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Stabilization of Uncertain Takagi-Sugeno Descriptors: A Fuzzy Lyapunov Approach

Tahar Bouarar, Kevin Guelton, *Member, IEEE*, Noureddine Manamanni, *Member, IEEE* and Patrice Billaudel

Abstract— **This paper presents stability conditions for Takagi-Sugeno (T-S) uncertain descriptors. These are based on a fuzzy Lyapunov approach and a non-PDC (Parallel Distributed Compensation) control scheme. To design the fuzzy controller, the stability conditions are derived into LMIs. Moreover, in order to reduce once more the conservatism of the proposed stability conditions, a relaxation scheme, allowing rewriting the triple summation structure, is introduced for T-S descriptors. A designed example illustrates the efficiency of the proposed approaches.**

I. INTRODUCTION

Control approaches based on Takagi-Sugeno fuzzy modelling have been applied to various processes and in several engineering systems. Indeed, a T-S fuzzy model may represent a non linear one by a set of linear time invariant models interconnected together with nonlinear membership functions [1]. In the last few decades, stability analysis for T-S fuzzy control systems has attracted a great consideration. Several works exist in literature dealing with stability and stabilization of T-S systems. For example, quadratic stability conditions with Parallel Distributed Compensation (PDC) control law have been proposed in [2][3]. The latter conditions being conservative, relaxed approaches have been proposed. For instance, one can deal with piecewise Lyapunov approaches [4][5][6], non quadratic Lyapunov approaches [7][8] and more recently fuzzy Lyapunov approaches [5][9]. Note that, these results, derived from classical T-S models are not directly applicable to a wider class of systems such as, for instance, singular systems [10]. A wider class of T-S systems, called T-S descriptors have been firstly studied in the case of control [11][12] or more recently for observer design [13][22]. In addition, more extensively to singular systems, descriptors are also the natural way to describe mechanical systems with time varying inertia. Indeed, in the later case, a descriptor modelling approach leads to less computational cost when solving a set of LMIs [13][21]. Following the goal of extending the results for a wider class of systems, robust stability conditions considering parametric uncertainties [14][15], external disturbances [16] or both [17] have been proposed. Nevertheless, [14] provides BMI conditions and [15][16][17] are based on a Lyapunov quadratic approach that still leading to conservative LMI conditions.

In this paper, a fuzzy Lyapunov approach, extending our previous results in [15], is proposed to improve the conservatism of the LMI conditions. Moreover, a typical relaxation scheme is proposed for descriptor systems. Finally, a numerical example illustrates the efficiency of the proposed approach and the interest of the suggested relaxation scheme.

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The authors are with the CReSTIC-URCA, Moulin de la House BP 1039, F-51687 Reims Cedex 2, France. Corresponding author: Kevin Guelton, phone: +33-3-26-91-32-61; Fax:+33-3-26-91-31-06; e-mail: kevin.guelton@univ-reims.fr.

II. USEFUL NOTATIONS, LEMMA AND COROLLARY

The following notations, lemma and corollary will be used respectively to clarify the mathematical expression and to provide the LMI stability conditions. Let us consider, for $k \in \{1,\dots,l\}$ and $i \in \{1,\dots,r\}$, the scalar functions $v_k(z)$ and $h_i(z)$, the matrices W_k, G_i and Q_{ik} with appropriate dimensions. We will denote $W_v = \sum_{k=1}^{l} v_k(z) W_k$, $G_h = \sum_{i=1}^{r} h_i(z) G_i$ and $Q_{hv} = \sum_{k=1}^{l}\sum_{i=1}^{r} v_k(z) h_i(z) Q_{ik}$. As usual a star $(*)$ indicates a transpose quantity in a symmetric matrix. Moreover, a nonlinear matrix $X(z(t))$ in its general formulation will be denoted X_z.

