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LMI BASED H_∞ CONTROLLER DESIGN FOR UNCERTAIN TAKAGI-SUGENO DESCRIPTORS SUBJECT TO EXTERNAL DISTURBANCES

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Abstract: This paper deals with the design of an H_∞ fuzzy controller for uncertain T-S fuzzy descriptor with external disturbances. The control scheme is based on the modified PDC control law and an H_∞ criterion to attenuate external disturbances. Stability conditions are obtained via a quadratic Lyapunov function and are given in terms of LMI. To illustrate the efficiency of the proposed approach, a design example is provided with the simulation of a planar two degrees of freedom robot. *Copyright © 2007 IFAC*

Keywords: Descriptors, Takagi-Sugeno, LMI, H_∞ , External disturbances, Uncertainties.

1. INTRODUCTION

Many results about stability conditions and robust stability conditions for classical Takagi-Sugeno (T-S) fuzzy systems (Takagi and Sugeno, 1985) have already been obtained in the literature, see e.g. (Tanaka and Wang, 2001), (Sala, et al., 2005) and reference therein. On other hand, T-S fuzzy descriptor systems were first introduced by (Taniguchi, et al., 1999). These ones depict a wider class of systems because of the extensive application in many engineering systems, such as singular systems (Dai, 1989), joint torques estimation in human standing (Guelton, et al., 2006) or for the modelling of a two degrees of freedom pneumatic robot (Schulte and Guelton, 2006). Since the introduction of T-S fuzzy descriptor systems by (Taniguchi, et al., 1999), few works have been devoted to the study of their stability conditions. For instance, stability conditions for T-S fuzzy descriptor with parameters uncertainties has been reported by (Ma and Sun, 2004)(Yue and Lam, 2004). Moreover, the design of an H_∞ controller without uncertainties was also considered by (Yoneyama and Ichikawa, 1999). Nevertheless, in the above mentioned studies, the suggested stability conditions have been obtained for restrictive classes of descriptors systems in order to obtain LMI or BMI conditions. Indeed, the left part ($E\dot{x}(t) = \dots$) was considered as linear time

invariant and wasn't take into account the uncertain term. Thus, these are not suitable for a wide class of engineering systems. For instance, mechanical systems with time varying inertia (at least two degree of freedom) can be "naturally" modelled as a descriptor (with $E(x(t))\dot{x}(t) = \dots$) (Guelton, et al., 2006)(Schulte and Guelton, 2006). Moreover, introducing uncertainties in the left part is also enlarging the class of complex systems studied. For instance, dynamical models where inertias are unknown or difficult to estimate can be considered ($(E(x(t)) + \Delta E(t))\dot{x}(t) = \dots$). In this way the class of fuzzy descriptor systems studied in this paper is more general than the ones already studied in literature. The aim of this paper is to propose a controller design based on an H_∞ criterion for uncertain fuzzy descriptor with external disturbances presented in section 2. Then, stability conditions, formulated in terms of bilinear matrix inequalities, are obtained via a quadratic Lyapunov function and a modified PDC control law. Afterward, in order to be solved by classical convex optimization algorithms, an approach is proposed to put these conditions into LMI added to a relaxation scheme proposed by (Tuan, et al., 2001). Finally, in section 3, a design example will illustrate the efficiency of the proposed approach.

2. LMI STABILITY CONDITIONS FORMULATION

2.1 Considered class of uncertain and disturbed T-S descriptors

Takagi and Sugeno have propose an elegant way to approximate nonlinear affine systems as a collection of Linear models blended together by nonlinear functions. In that way, a T-S formulation of uncertain and disturbed non linear descriptor system can be described by the following state space equation:

$$\begin{aligned} & \sum_{k=1}^l v_k(z(t)) [E_k + \Delta E_k(t)] \dot{x}(t) \\ &= \sum_{i=1}^r h_i(z(t)) \left\{ \begin{array}{l} [A_i + \Delta A_i(t)] x(t) \\ + [B_i + \Delta B_i(t)] u(t) \end{array} \right\} + \phi(t) \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $z(t) \in \mathbb{R}^p$ is the premises vector, $\phi(t) \in \mathbb{R}^n$ denote the unknown disturbances with a known upper bound $\phi_{up} \geq \|\phi(t)\|$, l and r represent respectively the number of the fuzzy rules at the left and right side of the state representation, and the state matrices $\Delta E_k \in \mathbb{R}^{n \times n}$, $\Delta A_i \in \mathbb{R}^{n \times n}$, $\Delta B_i \in \mathbb{R}^{n \times m}$ contain all the modelling uncertainties of $E_k \in \mathbb{R}^{n \times n}$, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$.

