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LMI BASED H_∞ CONTROLLER DESIGN FOR UNCERTAIN TAKAGI-SUGENO DESCRIPTORS SUBJECT TO EXTERNAL DISTURBANCES

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Abstract: This paper deals with the design of an H_∞ fuzzy controller for uncertain T-S fuzzy descriptor with external disturbances. The control scheme is based on the modified PDC control law and an H_∞ criterion to attenuate external disturbances. Stability conditions are obtained via a quadratic Lyapunov function and are given in terms of LMI. To illustrate the efficiency of the proposed approach, a design example is provided with the simulation of a planar two degrees of freedom robot.

Keywords: Descriptors, Takagi-Sugeno, LMI, H_∞, External disturbances, Uncertainties.

1. INTRODUCTION

Many results about stability conditions and robust stability conditions for classical Takagi-Sugeno (T-S) fuzzy systems (Takagi and Sugeno, 1985) have already been obtained in the literature, see e.g. (Tanaka and Wang, 2001), (Sala, et al., 2005) and reference therein. On other hand, T-S fuzzy descriptor systems were first introduced by (Taniguchi, *et al.*, 1999). These ones depict a wider class of systems because of the extensive application in many engineering systems, such as singular systems (Dai, 1989), joint torques estimation in human standing (Guelton, et al., 2006) or for the modelling of a two degrees of freedom pneumatic robot (Schulte and Guelton, 2006). Since the introduction of T-S fuzzy descriptor systems by (Taniguchi, *et al.*, 1999), few works have been devoted to the study of their stability conditions. For instance, stability conditions for T-S fuzzy descriptor with parameters uncertainties has been reported by (Ma and Sun, 2004)(Yue and Lam, 2004). Moreover, the design of an H_∞ controller without uncertainties was also considered by (Yoneyama and Ichikawa, 1999). Nevertheless, in the above mentioned studies, the suggested stability conditions have been obtained for restrictive classes of descriptors systems in order to obtain LMI or BMI conditions. Indeed, the left part ($E\dot{x}(t)=\dots$) was considered as linear time invariant and wasn't take into account the uncertain term. Thus, these are not suitable for a wide class of engineering systems. For instance, mechanical systems with time varying inertia (at least two degree of freedom) can be "naturally" modelled as a descriptor (with $E(x(t))\dot{x}(t)=\dots$) (Guelton, et al., 2006)(Schulte and Guelton, 2006). Moreover, introducing uncertainties in the left part is also enlarging the class of complex systems studied. For instance, dynamical models where inertias are unknown or difficult to estimate can be considered ($\left(E(x(t))+\Delta E(t)\right)\dot{x}(t)=\dots$). In this way the class of fuzzy descriptor systems studied in this paper is more general than the ones already studied in literature. The aim of this paper is to propose a controller design based on an H_∞ criterion for uncertain fuzzy descriptor with external disturbances presented in section 2. Then, stability conditions, formulated in terms of bilinear matrix inequalities, are obtained via a quadratic Lyapunov function and a modified PDC control law. Afterward, in order to be solved by classical convex optimization algorithms, an approach is proposed to put these conditions into LMI added to a relaxation scheme proposed by (Tuan, *et al.*, 2001). Finally, in section 3, a design example will illustrate the efficiency of the proposed approach.

2. LMI STABILITY CONDITIONS FORMULATION

2.1 Considered class of uncertain and disturbed T-S descriptors

Takagi and Sugeno have propose an elegant way to approximate nonlinear affine systems as a collection of Linear models blended together by nonlinear functions. In that way, a T-S formulation of uncertain and disturbed non linear descriptor system can be described by the following state space equation:

$$\sum_{k=1}^{l} v_k\left(z(t)\right)\left[E_k+\Delta E_k(t)\right]\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{\begin{matrix}\left[A_i+\Delta A_i(t)\right]x(t) \\ +\left[B_i+\Delta B_i(t)\right]u(t)\end{matrix}\right\}+\phi(t) \quad (1)$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input vector, $z(t)\in\mathbb{R}^p$ is the premises vector, $\phi(t)\in\mathbb{R}^n$ denote the unknown disturbances with a known upper bound $\phi_{up}\geq\left\|\phi(t)\right\|$, l and r represent respectively the number of the fuzzy rules at the left and right side of the state representation, and the state matrices $\Delta E_k\in\mathbb{R}^{n\times n}$, $\Delta A_i\in\mathbb{R}^{n\times n}$, $\Delta B_i\in\mathbb{R}^{n\times m}$ contain all the modelling uncertainties of $E_k\in\mathbb{R}^{n\times n}$, $A_i\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times m}$.