Lemma 1 [18]: For real matrices with appropriate dimensions X, Y and S, and a positive scalar δ, one has:

$$X^T Y + Y^T X \le \delta X^T X + \delta^{-1} Y^T Y \tag{1}$$

$$X^T Y + Y^T X \le X^T S^{-1} X + Y^T S Y \tag{2}$$

Corollary: For real matrices X, Y, T, R, M, and a regular matrix with appropriate dimensions $Q > 0$, we have:

$$\begin{bmatrix} R & (T+XY)^T \\ T+XY & M \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} R + YQ^{-1}Y^T & T^T \\ T & M + XQX^T \end{bmatrix} < 0 \tag{3}$$

Proof: For real matrices X, Y, T, R, M and a regular matrix Q with appropriate dimensions, let us consider:

$$\begin{bmatrix} R & (T+XY)^T \\ T+XY & M \end{bmatrix} = \begin{bmatrix} R & T^T \\ T & M \end{bmatrix} + \begin{bmatrix} 0 & Y^T X^T \\ XY & 0 \end{bmatrix} < 0$$

From (2), it exists a matrix $Q > 0$ such that:

$$\begin{bmatrix} 0 \\ X \end{bmatrix} \begin{bmatrix} Y & 0 \end{bmatrix} + \begin{bmatrix} Y^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 & X^T \end{bmatrix}$$
$$\leq \begin{bmatrix} 0 \\ X \end{bmatrix} Q \begin{bmatrix} 0 & X^T \end{bmatrix} + \begin{bmatrix} Y^T \\ 0 \end{bmatrix} Q^{-1} \begin{bmatrix} Y & 0 \end{bmatrix}$$

that leads to (3) and ends the proof. ∎

III. CLASS OF STUDIED T-S DESCRIPTORS

Let us consider the class of uncertain T-S descriptor systems described by:

$$\begin{aligned} &\sum_{k=1}^{l} v_k(z(t)) \{E_k + \Delta E_k(t)\} \dot{x}(t) = \\ &\quad \sum_{i=1}^{r} h_i(z(t)) \{(A_i + \Delta A(t)) x(t) + (B_i + \Delta B(t)) u(t)\} \end{aligned} \tag{4}$$

where l and r represent respectively the number of fuzzy rules in the left and the right part of the state equation. $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ represent respectively the state and the input vectors. $z(t) \in \mathbb{R}^f$ is the premise vector. $v_k(z(t))$ and $h_i(z(t))$ are positive membership functions associated to the fuzzy rules satisfying the convex sum proprieties $\sum_{i=1}^{r} h_i(z(t)) = 1$ and $\sum_{k=1}^{l} v_k(z(t)) = 1$. $E_k \in \mathbb{R}^{n\times n}$, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are real state matrices. $\Delta E_k(t) \in \mathbb{R}^{n\times n}$, $\Delta A_i(t) \in \mathbb{R}^{n\times n}$ and $\Delta B_i(t) \in \mathbb{R}^{n\times m}$ are unknown matrices containing the bounded uncertainties such that:

$$\Delta A_i(t) = H_{ai} \Delta_{ai}(t) N_{ai}, \ \Delta B_i(t) = H_{bi} \Delta_{bi}(t) N_{bi}$$
$$\text{and } \Delta E_k(t) = H_{ek} \Delta_{ek}(t) N_{ek} \tag{5}$$

with H_{ai}, H_{bi}, H_{ek}, N_{ai}, N_{bi}, and N_{ek} are known constant matrices and $\Delta_{ai}(t)$, $\Delta_{bi}(t)$, $\Delta_{ek}(t)$ are unknown matrices functions bounded as, for $\eta = e$ or a or b and $\mu = k$ or i, $\forall t$ we have:

$$\Delta_{\eta\mu}^T(t) \Delta_{\eta\mu}(t) \leq I \tag{6}$$

Let us consider the following non-PDC control law:

$$u(t) = -\left(\sum_{j=1}^{r} \sum_{k=1}^{l} v_k(z(t)) h_j(z(t)) S_{jk} \right) (X_z^1)^{-1} x(t) \tag{7}$$

As usual for descriptor systems, we consider the extended state vector $\tilde{x}(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}^T$. Thus, (4) can be rewritten with the above defined notations as:

$$\tilde{E} \dot{\tilde{x}}(t) = \tilde{A}_{hv} \tilde{x}(t) + \tilde{B}_h u(t) \tag{8}$$

with $\tilde{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{A}_{hv} = \begin{bmatrix} 0 & I \\ A_h + \Delta A_h(t) & -E_v - \Delta E_v(t) \end{bmatrix}$ and $\tilde{B}_h = \begin{bmatrix} 0 \\ B_h + \Delta B_h(t) \end{bmatrix}$.