We suppose that the uncertainties are bounded as described in (Zhou & Khargonekar, 1988) such that $\Delta E_k(t) = H_{ek} \Delta_{ek}(t) N_{ek}$, $\Delta A_i(t) = H_{ai} \Delta_{ai}(t) N_{ai}$, and $\Delta B_i(t) = H_{bi} \Delta_{bi}(t) N_{bi}$, where H_{ek} , H_{ai} , H_{bi} , N_{ek} , N_{ai} and N_{bi} are known constant real matrices with appropriate dimensions, $\Delta_{ek}(t)$, $\Delta_{ai}(t)$ and $\Delta_{bi}(t)$ are unknown matrices functions which are bounded such that $\forall t$ and the index $\xi = e, a$ or b and $\varpi = i$ or k , $\Delta_{\xi\varpi}^T(t) \Delta_{\xi\varpi}(t) \leq I$.

$\sum_{k=1}^l v_k(z(t))$ and $\sum_{i=1}^r h_i(z(t))$ are positive functions and satisfy the convex sum propriety $\sum_{k=1}^l v_k(z(t)) = 1$ and $\sum_{i=1}^r h_i(z(t)) = 1$.

2.2 Basic stability conditions

Let us consider the modified PDC control law (Taniguchi, *et al.*, 1999):

$$u(t) = - \sum_{i=1}^r \sum_{k=1}^l h_i(z(t)) v_k(z(t)) K_{ik} x(t) \quad (2)$$

where $K_{ik} \in \mathbb{R}^{m \times n}$ are the local feedback gains.

The closed-loop uncertain and disturbed T-S descriptor can be obtained, combining (2) and (1). This one is given by:

$$\begin{aligned} & \sum_{k=1}^l v_k(z(t)) [E_k + \Delta E_k(t)] \dot{x}(t) \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{k=1}^l h_i(z(t)) h_j(z(t)) v_k(z(t)) \Gamma_{ijk} x(t) + \phi(t) \end{aligned} \quad (3)$$

$$\text{with } \Gamma_{ijk} = [A_i + \Delta A_i(t)] - [B_i + \Delta B_i(t)] K_{jk}.$$

Recall that the state representation (3) takes into account the modelling uncertainties. Since, the system is subject to external disturbances $\phi(t)$, we propose to attenuate them using H_∞ performance. Thus, the problem treated below leads to a complete robust control approach.

Let us consider the H_∞ criterion given by :

$$\int_{t_0}^{t_f} x^T(t) Q x(t) \leq \eta^2 \int_{t_0}^{t_f} \phi^T(t) \phi(t) dt \quad (4)$$

where t_f is the final control time, Q is a positive definite weighting matrix and η is the prescribed attenuation level.

According to this criterion, basic robust stability conditions for the closed loop system (3) are summarized in the following theorem.

Theorem 1: The T-S uncertain and disturbed system (1) is quadratically stable via the modified PDC control law (2) and the H_∞ criterion (4) if there exist matrices $Z_1 = Z_1^T > 0$, Z_3 , Z_4 , an attenuation level η and the gain matrices K_{jk} such that the following conditions are satisfied:

For $i, j = 1, 2, \dots, r$ and $k = 1, 2, \dots, l$,

$$\begin{bmatrix} -Z_3 - Z_3^T + Z_1^T Q Z_1 & (*) & (*) & 0 \\ \Xi_{ijk} & \chi_k & 0 & (*) \\ I & 0 & -\eta^2 I & 0 \\ 0 & I & 0 & -\eta^2 I \end{bmatrix} \leq 0 \quad (5)$$

with $\chi_k = -(E_k + \Delta E_k) Z_4 - Z_4^T (E_k + \Delta E_k)^T$ and

$$\Xi_{ijk} = \begin{pmatrix} Z_4^T + (A_i + \Delta A_i) Z_1 \dots \\ -(B_i + \Delta B_i) K_{jk} Z_1 + (E_k + \Delta E_k) Z_3 \end{pmatrix}. \quad \text{As}$$

usual, a star (*) in a symmetric matrix indicates a transpose quantity.