We suppose that the uncertainties are bounded as described in (Zhou & Khargonekar, 1988) such that $\Delta E_k(t)=H_{ek}\Delta_{ek}(t)N_{ek}$, $\Delta A_i(t)=H_{ai}\Delta_{ai}(t)N_{ai}$, and $\Delta B_i(t)=H_{bi}\Delta_{bi}(t)N_{bi}$, where H_{ek}, H_{ai}, H_{bi}, N_{ek}, N_{ai} and N_{bi} are known constant real matrices with appropriate dimensions, $\Delta_{ek}(t)$, $\Delta_{ai}(t)$ and $\Delta_{bi}(t)$ are unknown matrices functions which are bounded such that $\forall t$ and the index $\xi=e,a$ or b and $\varpi=i$ or k, $\Delta_{\xi\varpi}^T(t)\Delta_{\xi\varpi}(t)\leq I$.

$\sum_{k=1}^{l} v_k\left(z(t)\right)$ and $\sum_{i=1}^{r} h_i\left(z(t)\right)$ are positive functions and satisfy the convex sum propriety $\sum_{k=1}^{l} v_k\left(z(t)\right)=1$ and $\sum_{i=1}^{r} h_i\left(z(t)\right)=1$.

2.2 Basic stability conditions

Let us consider the modified PDC control law (Taniguchi, *et al.*, 1999):

$$u(t)=-\sum_{i=1}^{r}\sum_{k=1}^{l} h_i\left(z(t)\right)v_k\left(z(t)\right)K_{ik}x(t) \quad (2)$$

where $K_{ik}\in\mathbb{R}^{m\times n}$ are the local feedback gains.

The closed-loop uncertain and disturbed T-S descriptor can be obtained, combining (2) and (1). This one is given by:

$$\sum_{k=1}^{l} v_k\left(z(t)\right)\left[E_k+\Delta E_k(t)\right]\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{l} h_i\left(z(t)\right)h_j\left(z(t)\right)v_k\left(z(t)\right)\Gamma_{ijk}x(t)+\phi(t) \quad (3)$$

with $\Gamma_{ijk}=\left[A_i+\Delta A_i(t)\right]-\left[B_i+\Delta B_i(t)\right]K_{jk}$.

Recall that the state representation (3) takes into account the modelling uncertainties. Since, the system is subject to external disturbances $\phi(t)$, we propose to attenuate them using H_∞ performance. Thus, the problem treated below leads to a complete robust control approach.

Let us consider the H_∞ criterion given by :

$$\int_{t_0}^{t_f} x^T(t)Qx(t)\leq\eta^2\int_{t_0}^{t_f}\phi^T(t)\phi(t)dt \quad (4)$$

where t_f is the final control time, Q is a positive definite weighting matrix and η is the prescribed attenuation level.

According to this criterion, basic robust stability conditions for the closed loop system (3) are summarized in the following theorem.

Theorem 1: The T-S uncertain and disturbed system (1) is quadratically stable via the modified PDC control law (2) and the H_∞ criterion (4) if there exist matrices $Z_1=Z_1^T>0$, Z_3, Z_4, an attenuation level η and the gain matrices K_{jk} such that the following conditions are satisfied:

For $i,j=1,2,...,r$ *and* $k=1,2,...,l$,

$$\begin{bmatrix} -Z_3-Z_3^T+Z_1^TQZ_1 & (*) & (*) & 0 \\ \Xi_{ijk} & \chi_k & 0 & (*) \\ I & 0 & -\eta^2 I & 0 \\ 0 & I & 0 & -\eta^2 I \end{bmatrix}\leq 0 \quad (5)$$

with $\chi_k=-\left(E_k+\Delta E_k\right)Z_4-Z_4^T\left(E_k+\Delta E_k\right)^T$ and $\Xi_{ijk}=\begin{pmatrix} Z_4^T+\left(A_i+\Delta A_i\right)Z_1... \\ -\left(B_i+\Delta B_i\right)K_{jk}Z_1+\left(E_k+\Delta E_k\right)Z_3\end{pmatrix}$. As usual, a star $(*)$ in a symmetric matrix indicates a transpose quantity.

