Redundancy approach for fuzzy Lyapunov stabilization of Takagi-Sugeno descriptors

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Abstract—**This paper presents new stability conditions for Takagi-Sugeno (T-S) descriptors. Thanks to the descriptor redundancy, LMI based controller design of less conservatism are derived. These conditions are obtained using a fuzzy Non Quadratic Lyapunov function (NQLF) approach with a non-Parallel-Distributed-Compensation (Non-PDC) control law. First, stabilization without constraints is considered for the sake of genericity. Then, H_∞ based controller design is proposed in order to deal with the stability of closed loop descriptors subject to external disturbances. Finally, a numerical example is proposed to illustrate the efficiency of the proposed approaches.**

I. Introduction

Takagi-Sugeno fuzzy systems [1] has attracted a great interest because of their ability to approximate a large class of nonlinear systems, for instance using the sector nonlinearity approach [2]. Since their introduction, a lot of works concerned with the stability and the stabilization of such systems have been reported; see e.g. [2-11]. In most of the cases, based on the Lyapunov theory, these conditions are expected to be written in term of Linear Matrix Inequalities (LMI). For instance, quadratic stability and stabilization of T-S models have been proposed by [2][3]. Nevertheless, these results based on a quadratic Lyapunov function (QLF) require computing a common Lyapunov matrix for a set of LMIs and, by consequence are conservative. A way to relax the above stability conditions is to consider another type of candidate Lyapunov function. In this way, stability conditions based on piecewise Lyapunov function (PWLF) have been proposed by [4][5][6]. More extensively, non-quadratic Lyapunov functions (NQLF) have been considered by [7][8][9]. Note that, from NQLF, one can distinguish fuzzy non quadratic Lyapunov function (NQLF) approaches that seem to be the "natural" way to be investigated within T-S fuzzy models [5][10]. Moreover, most of works have been proposed dealing with "standard" state space T-S systems. A more general class of nonlinear systems described by algebraic differential equations is called descriptor [12][13]. This latter was first studied in its T-S form by [14][15]. In other hand, descriptor systems are able to represent singular systems [13] and constitute a "natural" representation of mechanical systems with time varying inertia [16][17]. Another improvement of T-S descriptor systems is that they allow reducing the number of local models leading to stability conditions with less computational cost. Quadratic stabilization and stability conditions for uncertain T-S descriptors systems have been proposed in [18][19] and with a H_∞ criterion in [20][21]. Nevertheless, in [18], conditions are in a Bilinear Matrix Inequality (BMI) form. Moreover, these approaches are still conservative since they are based on a QLF. A first result dealing with Fuzzy Lyapunov Function (FLF) based stability of T-S descriptor systems leading to less conservative results has been proposed in [22]. Nevertheless, this result is still quadratic since the chosen FLF is partially common in order to avoid the derivative of the membership functions introduced by the use of a complete NQLF. A generalization to full non quadratic LMI conditions has been proposed in our previous study to tackle uncertain T-S descriptors [23]. On the other hand, descriptor redundancy has been used to derive new stability conditions for "standard" state space T-S systems [24]. The main interest of this approach is to reduce the number of LMI conditions to be solved leading to less computational cost.

In this paper, one proposes to take advantages of the descriptor redundancy to reduce the conservatism of previous fuzzy Lyapunov conditions for the stabilization of T-S descriptors [22][23]. Thus, new LMI based controller design for descriptor systems are proposed (without and then with external disturbances). A non-PDC control law is employed and LMI conditions are obtained using a NQLF. Then, the efficiency of the proposed approaches in comparison to the one proposed in [22] is illustrated through an academic example.