In the same way, the control law (7) can be rewritten as:

$$u(t) = -\tilde{K}_{hvz} \tilde{x}(t) \tag{9}$$

with $\tilde{K}_{hvz} = \begin{bmatrix} S_{hv} (X_z^1)^{-1} & 0 \end{bmatrix}$.

Substituting (9) into (8), the closed-loop uncertain T-S descriptor dynamics is given by:

$$\tilde{E} \dot{\tilde{x}}(t) = \left(\tilde{A}_{hv} - \tilde{B}_h \tilde{K}_{hvz} \right) \tilde{x}(t) \tag{10}$$

The goal is now to provide LMI stability conditions allowing to find the matrices S_{jk} and X_z^1 stabilizing (10). A solution is proposed in the following section.

IV. SUFFICIENT STABILITY CONDITIONS AND LMI FORMULATION

The main result of this paper is summarized in the following theorem.

Theorem 1: Assume that, $\forall z(t)$ $\xi \in \{1, \ldots, r-1\}$ $\dot{h}_\xi(z(t)) \geq \phi_\xi$ and $\psi \in \{1, \ldots, l-1\}$, $\dot{v}_\psi(z(t)) \geq \theta_\psi$. The uncertain T-S descriptor systems (4) is globally asymptotically stable via the non-PDC control law (7), if there exist the matrices $X_{jk}^1 = X_{jk}^{1\,T} > 0$, X_{ij}^3, X_{ij}^4 and S_{jk}, the positive scalars τ_{ijk}^1, τ_{ijk}^2, τ_{ijk}^3, τ_{ijk}^4 such that the following LMIs are satisfied:

For all combinations of $i, j = 1, \ldots, r$ and $k = 1, \ldots, l$,

$$\Phi_{ijk} < 0 \tag{11}$$

with:

$$\Phi_{ijk} = \begin{bmatrix} \Omega_{11ijk} & & & & & \\ N_{ai} X_{jk}^1 & -\tau_{ijk}^1 I & & (*) & & \\ N_{bi} S_{jk} & 0 & -\tau_{ijk}^2 I & & & \\ N_{ek} X_{ij}^3 & 0 & 0 & -\tau_{ijk}^3 I & & \\ \Omega_{51ijk} & 0 & 0 & 0 & \Omega_{55ijk} & \\ 0 & 0 & 0 & 0 & N_{ek} X_{ij}^4 & -\tau_{ijk}^4 I \end{bmatrix} \tag{12}$$

and $\Omega_{51ijk} = X_{ij}^{4T} + A_i X_{jk}^1 - E_k X_{ij}^3 - B_i S_{jk}$,

$$\Omega_{11ijk} = X_{ij}^3 + X_{ij}^{3T} - \left(\sum_{\xi=1}^{r-1} \phi_\xi \left(X_{\xi k}^1 - X_{rk}^1 \right) + \sum_{\psi=1}^{l-1} \theta_\psi \left(X_{i\psi}^1 - X_{il}^1 \right) \right),$$

$$\Omega_{55ijk} = -X_{ij}^{4T} E_k{}^T - E_k X_{ij}^4 + \tau_{ijk}^1 H_{ai} H_{ai}^T + \tau_{ijk}^2 H_{bi} H_{bi}^T + \tau_{ijk}^3 H_{ek} H_{ek}^T + \tau_{ijk}^4 H_{ek} H_{ek}^T$$

Proof: Let us consider the following candidate fuzzy Lyapunov function:

$$V\left(\tilde{x}(t)\right) = \tilde{x}^T(t) \tilde{E} \left(X_z\right)^{-1} \tilde{x}(t) \tag{13}$$

Note that, if it can be established that (13) is a Lyapunov functional, X_z is a non singular matrix and X_z^{-1} exists. In the sequel, for space convenience, the time t in a time varying variable will be omitted when there is no ambiguity.