Proof: In order to clarify the mathematical expressions, we will use the following notations. Let us consider the positive scalar functions $v_k\left(z(t)\right)$ for $k\in\{1,2,...,l\}$, $h_i\left(z(t)\right)$ for $i\in\{1,2,...,r\}$ and the matrices E_k for $k\in\{1,2,...,l\}$, Y_i for $i\in\{1,2,...,r\}$ and T_{ik} for $i\in\{1,2,...,r\}$ and $k\in\{1,2,...,l\}$. We will denote $E_v=\sum_{k=1}^{l} v_k\left(z(t)\right)E_k$, $Y_h=\sum_{i=1}^{r} h_i\left(z(t)\right)Y_i$

and $\mathrm{T}_{hv} = \sum_{k=1}^{l}\sum_{i=1}^{r} v_k\left(z(t)\right)h_i\left(z(t)\right)\mathrm{T}_{ik}$.

Now, let us consider the extended state vector $\tilde{x}(t) = \left[x^T(t) \quad \dot{x}^T(t)\right]^T$, (3) becomes:

$$\tilde{E}_v\dot{\tilde{x}}(t) = \left(\tilde{A}_{hv} - \tilde{B}_h\tilde{K}_{hv}\right)\tilde{x}(t) + \tilde{\phi}(t) \tag{6}$$

with $\tilde{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{A}_{hv} = \begin{bmatrix} 0 & I \\ A_h + \Delta A_h & -E_v - \Delta E_v \end{bmatrix}$, $\tilde{B}_h = \begin{bmatrix} 0 \\ B_h + \Delta B_h \end{bmatrix}$, $\tilde{K}_{hv} = \begin{bmatrix} K_{hv} & 0 \end{bmatrix}$ and $\tilde{\phi}(t) = \begin{bmatrix} 0_{n\times 1} & \phi^T(t) \end{bmatrix}^T$.

By extension, (4) can be rewritten as:

$$\int_{t_0}^{t_f} \tilde{x}^T(t)\tilde{Q}\tilde{x}(t)dt \le \eta^2 \int_{t_0}^{t_f} \tilde{\phi}^T(t)\tilde{\phi}(t)\,dt \tag{7}$$

with $\tilde{Q} = diag\begin{bmatrix} Q & 0 \end{bmatrix}$.

Now, consider the following quadratic Lyapunov function candidate

$$V\left(\tilde{x}(t)\right) = \tilde{x}^T(t)\tilde{E}^T P\tilde{x}(t) \tag{8}$$

with the following needed symmetric condition to be considered as a quadratic Lyapunov function candidate.

$$\tilde{E}^T P = P^T\tilde{E} \ge 0 \tag{9}$$

Let $P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$, where P_1, P_2, P_3 and $P_4 \in \mathbb{R}^{n\times n}$.

The symmetric condition (9), leads to $P_1 = P_1^T \ge 0$, $P_2 = 0$, P_3 and P_4 are free matrices. The stability of the closed-loop model (6) is satisfied under the H_∞ performance (7) with the attenuation level η if:

$$\dot{V}\left(\tilde{x}(t)\right) + \tilde{x}(t)\tilde{Q}\tilde{x}(t) - \eta^2\tilde{\phi}^T(t)\tilde{\phi}(t) \le 0 \tag{10}$$

That is to say if:

$$\begin{aligned} &\dot{\tilde{x}}^T(t)\tilde{E}^T P\tilde{x}(t) + \tilde{x}^T(t)\tilde{E}^T P\dot{\tilde{x}}(t) \\ &\qquad + \tilde{x}^T(t)\tilde{Q}\tilde{x}(t) - \eta^2\tilde{\phi}^T(t)\tilde{\phi}(t) \le 0 \end{aligned} \tag{11}$$