II. Notations

Let us consider, for $k \in \{1,\dots,l\}$ and $i \in \{1,\dots,r\}$, the scalar functions $v_k(z)$ and $h_i(z)$, the matrices N_k, G_i and F_{ik} with appropriate dimensions. We will denote $N_v = \sum_{k=1}^{l} v_k(z) N_k$, $G_h = \sum_{i=1}^{r} h_i(z) G_i$ and $F_{hv} = \sum_{k=1}^{l}\sum_{i=1}^{r} v_k(z) h_i(z) F_{ik}$. As usual a star $(*)$ indicates a transpose quantity in a symmetric matrix. In the sequel, for

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space convenience, the time t in a time varying variable will be omitted when there is no ambiguity. Moreover, in order to lighten the mathematical expressions, a non linear matrix $X(z)$ depending on the premises will be denoted X_z as its interconnection structure is not defined.

III. T-S DESCRIPTOR SYSTEMS AND PROBLEM STATEMENT

Let us consider the class of T-S descriptor systems with external disturbances described by:

$$\sum_{k=1}^{l} v_k\left(z(t)\right)E_k\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)A_i x(t) + B_i u(t) + W_i\gamma(t) \quad (1)$$

where l and r represent respectively the number of fuzzy rules at the left and right of the state representation. $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$, $\gamma(t)\in\mathbb{R}^d$ represent respectively the state vector, the input vector and the unknown external disturbances vector bounded such that $\gamma_{up}\geq\|\gamma(t)\|$. $z(t)\in\mathbb{R}^f$ is the premise vector, $v_k\left(z(t)\right)$ and $h_i\left(z(t)\right)$ are positive membership functions satisfying the convex sum proprieties $\sum_{i=1}^{r} h_i\left(z(t)\right)=1$ and $\sum_{k=1}^{l} v_k\left(z(t)\right)=1$. $E_k\in\mathbb{R}^{n\times n}$, $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times m}$ and $W_i\in\mathbb{R}^{d\times n}$ are real state matrices.

In the sequel, one assumes that (1) is regular and impulse free [13]. To stabilize (1), one considers the non-PDC control law given by:

$$u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{l} h_i\left(z(t)\right)v_k\left(z(t)\right)S_{ik}\left(X_z^1\right)^{-1}x(t) \quad (2)$$

with S_{ik} and X_z^1 are real gain matrices with appropriate dimensions to be synthesized. Note that the fuzzy structure of X_z^1 will be defined and justified in the following.

As usual for descriptor systems, let us consider $\bar{x}(t)=\left[x^T(t) \quad \dot{x}^T(t)\right]^T$. Thus, (1) and (2) can be rewritten with the notations defined below as:

$$\bar{E}\,\dot{\bar{x}}(t) = \bar{A}_{hv}\bar{x}(t) + \bar{B}_h u(t) + \bar{W}_h\gamma(t) \quad (3)$$

with $\bar{E}=\begin{bmatrix} I & 0\\ 0 & 0\end{bmatrix}$, $\bar{A}_{hv}=\begin{bmatrix} 0 & I\\ A_h & -E_v\end{bmatrix}$, $\bar{B}_h=\begin{bmatrix} 0\\ B_h\end{bmatrix}$, $\bar{W}_h=\begin{bmatrix} 0\\ W_h\end{bmatrix}$.

$$u(t) = -\bar{K}_{hvz}\bar{x}(t) \quad (4)$$

with $\bar{K}_{hvz}=\left[S_{hv}\left(X_z^1\right)^{-1} \quad 0\right]$.

In order to take advantage of the descriptor redundancy, (4) can be rewritten as:

$$0\dot{u}(t) = u(t) + \bar{K}_{hvz}\bar{x}(t) \quad (5)$$

with $0\in\mathbb{R}^{m\times m}$ is a zero matrix. Therefore, the descriptor redundancy closed loop dynamics, combining (3) and (5), is given by:

$$\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}_{hvz}\tilde{x}(t) + \tilde{W}_h\gamma(t) \quad (6)$$

with $\tilde{E}=\begin{bmatrix}\bar{E} & 0\\ 0 & 0\end{bmatrix}$, $\tilde{A}=\begin{bmatrix}\bar{A}_{hv} & \bar{B}_h\\ \bar{K}_{hvz} & I\end{bmatrix}$, $\tilde{W}_h=\begin{bmatrix}\bar{W}_h\\ 0\end{bmatrix}$ and $\tilde{x}(t)=\left[\bar{x}^T(t) \quad u^T(t)\right]^T$.