From (13), one needs:

$$\tilde{E} X_z^{-1} = X_z^{-T} \tilde{E} \geq 0 \tag{14}$$

Let us consider $X_z = \begin{bmatrix} X_z^1 & X_z^2 \\ X_z^3 & X_z^4 \end{bmatrix}$, (14) implies $X_z^2 = 0$ and $X_z^1 = X_z^{1T} \geq 0$, X_z^3 and X_z^4 are free matrices. The closed-loop system (10) is stable if:

$$\dot{V}(\tilde{x}) = \dot{\tilde{x}}^T \tilde{E} X_z^{-1} \tilde{x} + \tilde{x}^T \tilde{E} X_z^{-1} \dot{\tilde{x}} + \tilde{x}^T \tilde{E} \dot{X}_z^{-1} \tilde{x} < 0 \tag{15}$$

According to (14) and (10), (15) yields:

$$\left(\tilde{A}_{hv} - \tilde{B}_h \tilde{K}_{hvz} \right)^T X_z^{-1} + X_z^{-T} \left(\tilde{A}_{hv} - \tilde{B}_h \tilde{K}_{hvz} \right) + \tilde{E} \dot{X}_z^{-1} < 0 \tag{16}$$

Multiplying left and right respectively by X_z^T and X_z, and considering (14), (16) becomes:

$$X_z^T \left(\tilde{A}_{hv}{}^T - \tilde{K}_{hvz}{}^T \tilde{B}_h{}^T \right) + \left(\tilde{A}_{hv} - \tilde{B}_h \tilde{K}_{hvz} \right) X_z + \tilde{E} X_z \dot{X}_z^{-1} X_z < 0 \tag{17}$$

Note that:

$$\begin{aligned} -\dot{X}_z^{-1} &= \frac{d}{dt} \left\{ X_z^{-1} X_z \right\} X_z^{-1} - X_z^{-1} \\ &= \dot{X}_z^{-1} X_z X_z^{-1} + X_z^{-1} \dot{X}_z X_z^{-1} - \dot{X}_z^{-1} \\ &= X_z^{-1} \dot{X}_z X_z^{-1} \end{aligned} \tag{18}$$

Thus (17) becomes:

$$X_z^T \left(\tilde{A}_{hv}{}^T - \tilde{K}_{hvz}{}^T \tilde{B}_h{}^T \right) + \left(\tilde{A}_{hv} - \tilde{B}_h \tilde{K}_{hvz} \right) X_z - \tilde{E} \dot{X}_z < 0 \tag{19}$$

According to (5) and the matrices defined in (8), (19) can be rewritten as:

$$\begin{bmatrix} X_z^{3T} + X_z^3 - \dot{X}_z^1 & (*) \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} < 0 \tag{20}$$

With $\Gamma_{21} = X_z^{4T} + A_h X_z^1 - E_v X_z^3 - B_h S_{hv} + H_{ah} \Delta_{ah} N_{ah} X_z^1 - H_{ev} \Delta_{ev} N_{ev} X_z^3 - H_{bh} \Delta_{bh} N_{bh} S_{hv}$

and

$$\Gamma_{22} = -X_z^{4T} E_v{}^T - E_v X_z^4 - X_z^{4T} N_{ev}{}^T \Delta_{ev}{}^T H_{ev}{}^T - H_{ev} \Delta_{ev} N_{ev} X_z^4$$

Applying lemma 1 and its corollary, (20) is satisfied if:

$$\begin{bmatrix} \Theta - \dot{X}_z^1 & (*) \\ X_z^{4T} + A_h X_z^1 - E_v X_z^3 - B_h S_{hv} & \Xi \end{bmatrix} < 0 \tag{21}$$

with

$$\Xi = -X_z^{4T} E_v{}^T - E_v X_z^4 + \tau_{hz}^1 H_{ah} H_{ah}^T + \tau_{hhv}^2 H_{bh} H_{bh}^T + \tau_{zv}^{4\,-1} X_z^{4T} N_{ev}{}^T N_{ev} X_z^4 + \tau_{zv}^3 H_{ev} H_{ev}^T + \tau_{zv}^4 H_{ev} H_{ev}^T$$

and

$$\Theta = X_z^{3T} + X_z^3 + \tau_{hz}^{1\,-1} X_z^{1T} N_{ah}^T N_{ah} X_z^1 + \tau_{hhv}^{2\,-1} S_{hv}^T N_{bh}^T N_{bh} S_{hv} + \tau_{zv}^{3\,-1} X_z^{3T} N_{ev}^T N_{ev} X_z^3.$$