Proof: In order to clarify the mathematical expressions, we will use the following notations. Let us consider the positive scalar functions $v_k(z(t))$ for $k \in \{1, 2, \dots, l\}$, $h_i(z(t))$ for $i \in \{1, 2, \dots, r\}$ and the matrices E_k for $k \in \{1, 2, \dots, l\}$, Y_i for $i \in \{1, 2, \dots, r\}$ and T_{ik} for $i \in \{1, 2, \dots, r\}$ and $k \in \{1, 2, \dots, l\}$. We

will denote $E_v = \sum_{k=1}^l v_k(z(t)) E_k$, $Y_h = \sum_{i=1}^r h_i(z(t)) Y_i$

Corollary: for real matrices A, B, W, Y, μ and a regular matrix $Q > 0$ with appropriate dimensions one has:

$$\begin{bmatrix} Y & W^T + B^T A^T \\ W + AB & \mu \end{bmatrix} < 0 \\ \Rightarrow \begin{bmatrix} Y + B^T Q^{-1} B & W^T \\ W & \mu + A Q A^T \end{bmatrix} < 0$$

Now, the following theorem summarizes the sufficient LMI stability conditions. This constitutes the main contribution of this work.

Theorem 2: The T-S uncertain and disturbed system (1) is quadratically stable via the modified PDC control law (2) and the H_∞ criterion (4) if there exist matrices $Z_1 = Z_1^T > 0$, Z_3 , Z_4 , M_{jk} , positive scalars σ_1^{-1} , σ_2 , σ_3 and σ_4 and an attenuation level $\delta = \eta^2$ such that the following conditions are satisfied:

- For $i=1,2,\dots,r$ and $k=1,2,\dots,l$, $\Psi_{iik} < 0$ (17)

- For $i=1,2,\dots,r$, $1 \leq i \neq j \leq r$ and $k=1,2,\dots,l$,

$$\frac{1}{r-1} \Psi_{iik} + \frac{1}{2} (\Psi_{ijk} + \Psi_{jik}) < 0 \quad (18)$$

with

$$\Psi_{ijk} = \begin{bmatrix} -Z_3 - Z_3^T & (*) & (*) & (*) & (*) & (*) & 0 & (*) & 0 \\ N_{ai} Z_1 & -\sigma_2 I & 0 & 0 & 0 & 0 & 0 & 0 & (*) \\ N_{bi} M_{jk} & 0 & -\sigma_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ N_{ek} Z_3 & 0 & 0 & -\sigma_4 I & 0 & 0 & 0 & 0 & 0 \\ Z_1 & 0 & 0 & 0 & -Q^{-1} I & 0 & 0 & 0 & 0 \\ \Gamma_{ijk} & 0 & 0 & 0 & 0 & \Upsilon_{ik} & (*) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{ek} Z_4 & -\sigma_1^{-1} I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 & 0 & -\delta I & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & -\delta I \end{bmatrix}$$

$$\Upsilon_{ik} = \begin{pmatrix} \sigma_2 H_{ai} H_{ai}^T + \sigma_3 H_{bi} H_{bi}^T + \sigma_4 H_{ek} H_{ek}^T \\ -E_k Z_4 + \sigma_1^{-1} H_{ek} H_{ek}^T - Z_4^T E_k^T \end{pmatrix} \text{ and}$$

$$\Gamma_{ijk} = Z_4^T + A_i Z_1 - B_i M_{jk} + E_k Z_3$$

Proof: Let us start from equation (16). With the bijective change of variables $M_{hv} = K_{hv} Z_1$, $\delta = \eta^2$ and substituting $\Delta E_v(t)$, $\Delta A_v(t)$, $\Delta B_v(t)$ by their bounded quantities as described in (Zhou and Khargonekar, 1988) (see below equation (1)), (5) can be rewritten as:

$$\begin{bmatrix} (-Z_3 - Z_3^T + Z_1^T Q Z_1) & (*) & (*) & 0 \\ \mathfrak{U}_{hhv} & \Lambda_v & 0 & (*) \\ I & 0 & -\delta I & 0 \\ 0 & I & 0 & -\delta I \end{bmatrix} \leq 0 \quad (19)$$

with

$$\Lambda_v = \begin{pmatrix} -E_v Z_4 - Z_4^T E_v^T \\ -\{H_e \Delta_e N_e\}_v Z_4 - Z_4^T \{H_e \Delta_e N_e\}_v^T \end{pmatrix},$$

$$\mathfrak{U}_{hhv} = \begin{pmatrix} Z_4^T + A_h Z_1 - B_h M_{hv} + E_v Z_3 + \{H_a \Delta_a N_a\}_h Z_1 \\ -\{H_b \Delta_b N_b\}_h M_{hv} + \{H_e \Delta_e N_e\}_v Z_3 \end{pmatrix},$$

and with the notations: for $\xi = e, a$ or b and $\varphi = h$

$$\text{or } v, \{H_\xi \Delta_\xi N_\xi\}_\varphi = \sum_{i=1}^r \varphi(z(t)) (H_{\xi i} \Delta_{\xi i}(t) N_{\xi i}).$$

In order to major the uncertain terms $\Delta_{\xi i}(t)$ contained in (19), the lemma 2 and the corollary defined bellow will be used. Then, considering that $\Delta_{\xi i}^T(t) \Delta_{\xi i}(t) \leq I$, the inequality (19) yields to:

$$\begin{bmatrix} \lambda_{hhv} & (*) & (*) & 0 \\ \begin{pmatrix} Z_4^T + A_h Z_1 \\ -B_h M_{hv} + E_v Z_3 \end{pmatrix} & \Omega_{hv} & 0 & (*) \\ I & 0 & -\delta I & 0 \\ 0 & I & 0 & -\delta I \end{bmatrix} \leq 0 \quad (20)$$

with

$$\lambda_{hhv} = \begin{pmatrix} -Z_3 - Z_3^T + Z_1^T Q Z_1 + \sigma_3^{-1} M_{hv}^T \{N_b^T N_b\}_h M_{hv} \\ + \sigma_2^{-1} Z_1^T \{N_a^T N_a\}_v Z_1 + \sigma_4^{-1} Z_3^T \{N_e^T N_e\}_v Z_3 \end{pmatrix},$$

$$\Omega_{hv} = \begin{pmatrix} \sigma_2 \{H_a H_a^T\}_h + \sigma_3 \{H_b H_b^T\}_h + \sigma_1 Z_4^T \{N_e^T N_e\}_v Z_4 \\ + \sigma_4 \{H_e H_e^T\}_v + \sigma_1^{-1} \{H_e H_e^T\}_v - E_v Z_4 - Z_4^T E_v^T \end{pmatrix}$$

Then, applying successfully the Schur complement on λ_{hhv} and Ω_{hv} , the conditions of theorem 2 hold. ■

3. EXAMPLE AND SIMULATION OF A PLANAR TWO DEGREES OF FREEDOM ROBOT

In order to illustrate the efficiency of the proposed approach, we consider the planar two degrees of freedom robot presented in Figure 1. The dynamic equation of this robot is given by:

$$M(q) \ddot{q}(t) + C(q, \dot{q}) \dot{q}(t) + G(q) q(t) = Ru(t) + \phi(t) \quad (21)$$

where, θ_1 , θ_2 , $\dot{\theta}_1$, $\dot{\theta}_2$ denotes respectively the angular positions and the angular velocities. $q = [\theta_1 \ \theta_2]^T$ is the vector of generalized coordinates, $\phi(t)$ is the vector of external

disturbances, $M(q) = \begin{bmatrix} a & c \cos(\theta_1 - \theta_2) \\ c \cos(\theta_1 - \theta_2) & b \end{bmatrix}$

is the inertia matrix, $C(q, \dot{q}) = \begin{bmatrix} 0 & c \dot{\theta}_2 \sin(\theta_1 - \theta_2) \\ -c \dot{\theta}_1 \sin(\theta_1 - \theta_2) & 0 \end{bmatrix}$ is the

