -3.6790 \\ -3.6790 & 3.7428 \end{bmatrix}$.

Moreover, from the LMI computation, the scalars $\rho_{12}^1 = 33.6574$, $\rho_{12}^2 = 33.6574$, $\rho_{21}^1 = 31.7767$ and $\rho_{21}^2 = 29.4103$ complete the solution of corollary 1.

Fig. 1 show the close-loop subsystem dynamics and the control laws simulated with initial states $x_1(0) = [-1 \quad 1]^T$ and $x_2(0) = [-2 \quad 1]^T$.

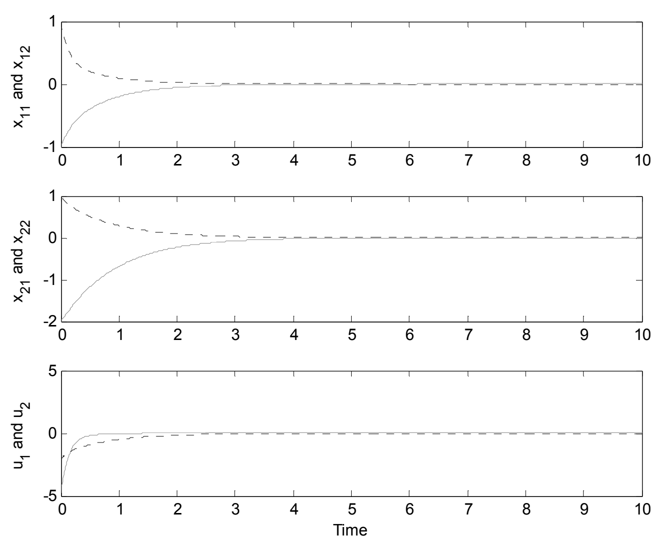

Fig.1. States dynamics of the closed loop interconnected descriptor system and control signals, (x_{11}, x_{21}, u_1) dotted line, (x_{12}, x_{22}, u_2) solid line.

V. Conclusion

In this paper, new LMI stabilization conditions are proposed for T-S interconnected descriptor systems. These LMI conditions are able to stabilize a wider class of descriptors enlarging the results found in the literature [23]. One more improvement is the use of a multiple fuzzy Lyapunov function making the results less conservative than classical quadratic ones. These LMI conditions constitute a convenient design methodology for decentralized non-PDC controller composed of n control law for n subsystem. In order to illustrate the proposed approach, a non linear system composed of two nonlinear interconnected subsystems is given. This one is rewritten as the T-S fuzzy model via the sector nonlinearity approach and then a decentralized PDC control law is designed. Finally, the efficiency of the proposed approach has been shown in simulation.

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VII. References

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