Applying the Schur complement [19], one obtains:

$$\begin{bmatrix} \Lambda_{11} - \dot{X}_z^1 & & & & & \\ N_{ah} X_z^1 & -\tau_{hz}^1 I & & (*) & & \\ N_{bh} S_{hv} & 0 & -\tau_{hhv}^2 I & & & \\ N_{ev} X_z^3 & 0 & 0 & -\tau_{vz}^3 I & & \\ \Lambda_{51} & 0 & 0 & 0 & \Lambda_{55} & \\ 0 & 0 & 0 & 0 & N_{ev} X_z^4 & -\tau_{vz}^4 I \end{bmatrix} < 0 \tag{22}$$

with

$\Lambda_{11} = X_z^3 + X_z^{3T}$, $\quad \Lambda_{51} = X_z^{4T} + A_h X_z^1 - E_v X_z^3 - B_h S_{hv}$ and

$$\Lambda_{55} = -X_z^{4T} E_v{}^T - E_v X_z^4 + \tau_{hz}^1 H_{ah} H_{ah}^T + \tau_{hhv}^2 H_{bh} H_{bh}^T + \tau_{vz}^3 H_{ev} H_{ev}^T + \tau_{vz}^4 H_{ev} H_{ev}^T.$$

Note that the minimal interconnection structure for (22) is a triple sum (hhv). Thus, a convenient way to run a less conservative LMI conditions is to choose:

$X_z^1 = X_{hv}^1 = \sum_{j=1}^{r} \sum_{k=1}^{l} h_j v_k X_{jk}^1$, $X_z^3 = X_{hh}^3 = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j X_{ij}^3$, $X_z^4 = X_{hh}^4 = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j X_{ij}^4$ and so $\tau_{hz}^1 = \tau_{hhv}^1$, $\tau_{zv}^3 = \tau_{hhv}^3$ and $\tau_{zv}^4 = \tau_{hhv}^4$.

In that case $\dot{X}_z^1 = \dot{X}_{hv}^1 = \sum_{j=1}^{r} \sum_{k=1}^{l} \dot{h}_j v_k X_{jk}^1 + \sum_{j=1}^{r} \sum_{k=1}^{l} h_j \dot{v}_k X_{jk}^1$, can be rewritten with the convex sum property of the membership functions as:

$$\dot{X}_{hv}^{1}=\sum_{k=1}^{l}\sum_{j=1}^{r}h_{j}v_{k}\left(\sum_{\xi=1}^{r}\dot{h}_{\xi}X_{\xi k}^{1}+\sum_{\psi=1}^{l}\dot{v}_{\psi}X_{j\psi}^{1}\right) \tag{23}$$

Moreover, from the convex property of the membership functions, one can write $\dot{h}_r\left(z(t)\right)=-\sum_{\xi=1}^{r-1}\dot{h}_\xi\left(z(t)\right)$ and $\dot{v}_l\left(z(t)\right)=-\sum_{\psi=1}^{l-1}\dot{v}_\psi\left(z(t)\right)$. Therefore, without loss of generality, one can rewrite (23) with less conservatism as:

$$\begin{aligned}\dot{X}_{hv}^{1}&=\sum_{k=1}^{l}\sum_{j=1}^{r}h_{j}v_{k}\left(\left(\sum_{\xi=1}^{r-1}\dot{h}_{\xi}X_{\xi k}^{1}\right)+\dot{h}_rX_{rk}^{1}+\left(\sum_{\psi=1}^{l-1}\dot{v}_{\psi}X_{j\psi}^{1}\right)+\dot{v}_lX_{jl}^{1}\right)\\&=\sum_{k=1}^{l}\sum_{j=1}^{r}h_{j}v_{k}\left(\sum_{\xi=1}^{r-1}\dot{h}_{\xi}\left(X_{\xi k}^{1}-X_{rk}^{1}\right)+\sum_{\psi=1}^{l-1}\dot{v}_{\psi}\left(X_{j\psi}^{1}-X_{jl}^{1}\right)\right)\end{aligned} \tag{24}$$

Note that, for $i=1,...,r$, $h_i\left(z(t)\right)$ and, for $k=1,...,l$, $v_k\left(z(t)\right)$ are required to be at least C^1. This is obviously satisfied for fuzzy models constructed via a sector nonlinearity approach [2] if the system (4) is at least C^1 or, for instance when membership functions are chosen with a smoothed Gaussian shape. Thus, one can consider, for $i=1,...,r-1$, ϕ_i the lower bounds of $\dot{h}_i\left(z(t)\right)$ and, for $k=1,...,l-1$, θ_k the lower bounds of $\dot{v}_k\left(z(t)\right)$. According to that, one can write:

$$-\dot{X}_{hv}^{1}\le-\sum_{k=1}^{l}\sum_{j=1}^{r}h_{j}v_{k}\left(\sum_{\xi=1}^{r-1}\phi_{\xi}\left(X_{\xi k}^{1}-X_{rk}^{1}\right)+\sum_{\psi=1}^{l-1}\theta_{\psi}\left(X_{j\psi}^{1}-X_{jl}^{1}\right)\right) \tag{25}$$

and so, from (22) and (25), one has:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{l}h_ih_jv_k\Phi_{ijk}<0 \tag{26}$$

with the Φ_{ijk} defined in (12) and (26).