By substituting (6) into (11), one obtains:

$$\begin{bmatrix} \tilde{x}(t) \\ \tilde{\phi}(t) \end{bmatrix}^T \begin{bmatrix} \zeta_{hv} & P^T \\ P & -\eta^2 I \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{\phi}(t) \end{bmatrix} \le 0 \tag{12}$$

with $\zeta_{hv} = \left(\tilde{A}_{hv}{}^T - \tilde{K}_{hv}{}^T\tilde{B}_h{}^T\right)P + P^T\left(\tilde{A}_{hv} - \tilde{B}_h\tilde{K}_{hv}\right) + \tilde{Q}$

Thus, (12) is equivalent to:

$$\begin{bmatrix} \zeta_{hv} & P^T \\ P & -\eta^2 I \end{bmatrix} \le 0 \tag{13}$$

We consider the following change of variables:

$$X = P^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -P_4^{-1}P_3P_1^{-1} & P_4^{-1} \end{bmatrix} = \begin{bmatrix} Z_1 & 0 \\ -Z_3 & Z_4 \end{bmatrix} \tag{14}$$

After multiplying (13) left and right respectively by $diag\begin{bmatrix} X^T & I \end{bmatrix}$ and $diag\begin{bmatrix} X & I \end{bmatrix}$, one obtains:

$$\begin{bmatrix} \begin{pmatrix} X^T\left(\tilde{A}_{hv}{}^T - \tilde{K}_{hv}{}^T\tilde{B}_h{}^T\right) \\ +\left(\tilde{A}_{hv} - \tilde{B}_h\tilde{K}_{hv}\right)X + X^T\tilde{Q}X \end{pmatrix} & I \\ I & -\eta^2 I \end{bmatrix} \le 0 \tag{15}$$

Then, substituting (14) and the matrices $\tilde{A}_{hv}$, $\tilde{B}_h$, $\tilde{K}_{hv}$ and $\tilde{Q}$ defined bellow into (15), it yields:

$$\begin{bmatrix} -Z_3 - Z_3^T + Z_1^T Q Z_1 & (*) & (*) & 0 \\ \Xi_{hhv} & \chi_v & 0 & (*) \\ I & 0 & -\eta^2 I & 0 \\ 0 & I & 0 & -\eta^2 I \end{bmatrix} \le 0 \tag{16}$$

with $\chi_v = -\left(E_v + \Delta E_v\right)Z_4 - Z_4^T\left(E_v + \Delta E_v\right)^T$ and

$$\Xi_{hhv} = \begin{pmatrix} Z_4^T + \left(A_h + \Delta A_h\right)Z_1 \\ \qquad -\left(B_h + \Delta B_h\right)K_{hv}Z_1 + \left(E_v + \Delta E_v\right)Z_3 \end{pmatrix}.$$

which is obviously satisfied if the condition (5) holds. ■

2.3 LMI formulation of stability conditions

The conditions provided by theorem 1 are formulated in terms of BMI (bilinear matrix inequalities). In order to be solved by classical convex optimisation algorithms (Gahinet, et al., 1995), they have to be put into LMI. Moreover, these conditions are conservative. In order to relax them, many schemes can be employed (Tuan, *et al.*, 2001), (Tanaka, *et al.*, 1998), (Liu and Zhang, 2003). In our case, we choose to apply the relaxation scheme given in (Tuan, *et al.*, 2001) since it seems to be the most efficient for most of the cases.

Lemma 1 (Tuan, *et al.*, 2001): For $i, j = 1, 2, \ldots, r$ and $k = 1, 2, \ldots, l$, we have $\Upsilon_{ijk} < 0$. These conditions are equivalent to:

- For $i = 1, 2, \ldots, r$ and $k = 1, 2, \ldots, l$, $\Upsilon_{iik} < 0$
- For $i = 1, 2, \ldots, r$, $1 \le i \ne j \le r$ and $k = 1, 2, \ldots, l$,

$$\frac{1}{r-1}\Upsilon_{iik} + \frac{1}{2}\left(\Upsilon_{ijk} + \Upsilon_{jik}\right) < 0$$

In order to put the provided stability conditions into LMI, we also need the following lemma and corollary.