Note that, the descriptor redundancy approach allows removing crossing terms between the control gains and the input matrix. Then, the goal is to derive LMI stability conditions with external disturbances attenuation for (6).

IV. STABILITY CONDITIONS AND LMI FORMULATIONS

In this section, the stability of (6) without external disturbances (i.e. $\gamma(t)=0$) will be first considered. The solution of this problem is summarized in the following theorem.

Theorem 1: Assume that $\forall z(t)$, $\mu\in\{1,...,r-1\}$, $\varepsilon\in\{1,...,l-1\}$, $\dot{h}_\mu\left(z(t)\right)\geq\lambda_\mu$ and $\dot{v}_\varepsilon\left(z(t)\right)\geq\varpi_\varepsilon$. The T-S descriptor systems (1) with ($\gamma(t)=0$) is globally asymptotically stable via the non-PDC control law (2), if there exist: the matrices $X_{jk}^1 = X_{jk}^{1\,T}>0$, X_{ij}^4, X_{ij}^5, X_{ij}^6, X_{jk}^7, X_{jk}^8, X_{jk}^9 and S_{jk} such that, for all $i,j\in\{1,...,r\}$ and $k=1,...,l$:

$$\Psi_{ijk}<0 \quad (7)$$

$$X_{j\varepsilon}^1 - X_{jl}^1\geq 0 \text{ and } X_{\mu k}^1 - X_{rk}^1\geq 0 \quad (8)$$

with

$$\Psi_{ijk}=\begin{bmatrix} X_{ij}^{4T}+X_{ij}^4+\Omega & (*) & (*)\\ \begin{pmatrix} X_{ij}^{5T}+A_iX_{jk}^1-\\ E_kX_{ij}^4+B_iX_{jk}^7\end{pmatrix} & \begin{pmatrix} -X_{ij}^{5T}E_k^T-E_kX_{ij}^5+\\ X_{jk}^{8\,T}B_i^T+B_iX_{jk}^8\end{pmatrix} & (*)\\ X_{ij}^{6T}+S_{jk}+X_{jk}^7 & \begin{pmatrix} -X_{ij}^{6T}E_k^T+\\ X_{jk}^{9\,T}B_i^T+X_{jk}^8\end{pmatrix} & \begin{pmatrix} X_{jk}^{9\,T}+\\ X_{jk}^9\end{pmatrix}\end{bmatrix}$$

and $\Omega = -\sum_{\varepsilon=1}^{l-1} \varpi_\varepsilon(z)\left(X_{j\varepsilon}^1 - X_{jl}^1\right) - \sum_{\mu=1}^{r-1} \lambda_\mu(z)\left(X_{\mu k}^1 - X_{rk}^1\right)$.

Proof: Let us consider the following candidate fuzzy NQLF:

$$V\left(\tilde{x}(t)\right) = \tilde{x}^T(t)\tilde{E}X_z^{-1}\tilde{x}(t) \tag{9}$$

with X_z is a time varying non singular matrix and

$$\tilde{E}X_z^{-1} = X_z^{-T}\tilde{E} > 0 \tag{10}$$

Let us consider $X_z = \begin{bmatrix} X_z^1 & X_z^2 & X_z^3 \\ X_z^4 & X_z^5 & X_z^6 \\ X_z^7 & X_z^8 & X_z^9 \end{bmatrix}$, from (10), we obtain:

$X_z^1 = X_z^{1T} > 0$ and $X_z^2 = X_z^3 = 0$.