Thus, sufficiently, (26) is obviously satisfied if (11) is verified. That ends the proof. ■

V. TYPICAL RELAXATION SCHEME FOR T-S DESCRIPTOR

Note that, in most of the cases, stability conditions for T-S descriptor are sufficiently obtained from a triple sum interconnection structure such like (26) (see e.g. [11][15][17]). Indeed, the set of LMIs are obtained from (26) considering that all the "Φ_{ijk} s" should be negative. This obviously leads to conservatism. A way to improve the conservatism is to rewrite the triple sum structure. Note that this way is often followed to rewrite classical T-S models' stability conditions that are written in a double summation structure [2]. Moreover, let us point out that, in our knowledge, there is no relaxation schemes dedicated to T-S descriptor systems. The following lemma provides a first purpose to achieve this point.

Lemma 2: Let us consider $q=\min\{r,l\}$, the inequality $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{l}h_ih_jv_k\Psi_{ijk}<0$ is satisfied if:

$$\Psi_{iii}<0 \text{ for } i=1,...,q \tag{27}$$

$$\Psi_{iik}<0 \text{ for } i=1,...,r,\ k=1,...,l \text{ and } k\neq i \tag{28}$$

$$\Psi_{iji}+\Psi_{jii}<0 \text{ for } i=1,...,q,\ j=1,...,r \text{ and } j<i \tag{29}$$

$$\Psi_{ijk}+\Psi_{jik}<0 \text{ for } i,j=1,...,r,\ k=1,...,l,\ j<i \text{ and } k\neq i \tag{30}$$

Proof:

Let us consider the following proprieties:

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\varphi_{ij}=\sum_{i=1}^{r}h_i^2\varphi_{ii}+\sum_{i=1}^{r}\sum_{\substack{j=1\\j<i}}^{r}h_ih_j\left(\varphi_{ij}+\varphi_{ji}\right) \tag{31}$$

$$\sum_{k=1}^{l}\sum_{i=1}^{r}v_kh_i\varphi_{ik}=\sum_{i=1}^{q}v_ih_i\varphi_{ii}+\sum_{\substack{k=1\\k\neq i}}^{l}\sum_{i=1}^{r}v_kh_i\varphi_{ik} \tag{32}$$

with φ_{ij} are matrices with appropriate dimensions and $q=\min\{r,l\}$. Developing (26) by (31), one obtains:

$$\begin{aligned}&\sum_{k=1}^{l}\sum_{i=1}^{r}\sum_{j=1}^{r}v_kh_ih_j\Psi_{ijk}\\&=\sum_{k=1}^{l}\sum_{i=1}^{r}v_kh_i^2\Psi_{iik}+\sum_{k=1}^{l}\sum_{i=1}^{r}\sum_{\substack{j=1\\j<i}}^{r}v_kh_ih_i\left(\Psi_{ijk}+\Psi_{jik}\right)\end{aligned} \tag{33}$$

One more time, developing (33) by (32), one obtains:

$$\begin{aligned}&\sum_{k=1}^{l}\sum_{i=1}^{r}\sum_{j=1}^{r}v_kh_ih_j\Psi_{ijk}\\&=\sum_{i=1}^{q}v_ih_i^2\Psi_{iii}+\sum_{\substack{k=1\\k\neq i}}^{l}\sum_{i=1}^{r}v_kh_i^2\Psi_{iik}\\&\quad+\sum_{i=1}^{q}\sum_{\substack{j=1\\j<i}}^{r}v_ih_ih_j\left(\Psi_{iji}+\Psi_{jii}\right)+\sum_{\substack{k=1\\k\neq i}}^{l}\sum_{i=1}^{r}\sum_{\substack{j=1\\j<i}}^{r}v_kh_ih_i\left(\Psi_{ijk}+\Psi_{jik}\right)\end{aligned} \tag{34}$$