Lemma 2 (Zhou and Khargonekar, 1988): For real matrices X and Y**,** with appropriate dimensions and a positive scalar τ, one has :

$$X^T Y + Y^T X \le \tau X^T X + \tau^{-1} Y^T Y$$

Corollary: for real matrices A, B, W, Y, μ and a regular matrix $Q>0$ with appropriate dimensions one has:

$$\begin{bmatrix} Y & W^T+B^TA^T \\ W+AB & \mu \end{bmatrix}<0 \Rightarrow \begin{bmatrix} Y+B^TQ^{-1}B & W^T \\ W & \mu+AQA^T \end{bmatrix}<0$$

Now, the following theorem summarizes the sufficient LMI stability conditions. This constitutes the main contribution of this work.

Theorem 2: The T-S uncertain and disturbed system (1) is quadratically stable via the modified PDC control law (2) and the H_∞ criterion (4) if there exist matrices $Z_1=Z_1^T>0$, Z_3, Z_4, M_{jk}, positive scalars σ_1^{-1}, σ_2, σ_3 and σ_4 and an attenuation level $\delta=\eta^2$ such that the following conditions are satisfied:

- For $i=1,2,...,r$ and $k=1,2,...,l$, $\Psi_{iik}<0$ (17)
- For $i=1,2,...,r$, $1\le i\ne j\le r$ and $k=1,2,...,l$,

$$\frac{1}{r-1}\Psi_{iik}+\frac{1}{2}\left(\Psi_{ijk}+\Psi_{jik}\right)<0 \quad (18)$$

with

$$\Psi_{ijk}=\begin{bmatrix} -Z_3-Z_3^T & (*) & (*) & (*) & (*) & (*) & 0 & (*) & 0 \\ N_{ai}Z_1 & -\sigma_2 I & 0 & 0 & 0 & 0 & 0 & 0 & (*) \\ N_{bi}M_{jk} & 0 & -\sigma_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ N_{ek}Z_3 & 0 & 0 & -\sigma_4 I & 0 & 0 & 0 & 0 & 0 \\ Z_1 & 0 & 0 & 0 & -Q^{-1}I & 0 & 0 & 0 & 0 \\ \Gamma_{ijk} & 0 & 0 & 0 & 0 & \Upsilon_{ik} & (*) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{ek}Z_4 & -\sigma_1^{-1}I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -\delta I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & -\delta I \end{bmatrix}$$

$$\Upsilon_{ik}=\begin{pmatrix} \sigma_2 H_{ai}H_{ai}^T+\sigma_3 H_{bi}H_{bi}^T+\sigma_4 H_{ek}H_{ek}^T \\ -E_kZ_4+\sigma_1^{-1}H_{ek}H_{ek}^T-Z_4^TE_k^T \end{pmatrix}$$ and

$$\Gamma_{ijk}=Z_4^T+A_iZ_1-B_iM_{jk}+E_kZ_3$$

Proof: Let us start from equation (16). With the bijective change of variables $M_{hv}=K_{hv}Z_1$, $\delta=\eta^2$ and substituting $\Delta E_v(t)$, $\Delta A_v(t)$, $\Delta B_v(t)$ by their bounded quantities as described in (Zhou and Khargonekar, 1988) (see below equation (1)), (5) can be rewritten as:

$$\begin{bmatrix} \left(-Z_3-Z_3^T+Z_1^TQZ_1\right) & (*) & (*) & 0 \\ \mho_{hhv} & \Lambda_v & 0 & (*) \\ I & 0 & -\delta I & 0 \\ 0 & I & 0 & -\delta I \end{bmatrix}\le 0 \quad (19)$$

with

$$\Lambda_v=\begin{pmatrix} -E_vZ_4-Z_4^TE_v^T \\ -\{H_e\Delta_eN_e\}_vZ_4-Z_4^T\{H_e\Delta_eN_e\}_v^T \end{pmatrix},$$

$$\mho_{hhv}=\begin{pmatrix} Z_4^T+A_hZ_1-B_hM_{hv}+E_vZ_3+\{H_a\Delta_aN_a\}_hZ_1 \\ -\{H_b\Delta_bN_b\}_hM_{hv}+\{H_e\Delta_eN_e\}_vZ_3 \end{pmatrix},$$

and with the notations: for $\xi=e,a$ or b and $\varphi=h$ or v, $\{H_\xi\Delta_\xi N_\xi\}_\varphi=\sum_{i=1}^{r}\varphi(z(t))\left(H_{\xi i}\Delta_{\xi i}(t)N_{\xi i}\right)$.