The T-S descriptor system (6) is stable if the derivative of (9) along the trajectory of (6) is negative, i.e.:

$$\dot{V}(\tilde{x}) = \dot{\tilde{x}}^T\tilde{E}X_z^{-1}\tilde{x} + \tilde{x}^T\tilde{E}X_z^{-1}\dot{\tilde{x}} + \tilde{x}^T\tilde{E}\dot{X}_z^{-1}\tilde{x} < 0 \tag{11}$$

According to (10) and substituting (6) in (11), one obtains:

$$\tilde{A}^T X_z^{-1} + X_z^{-T}\tilde{A} + \tilde{E}\dot{X}_z^{-1} < 0 \tag{12}$$

Multiplying left and right respectively by X_z^T and X_z, and considering (10), (12) yields:

$$X_z^T\tilde{A}^T + \tilde{A}X_z + \tilde{E}X_z\dot{X}_z^{-1}X_z < 0 \tag{13}$$

Note that:

$$\begin{aligned} &\frac{d}{dt}\left\{X_z^{-1}X_z\right\} = \frac{d}{dt}\{I\} = \frac{d}{dt}\left\{X_z^{-1}\right\}X_z + X_z^{-1}\frac{d}{dt}\left\{X_z\right\} = 0 \\ &\Leftrightarrow -\frac{d}{dt}\left\{X_z^{-1}\right\} = X_z^{-1}\frac{d}{dt}\left\{X_z\right\}X_z^{-1} \end{aligned} \tag{14}$$

Thus (13) becomes:

$$X_z^T\tilde{A}^T + \tilde{A}X_z - \tilde{E}\dot{X}_z < 0 \tag{15}$$

This can be rewritten in its extended form as:

$$\begin{bmatrix} X_z^{4T} + X_z^4 - \dot{X}_z^1 & (*) & (*) \\ \begin{pmatrix} X_z^{5T} + A_h X_z^1 - \\ E_v X_z^4 + B_h X_z^7 \end{pmatrix} & \begin{pmatrix} -X_z^{5T}E_v^T - E_v X_z^5 + \\ X_z^{8T}B_h^T + B_h X_z^8 \end{pmatrix} & (*) \\ X_z^{6T} + S_{hv} + X_z^7 & X_z^{9T}B_h^T - X_z^{6T}E_v^T + X_z^8 & X_z^{9T} + X_z^9 \end{bmatrix} < 0 \tag{16}$$

Regarding to (16), a way to derive less conservative LMI stability conditions is to choose an interconnection fuzzy structure as a triple sum. Thus, one can choose:

$X_z^1 = X_{hv}^1 = \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k X_{jk}^1$, $X_z^4 = X_{hh}^4 = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}^4$,

$X_z^5 = X_{hh}^5 = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}^5$, $X_z^6 = X_{hh}^6 = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}^6$,

$X_z^7 = X_{hv}^7 = \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k X_{jk}^7$, $X_z^8 = X_{hv}^8 = \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k X_{jk}^8$,

and $X_z^9 = X_{hv}^9 = \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k X_{jk}^9$.

In this case:

$$\dot{X}_z^1 = \dot{X}_{hv}^1 = \sum_{j=1}^{r}\sum_{k=1}^{l} \dot{h}_j v_k X_{jk}^1 + \sum_{j=1}^{r}\sum_{k=1}^{l} h_j \dot{v}_k X_{jk}^1 \tag{17}$$

From the convex sum propriety, one has $\dot{h}_r(z) = -\sum_{\mu=1}^{r-1}\dot{h}_\mu(z)$ and $\dot{v}_l(z) = -\sum_{\varepsilon=1}^{l-1}\dot{v}_\varepsilon(z)$. Therefore, (17) can be rewritten as:

$$\begin{aligned} \dot{X}_{hv}^1 &= \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k \left\{\left(\sum_{\varepsilon=1}^{l-1}\dot{v}_\varepsilon X_{j\varepsilon}^1 + \dot{v}_l X_{jl}^1\right) + \left(\sum_{\mu=1}^{r-1}\dot{h}_\mu X_{\mu k}^1 + \dot{h}_r X_{rk}^1\right)\right\} \\ &= \sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k \left\{\sum_{\varepsilon=1}^{l-1}\dot{v}_\varepsilon\left(X_{j\varepsilon}^1 - X_{jl}^1\right) + \sum_{\mu=1}^{r-1}\dot{h}_\mu\left(X_{\mu k}^1 - X_{rk}^1\right)\right\} \end{aligned} \tag{18}$$