Thus, $\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{l}h_ih_jv_k\Psi_{ijk}<0$ is satisfied if (27), (28) , (29) and (30) are verified. That ends the proof. ■

VI. SIMULATION EXAMPLE AND RESULTS

Let us consider the following nonlinear descriptor:

$$E(x(t))\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) \tag{35}$$

where $x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix}^T$,

$E(x(t)) = \begin{bmatrix} -5 & 5\sin(x_1(t)) \\ \cos^2(x_2(t)) & -4 \end{bmatrix}$,

$A(x(t)) = \begin{bmatrix} 1 & 5\cos(x_1(t)) \\ -2 & \dfrac{\sin(x_2(t))}{x_2(t)} \end{bmatrix}$,

and $B(x_2(t)) = \begin{bmatrix} 0 \\ -\tanh^2(x_2(t)) \end{bmatrix}$.

An exact T-S descriptor for (35) can be obtained using the sector non linearity approach [2]. In that case, one has to consider:

- Two nonlinearities in the left part of (35): $\sin(x_1)$ and $\cos^2(x_2)$, leading to $l=4$.
- Three nonlinearities in the right part of (35): $\cos(x_1)$, $\dfrac{\sin(x_2)}{x_2}$ and $\tanh^2(x_2)$, leading to $r=8$.

The number of LMI conditions that have to be satisfied in this case is 256 using theorem 1 and 154 after applying lemma 2. As a consequence, using actual solvers and computers, the computational cost would be considerable and in some cases doesn't lead to a solution. In order to reduce the number of LMIs, one can choose to put all the nonlinear terms depending on the state variable x_2 into uncertainties. In this case, $\sin(x_1)$ and $\cos(x_1)$ are still to be splited using the sector nonlinearity approach. This leads to $l=2$ and $r=2$ and so 7 LMIs to be satisfied. Let us consider, for $x_1 \in \mathbb{R}$:

$$\sin(x_1) = \frac{1-\sin(x_1)}{2}(-1) + \frac{\sin(x_1)+1}{2}(1) \tag{36}$$

and
$$\cos(x_1) = \frac{1-\cos(x_1)}{2}(-1) + \frac{\cos(x_1)+1}{2}(1) \tag{37}$$

Also for the uncertain part, one can write:

$$\cos^2(x_2) = \frac{1}{2} + \frac{1}{2}\Delta_1(t), \tag{38}$$

$$\frac{\sin(x_2)}{x_2} = \frac{1-\lambda}{2} + \frac{1+\lambda}{2}\Delta_2(t), \tag{39}$$

$$\tanh^2(x_2) = \frac{1}{2} + \frac{1}{2}\Delta_3(t), \tag{40}$$

with $\lambda = -0.2172$, the minimum value of $\dfrac{\sin(x_2)}{x_2}$ for $x_2 \in \mathbb{R}$ and $\Delta_1(t)$, $\Delta_2(t)$ and $\Delta_3(t)$, uncertain bounded functions such that $\Delta_1^2(t) \le 1$, $\Delta_2^2(t) \le 1$ and $\Delta_3^2(t) \le 1$.

According to (36), (37), (39), (38), (40), an uncertain T-S descriptor representation of (35) is given as:

$$\sum_{k=1}^{2} v_k(E_k + \Delta E_k)\dot{x} = \sum_{i=1}^{2} h_i\big((A_i + \Delta A_i)\dot{x} + (B_i + \Delta B_i)u\big) \tag{41}$$

with: $E_1 = \begin{bmatrix} -5 & -5 \\ 0.5 & -4 \end{bmatrix}$, $E_2 = \begin{bmatrix} -5 & 5 \\ 0.5 & -4 \end{bmatrix}$,

$A_1 = \begin{bmatrix} 1 & -5 \\ -2 & \dfrac{1-\lambda}{2} \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 5 \\ -2 & \dfrac{1-\lambda}{2} \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix}$,

and where $\Delta E(t) = H_e \Delta_1(t) N_e$, $\Delta A(t) = H_a \Delta_2(t) N_a$, $\Delta B(t) = H_b \Delta_3(t) N_b$ with $N_e = \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix}$,

$N_a = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1+\lambda}{2} \end{bmatrix}$, $N_b = -0.5$, $H_b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and

$H_e = H_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and finally $h_1(z) = \dfrac{1-\cos(x_1)}{2}$,

$h_2(z) = \dfrac{\cos(x_1)+1}{2}$, $v_1(z) = \dfrac{1-\sin(x_1)}{2}$

and $v_2(z) = \dfrac{\sin(x_1)+1}{2}$.