In order to major the uncertain terms $\Delta_{\xi i}(t)$ contained in (19), the lemma 2 and the corollary defined bellow will be used. Then, considering that $\Delta_{\xi i}^T(t)\Delta_{\xi i}(t)\le I$, the inequality (19) yields to:

$$\begin{bmatrix} \lambda_{hhv} & (*) & (*) & 0 \\ \begin{pmatrix} Z_4^T+A_hZ_1 \\ -B_hM_{hv}+E_vZ_3 \end{pmatrix} & \Omega_{hv} & 0 & (*) \\ I & 0 & -\delta I & 0 \\ 0 & I & 0 & -\delta I \end{bmatrix}\le 0 \quad (20)$$

with

$$\lambda_{hhv}=\begin{pmatrix} -Z_3-Z_3^T+Z_1^TQZ_1+\sigma_3^{-1}M_{hv}^T\{N_b^TN_b\}_hM_{hv} \\ +\sigma_2^{-1}Z_1^T\{N_a^TN_a\}_vZ_1+\sigma_4^{-1}Z_3^T\{N_e^TN_e\}_vZ_3 \end{pmatrix},$$

$$\Omega_{hv}=\begin{pmatrix} \sigma_2\{H_aH_a^T\}_h+\sigma_3\{H_bH_b^T\}_h+\sigma_1Z_4^T\{N_e^TN_e\}_vZ_4 \\ +\sigma_4\{H_eH_e^T\}_v+\sigma_1^{-1}\{H_eH_e^T\}_v-E_vZ_4-Z_4^TE_v^T \end{pmatrix}$$

Then, applying successfully the Schur complement on λ_{hhv} and Ω_{hv}, the conditions of theorem 2 hold.■

3. EXAMPLE AND SIMULATION OF A PLANAR TWO DEGREES OF FREEDOOM ROBOT

In order to illustrate the efficiency of the proposed approach, we consider the planar two degrees of freedom robot presented in Figure 1. The dynamic equation of this robot is given by:

$$M(q)\ddot{q}(t)+C(q,\dot{q})\dot{q}(t)+G(q)q(t)=Ru(t)+\phi(t) \quad (21)$$

where, θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$ denotes respectively the angular positions and the angular velocities. $q=\begin{bmatrix}\theta_1 & \theta_2\end{bmatrix}^T$ is the vector of generalized coordinates, $\phi(t)$ is the vector of external disturbances, $M(q)=\begin{bmatrix} a & c\cos(\theta_1-\theta_2) \\ c\cos(\theta_1-\theta_2) & b \end{bmatrix}$ is the inertia matrix, $C(q,\dot{q})=\begin{bmatrix} 0 & c\dot{\theta}_2\sin(\theta_1-\theta_2) \\ -c\dot{\theta}_1\sin(\theta_1-\theta_2) & 0 \end{bmatrix}$ is the Coriolis anti-symetric matrix, $G(q)=\begin{bmatrix} d\sin c(\theta_1) & 0 \\ 0 & e\sin c(\theta_2) \end{bmatrix}$ is the gravitational matrix and $R=\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ is the matrix linking joint torque to the generalized torque $u(t)=\begin{bmatrix}u_1 & u_2\end{bmatrix}^T$, and with $a=m_1K^2L_1^2+m_2L_1^2+I_1$, $b=m_2L_2^2+I_2$, $c=m_2L_1L_2$, $d=(m_2+m_1K)gL_1$, $e=m_2gL_2$ and

$m_1 = 1.4\,kg$, $m_2 = 0.7\,kg$, $L_1 = 0.6\,m$, $L_2 = 0.3\,m$, $K = 0.5$, $g = 9.81\,m\,s^{-2}$, $I_1 = I_2 = 0.25\,kg\,m^2$.

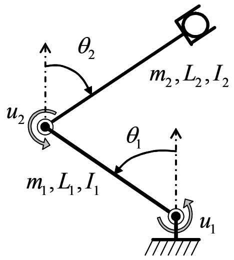

Fig.1. Planar two degrees of freedom robot.