For $j = 1,...,r$, $k = 1,...,l$, $h_j(z)$ and $v_k(z)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach [2] if the system (1) is at least C^1 or, for instance when membership functions are chosen with a smoothed Gaussian shape. Let us assume that, for $j = 1,...,r-1$, $k = 1,...,l-1$, λ_j and ϖ_k are the lower bound of $\dot{h}_j(z)$ and $\dot{v}_k(z)$. One can write:

$$-\dot{X}_{hv}^1 \le -\sum_{j=1}^{r}\sum_{k=1}^{l} h_j v_k \left\{\sum_{\varepsilon=1}^{l-1}\varpi_\varepsilon\left(X_{j\varepsilon}^1 - X_{jl}^1\right) + \sum_{\mu=1}^{r-1}\lambda_\mu\left(X_{\mu k}^1 - X_{rk}^1\right)\right\} \tag{19}$$

with $X_{j\varepsilon}^1 - X_{jl}^1 \ge 0$ and $X_{\mu k}^1 - X_{rk}^1 \ge 0$.

Then, from (16) and (19); one obtains:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{l} h_i(z)h_j(z)v_k(z)\Psi_{ijk} < 0 \tag{20}$$

with the Ψ_{ijk} defined in (7).

Finally, (20) is sufficiently satisfied if (7) is verified. That ends the proof. ■

V. H_∞ PERFORMANCES

The stability conditions proposed in the previous section (theorem 1) concerns the class of T-S descriptor systems without external disturbances ($\gamma(t)=0$). Now, in this section, the aim is to propose a H_∞ based controller design the ensure the attenuation of the external disturbances. Let us consider the H_∞ criterion given by:

$$\int_{t_0}^{t_f} \tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - \eta^2 \int_{t_0}^{t_f} \tilde{\gamma}^T(t)\tilde{\gamma}(t) \le 0 \tag{21}$$

where t_0 and t_f are respectively the initial time and the final time. $\tilde{Q} = \begin{bmatrix} Q & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} > 0$ with Q a weighting matrix. η is the attenuation level.

Theorem 2: Assume that $\forall z(t)$, $\mu \in \{1,...,r-1\}$, $\varepsilon \in \{1,...,l-1\}$, $\dot{h}_\mu(z(t)) \ge \lambda_\mu$ and $\dot{v}_\varepsilon(z(t)) \ge \varpi_\varepsilon$. The T-S descriptor system (1) is globally asymptotically stable via the non-PDC control law (2) and attenuates the external disturbance with $\gamma(t) \ne 0$ with the attenuation level η if there exist the matrices $X_{jk}^1 = X_{jk}^{1\,T} > 0$, X_{ij}^4, X_{ij}^5, X_{ij}^6, X_{jk}^7, X_{jk}^8, X_{jk}^9 and S_{jk}, the positive scalar $\rho = \eta^2$, such that, for all $i, j \in \{1,...,r\}$ and $k = 1,...,l$:

$$\left[\begin{array}{cccc|c} & \Psi_{ijk} & & & (*) \\ \hline X_{jk}^1 & 0 & 0 & -Q^{-1} & 0 \\ 0 & W_i^T & 0 & 0 & -\rho I \end{array}\right] < 0 \tag{22}$$

$$X_{j\varepsilon}^1 - X_{jl}^1 \ge 0 \text{ and } X_{\mu k}^1 - X_{rk}^1 \ge 0 \tag{23}$$