Using the MATLAB LMI Toolbox [20], feasibility area obtained from the initial stability conditions (theorem 1) and the ones obtained from relaxed stability conditions (lemma2) for $\theta_1 \in [-1 \quad 1]$ and $\phi_1 \in [-2 \quad 1]$ are compared in fig 1. Therefore, one can see the interest of the relaxation since the feasibility area of the relaxed stability conditions is broader than that of non-relaxed results.

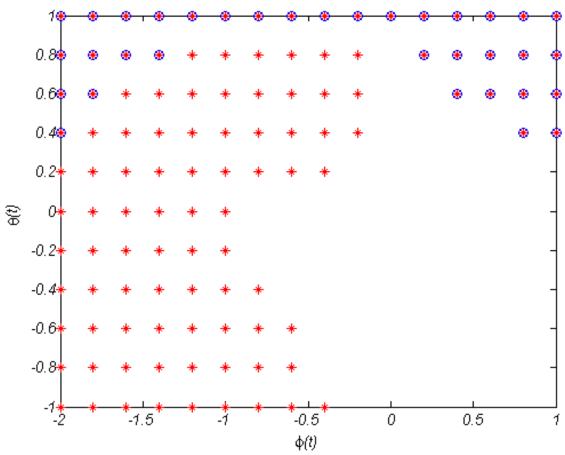

Fig 1: Feasibility areas of the stability conditions with and without relaxation. *"o"* indicate area of stability conditions (theorem 1) and *"*"* indicate area of relaxed stability conditions (lemma 2).

Note that, in the present example, $E(x(t))$ is chosen well-define in order to facilitate numerical simulation. However the proposed LMI conditions are still workable for some descriptor systems where $E(x(t))$ is not invertible. Fig 2, illustrates the convergence of the state vector and the control signal when simulating (35) in closed loop after solving (via the Matlab LMI Toolbox [21]) the relaxed stability conditions (lemma 2) with $\phi_1 = -3.5$, $\theta_1 = -5$ and the initial state $x(0)^T = [1.7 \quad 1.5]^T$. Note that ϕ_1 (resp. θ_1) have been chosen as a wider values (regarding to the lower bound of $\dot{h}(z(t))$ and $\dot{w}(z(t))$) in order to illustrate the efficiency of the proposed approach. Moreover, note that an analytical method has been proposed to find the values of these bound [5].

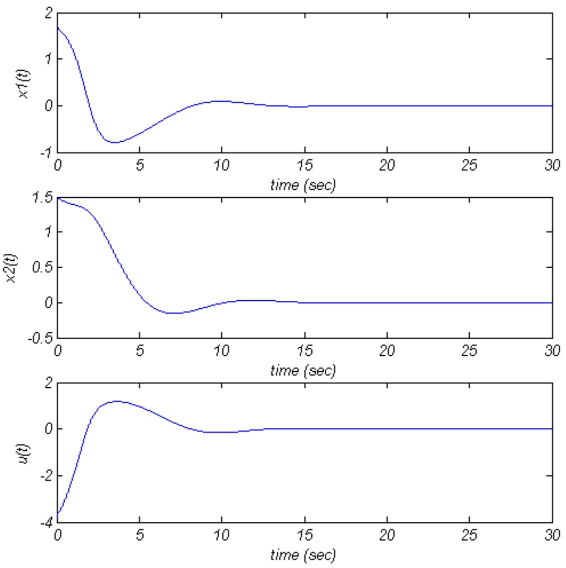

Fig 2: simulation results

VII. CONCLUSION

In this paper, new stability conditions for uncertain T-S descriptor systems have been proposed. These are based on a fuzzy Lyapunov function and a non-PDC (Lyapunov dependant) control law. The proposed LMI stability conditions are less conservative than the quadratic case since they require a set of Lyapunov matrices instead of common one. In order to improve once more the proposed stability conditions a typical relaxation scheme for T-S descriptor systems was proposed. Finally, in order to illustrate the efficiency of the proposed fuzzy Lyapunov approach, and the interest of proposed relaxation scheme, an academic example has been studied.

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