Let $x = \begin{bmatrix} \theta_1 & \theta_2 & \dot{\theta}_1 & \dot{\theta}_2 \end{bmatrix}$ be the state vector of the planar robot. The dynamical model (21) can be rewritten as a non linear descriptor as:

$$E(x(t))\dot{x}(t) = A(x(t))x(t) + Bu(t) + \tilde{\phi}(t) \qquad (22)$$

with $E(x(t)) = \begin{bmatrix} I & 0 \\ 0 & M(x(t)) \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ R \end{bmatrix}$, $A(x(t)) = \begin{bmatrix} 0 & I \\ G(x(t)) & -C(x(t)) \end{bmatrix}$ and $\tilde{\phi}(t) = \begin{bmatrix} 0 \\ \phi(t) \end{bmatrix}$.

Note that (22) contains one non linear term $\eta_e(x) = \cos(\theta_1 - \theta_2)$ in $E(x(t))$ and four ones in $A(x(t))$: $\eta_{a1}(x) = \dfrac{\sin\theta_1}{\theta_1}$, $\eta_{a2}(x) = \dfrac{\sin\theta_2}{\theta_2}$, $\eta_{a3}(x) = \dot{\theta}_2 \sin(\theta_1 - \theta_2)$ and $\eta_{a4}(x) = \dot{\theta}_1 \sin(\theta_1 - \theta_2)$. Using the sector nonlinearity approach (Tanaka & Wang, 2001), this leads to $l = 2$ and $r = 16$ rules for the left and the right part of the T-S fuzzy model. Then, to ensure the stability of the descriptor system, $l.r.(r+1)/2 = 272$ LMI conditions have to be verified. Consequently, this should be conservative with classical stability conditions for T-S descriptors (Taniguchi, *et al.*, 1999). In order to reduce the conservatism, some non linear terms will be put to uncertainties. Indeed, the motion of the system being physically restricted, the velocities can be considered as bounded such that $\eta_{a3}(x(t)) < \alpha$ and $\eta_{a4}(x(t)) < \beta$ with $\alpha = \beta = 2\pi\, rad \cdot s^{-1}$. The non linear descriptor (22) becomes:

$$E(x(t))\dot{x}(t) = \left[\tilde{A}(x(t)) + \Delta\tilde{A}(t)\right]x(t) + Bu(t) + \tilde{\phi}(t) \qquad (23)$$

with $\tilde{A}(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\eta_{a1}(x(t)) & 0 & 0 & 0 \\ 0 & e\eta_{a2}(x(t)) & 0 & 0 \end{bmatrix}$ and

$$\Delta\tilde{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c\eta_{a3}(t) \\ 0 & 0 & c\eta_{a4}(t) & 0 \end{bmatrix}.$$

$\eta_{a1}(x(t))$ and $\eta_{a2}(x(t))$ being bounded by 1 and $\rho = \min(\eta_{a1}(x(t)))$, one can write:

$$\eta_{a1}(x(t)) = \underbrace{\frac{\eta_{a1}(x(t)) - \rho}{1-\rho}}_{w_{11}}(1) + \underbrace{\frac{1 - \eta_{a1}(x(t))}{1-\rho}}_{w_{12}}(\rho) \quad (24)$$

$$\eta_{a2}(x(t)) = \underbrace{\frac{\eta_{a2}(x(t)) - \rho}{1-\rho}}_{w_{21}}(1) + \underbrace{\frac{1 - \eta_{a2}(x(t))}{1-\rho}}_{w_{22}}(\rho) \quad (25)$$

Finally, the uncertain and disturbed T-S model of the planar two degrees of freedom robot is:

$$\sum_{k=1}^{2} v_k(z(t)) E_k \dot{x}(t) = \sum_{i=1}^{4} h_i(z(t))\left[\left\{\tilde{A}_i + \Delta\tilde{A}(t)\right\}x(t)\right] + Bu(t) + \tilde{\phi}(t)$$

with $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & c \\ 0 & 0 & c & b \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & -c \\ 0 & 0 & -c & b \end{bmatrix}$,