Proof: The closed loop dynamics subject to external disturbances (6) is stable under the H_∞ performance (21) if:

$$\dot{V}(\tilde{x}) + \tilde{x}^T\tilde{Q}\tilde{x} - \eta^2\gamma^T(t)\gamma(t) < 0 \tag{24}$$

According to (10), (11) and substituting (6) in (24), one obtains:

$$\begin{aligned} &\tilde{x}^T\left(\tilde{A}^T X_z^{-1} + X_z^{-T}\tilde{A} + \tilde{Q} + \tilde{E}\dot{X}_z^{-1}\right)\tilde{x} + \\ &\qquad \gamma^T \tilde{W}_h^T \tilde{x} + \tilde{x}^T \tilde{W}_h \gamma - \eta^2 \gamma^T \gamma < 0 \end{aligned} \tag{25}$$

That is to say:

$$\begin{bmatrix} \tilde{x}^T \\ \gamma^T \end{bmatrix} \begin{bmatrix} \tilde{A}^T X_z^{-1} + X_z^{-T}\tilde{A} + \tilde{Q} + \tilde{E}\dot{X}_z^{-1} & (*) \\ \tilde{W}_h^T X_z^{-1} & -\eta^2 I \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \gamma \end{bmatrix} < 0 \tag{26}$$

Thus (26) is equivalent to:

$$\begin{bmatrix} \tilde{A}^T X_z^{-1} + X_z^{-T}\tilde{A} + \tilde{Q} + \tilde{E}\dot{X}_z^{-1} & (*) \\ \tilde{W}_h^T X_z^{-1} & -\eta^2 I \end{bmatrix} < 0 \tag{27}$$

Multiplying (27) left and right respectively by $\begin{bmatrix} X_z^T & 0 \\ 0 & I \end{bmatrix}$ and $\begin{bmatrix} X_z & 0 \\ 0 & I \end{bmatrix}$, one obtains:

$$\begin{bmatrix} X_z^T\tilde{A}^T + \tilde{A}X_z + X_z^T\tilde{Q}X_z + X_z^T\tilde{E}\dot{X}_z^{-1}X_z & (*) \\ \tilde{W}_h^T & -\eta^2 I \end{bmatrix} < 0 \tag{28}$$

Following the same steps as for the proof of theorem 1 and applying the Schur complement, one obtains obviously the conditions given in theorem 2, with $\rho = \eta^2$. ■

VI. EXAMPLE AND SIMULATION.

To illustrate the efficiency of the proposed approaches, we consider the following nonlinear disturbed descriptor system:

$$\sum_{k=1}^{2} v(z(t)) E_k \dot{x}(t) = \sum_{i=1}^{2} h_i(z(t)) \{A_i x(t) + B_i u(t) + W_i \gamma(t)\} \tag{29}$$

where $E_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -4.3 & 4.8 \\ -1.7 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} a & -4.6 \\ 3.9 & -1.9 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0.9 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ b \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $W_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ and $W_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$. a and b are two real parameters. For simulations realization the membership functions are chosen as: $v_1(x_1(t)) = \dfrac{1+\sin(x_1(t))}{2}$, $v_2(x_1(t)) = \dfrac{1-\sin(x_1(t))}{2}$, $h_1(x_1(t)) = \dfrac{1-\cos(x_1(t))}{2}$ and $h_2(x_1(t)) = \dfrac{1+\cos(x_1(t))}{2}$.

Remark 1: In the following simulation results, the lower bounds values of ϖ_ε and λ_μ associated respectively to $\dot{v}(z(t))$ and $\dot{h}(z(t))$ are chosen as: $\varpi_\varepsilon = \lambda_\mu = -5$.

The following results are obtained by using the Matlab LMI Toolbox [25]. Fig. 1 shows the feasibility area using theorem 1, the one using the relaxed quadratic stability conditions given in [22] and the standard quadratic case ([19] without uncertainties), for several values of parameters $a \in [-13.5 \quad -5]$ and $b \in [-6 \quad -4.9]$. From Fig 1, we remark that the stability conditions proposed in theorem 1 outperforms the existing ones. Indeed, the feasibility region provided by theorem 1 is broader than the one obtained via the result in [22] and the quadratic case.

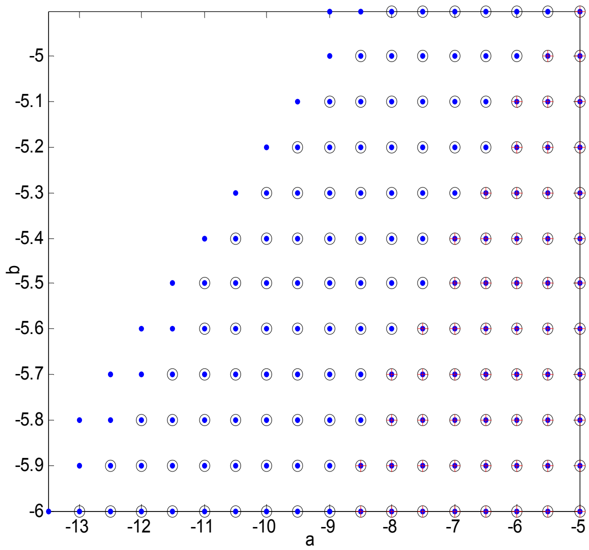

Fig 1: Feasibility area ("." Theorem 1 (NQLF), "o" [22] (Relaxed QLF) and "+" [19] (QLF))

Let us, for example, consider $a = -13$ and $b = -5.8$. Note that for these values, no solution can be obtained using the result given in [22] and [19]. On the other hand, for same values of a and b, (29) can be stabilized with a fuzzy controller of the form (2) synthesized by solving the set of LMIs given in theorem 1. In that case, fig 2 illustrates the convergence of the state vector and the evolution of the control signal for the initial condition $x(0) = [-2.5 \quad 2]^T$. The non-PDC controller design is given by the gain matrices:

$$S_{11} = [\,-17.1698 \quad 268.9822\,], S_{12} = [-5.1359 \quad 268.0605],$$

$$S_{21} = [-12.3 \quad -1892.3], S_{22} = [-12.5 \quad -1892.1],$$

$$X_{11}^1 = \begin{bmatrix} 47.4879 & 41.8354 \\ 41.8354 & 51.1540 \end{bmatrix}, \quad X_{12}^1 = \begin{bmatrix} 46.4548 & 41.3193 \\ 41.3193 & 50.8946 \end{bmatrix},$$

$$X_{21}^1 = \begin{bmatrix} 46.8401 & 41.5333 \\ 41.5333 & 51.0108 \end{bmatrix} \text{ and } X_{22}^1 = \begin{bmatrix} 46.2339 & 41.229 \\ 41.229 & 50.8554 \end{bmatrix}.$$

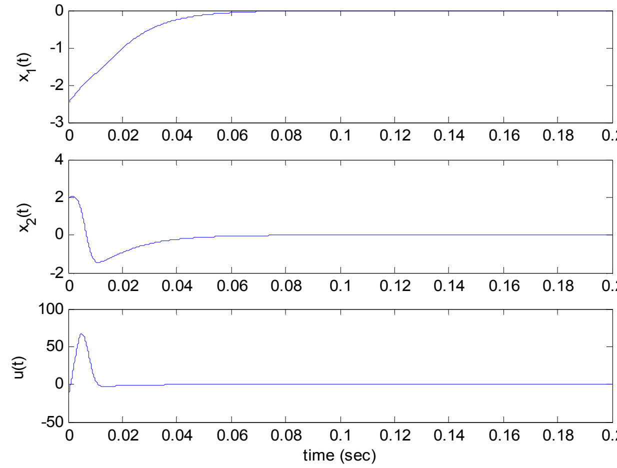

Fig. 2: Simulation results of the theorem 1.