$\tilde{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 \\ 0 & e & 0 & 0 \end{bmatrix}$, $\tilde{A}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 \\ 0 & e\rho & 0 & 0 \end{bmatrix}$,

$\tilde{A}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\rho & 0 & 0 & 0 \\ 0 & e & 0 & 0 \end{bmatrix}$, $\tilde{A}_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\rho & 0 & 0 & 0 \\ 0 & e\rho & 0 & 0 \end{bmatrix}$,

To apply the LMI given by the theorem 2, one have to rewrite the uncertain matrix as $\Delta\tilde{A}(t) = H_a \Delta_a(t) N_a$ with $N_a = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c\beta \\ 0 & 0 & c\alpha & 0 \end{bmatrix}$,

$H_a = I$ and $\Delta a(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_1(t) \\ 0 & 0 & \Delta_2(t) & 0 \end{bmatrix}$.

The solutions of the conditions given by theorem 2 are now derived for the planar robot using the MATLAB LMI Toolbox (Gahinet, *et al.*,1995). The theoretical attenuation level is $\eta = 0.7501$ for a weighting matrix:

$\tilde{Q}$ = 1.0e-3 *[14 0 0 0;0 9.89 0 0;0 0 9.82 0;0 0 0 16];

The obtained gain matrices are given by:

$$K_{11} = \begin{bmatrix} 2403 & 522 \\ 3344.5 & 4595.5 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, \quad K_{12} = \begin{bmatrix} 1852 & 303 \\ 3350.8 & 5222 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{21}=\begin{bmatrix} 2403 & 522 \\ 3344.3 & 4595.3 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{22}=\begin{bmatrix} 1852 & 303 \\ 3350.6 & 5221.7 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{31}=\begin{bmatrix} 2393 & 522 \\ 3344.5 & 4595.5 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{32}=\begin{bmatrix} 1852 & 303 \\ 3350.8 & 5222 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{41}=\begin{bmatrix} 2393 & 522 \\ 3344.3 & 4595.3 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{42}=\begin{bmatrix} 1842 & 303 \\ 3350.6 & 5221.7 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T$$

The simulation of the non linear system (21) controlled via the obtained modified PDC control law was performed with the initial state conditions $x(0)=\begin{bmatrix}\pi & \pi & 0 & 0\end{bmatrix}^T (rad)$ and the external disturbances vector given by:

$$\tilde{\phi}(t)=\begin{bmatrix}0 & 0 & 20\sin(7\pi t) & 25\sin(7\pi t)\end{bmatrix}^T$$

Figure 2 illustrates the convergence of the state vector and the evolution of the control signals. The stability is achieved in less than 10 Sec. and the external disturbances are successfully balanced by the control signal.

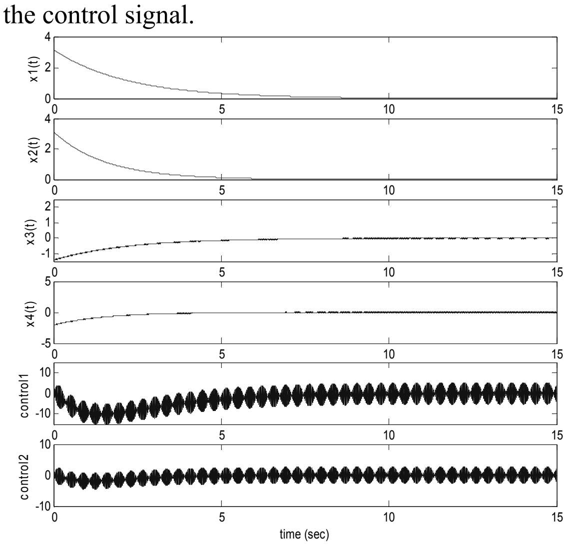

Fig. 2. Simulation results

CONCLUSION

In this paper, a fuzzy H_∞ controller design for a general class of uncertain fuzzy descriptor with external disturbances is proposed. The stability conditions have been obtained via a quadratic Lyapunov function and the modified PDC control law. The LMI formulation has been provided and a relaxation scheme due to (Tuan, *et al.*, 2001) has been applied. At last, a practical example has illustrated the efficiency of the proposed approach.

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