Remark 2: The LMI conditions proposed in theorem 1 and 2 are depending on the lower bounds of $\dot{h}_\mu(z(t))$ and $\dot{v}_\varepsilon(z(t))$ for $\mu \in \{1,...,r-1\}$ and $\varepsilon \in \{1,...,l-1\}$. This is often pointed out as a criticism to fuzzy Lyapunov approach since these parameters may be difficult to choose. Notice that a way to choose them has been proposed in [5]. Moreover, one can state that the plant's physical bounds may also be considered when a known engineering example is studied. Nevertheless, to reduce the number of functions to be bounded, one can also choose an interconnection fuzzy structure as a double sum for (16) and (28) by choosing:

$$X_z^1 = X_v^1 = \sum_{k=1}^{l} v_k X_k^1, \quad X_z^4 = X_h^4 = \sum_{i=1}^{r} h_i X_i^4,$$

$$X_z^5 = X_h^5 = \sum_{i=1}^{r} h_i X_i^5, \quad X_z^6 = X_h^6 = \sum_{i=1}^{r} h_i X_i^6,$$

$$X_z^7 = X_v^7 = \sum_{k=1}^{l} v_k X_k^7, \quad X_z^8 = X_v^8 = \sum_{k=1}^{l} v_k X_k^8$$

and $X_z^9 = X_v^9 = \sum_{k=1}^{l} v_k X_k^9$.

In this special case $\dot{X}_v^1 = \sum_{k=1}^{l} v_k(z) \left\{ \sum_{\varepsilon=1}^{l-1} \varpi_\varepsilon \left(X_\varepsilon^1 - X_l^1 \right) \right\}$, the only functions to be bounded are $\dot{v}_\varepsilon(z(t))$ for $\varepsilon \in \{1,...,l-1\}$. As shown in the following example, the price to pay is a small increase of the conservatism regarding to the conditions proposed in theorems 1 and 2 (based on the triple sum) but with a decrease of the computational cost since the number of decision variable is reduced. Indeed, the numbers of LMI conditions to be verified from theorem 1 and remark 2 are respectively $r \times l \times (r+1) + r + l - 2$ and $l \times (r+2) - 1$. Finally, note that even if a double sum structure is employed, the conditions are still less conservative than the one proposed in the previous literature.

Another way to show the gain, in terms of conservatism, of the non-quadratic conditions proposed in this paper and the relaxed quadratic ones proposed in [22] is to compare the obtained H_∞ performance. Table 1 shows the attenuation level $\rho = \eta^2$ computed from theorem 2, remark 2 and the second theorem in [22] for $a \in [-14 \quad 0]$, $b = -1$ and $Q = I_{2\times 2}$. As expected, the attenuation levels obtained from theorem 2 are always lower than those from [22]. Moreover, with the proposed LMI conditions, these values are closer to zero which means that the effect of the external disturbances is more attenuated.

TABLE I

H_∞ PERFORMANCE COMPUTED FROM THEOREM. 2, REMARK 3 AND [22]

a	-14	-12	-10	-8	-6	-4	-2	0
ρ (Th 2)	1. 9	1. 93	1. 95	1. 99	2.04	2.13	2.28	2.58
ρ (Rem. 2)	5.7	5.86	6.07	6. 36	6.79	7..52	8.93	12.8
ρ ([22])	6.46	6.7	7.02	7.45	9.01	10.6	14.1	25.2

VII. CONCLUSION

In this paper, LMI based controller design of less conservatism has been proposed for T-S descriptors using a redundant formulation of closed-loop dynamics. The proposed LMI conditions are obtained via a NQLF and a non-PDC control law. Note that, using a redundancy approach, LMI formulation is easier since change of variables is no more required. Finally, a numerical example has been proposed to illustrate the efficiency of the proposed approaches regarding to previously published LMI stability conditions for T-S descriptors.

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