Coriolis anti-symmetric matrix, $G(q) = \begin{bmatrix} d \sin c(\theta_1) & 0 \\ 0 & e \sin c(\theta_2) \end{bmatrix}$ is the gravitational

matrix and $R = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ is the matrix linking joint

torque to the generalized torque $u(t) = [u_1 \ u_2]^T$, and with $a = m_1 K^2 L_1^2 + m_2 L_1^2 + I_1$, $b = m_2 L_2^2 + I_2$, $c = m_2 L_1 L_2$, $d = (m_2 + m_1 K) g L_1$, $e = m_2 g L_2$ and

$m_1 = 1.4 \text{ kg}$, $m_2 = 0.7 \text{ kg}$, $L_1 = 0.6 \text{ m}$, $L_2 = 0.3 \text{ m}$,
 $K = 0.5$, $g = 9.81 \text{ m s}^{-2}$, $I_1 = I_2 = 0.25 \text{ kg m}^2$.

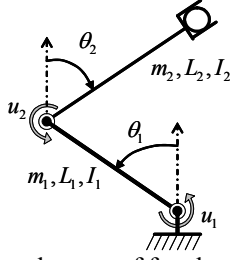


Fig.1. Planar two degrees of freedom robot.

Let $x = [\theta_1 \ \theta_2 \ \dot{\theta}_1 \ \dot{\theta}_2]^T$ be the state vector of the planar robot. The dynamical model (21) can be rewritten as a non linear descriptor as:

$$E(x(t))\dot{x}(t) = A(x(t))x(t) + Bu(t) + \tilde{\phi}(t) \quad (22)$$

with $E(x(t)) = \begin{bmatrix} I & 0 \\ 0 & M(x(t)) \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ R \end{bmatrix}$,

$$A(x(t)) = \begin{bmatrix} 0 & I \\ G(x(t)) & -C(x(t)) \end{bmatrix} \text{ and } \tilde{\phi}(t) = \begin{bmatrix} 0 \\ \phi(t) \end{bmatrix}.$$

Note that (22) contains one non linear term $\eta_e(x) = \cos(\theta_1 - \theta_2)$ in $E(x(t))$ and four ones in $A(x(t))$: $\eta_{a1}(x) = \frac{\sin \theta_1}{\theta_1}$, $\eta_{a2}(x) = \frac{\sin \theta_2}{\theta_2}$,
 $\eta_{a3}(x) = \dot{\theta}_2 \sin(\theta_1 - \theta_2)$ and $\eta_{a4}(x) = \dot{\theta}_1 \sin(\theta_1 - \theta_2)$. Using the sector nonlinearity approach (Tanaka & Wang, 2001), this leads to $l=2$ and $r=16$ rules for the left and the right part of the T-S fuzzy model. Then, to ensure the stability of the descriptor system, $l.r.(r+1)/2 = 272$ LMI conditions have to be verified. Consequently, this should be conservative with classical stability conditions for T-S descriptors (Taniguchi, *et al.*, 1999). In order to reduce the conservatism, some non linear terms will be put to uncertainties. Indeed, the motion of the system being physically restricted, the velocities can be considered as bounded such that $\eta_{a3}(x(t)) < \alpha$ and $\eta_{a4}(x(t)) < \beta$ with $\alpha = \beta = 2\pi \text{ rad} \cdot \text{s}^{-1}$. The non linear descriptor (22) becomes:

$$E(x(t))\dot{x}(t) = [\tilde{A}(x(t)) + \Delta\tilde{A}(t)]x(t) + Bu(t) + \tilde{\phi}(t) \quad (23)$$

with $\tilde{A}(x) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\eta_{a1}(x(t)) & 0 & 0 & 0 \\ 0 & e\eta_{a2}(x(t)) & 0 & 0 \end{bmatrix}$ and

$$\Delta\tilde{A}(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c\eta_{a3}(t) \\ 0 & 0 & c\eta_{a4}(t) & 0 \end{bmatrix}.$$

$\eta_{a1}(x(t))$ and $\eta_{a2}(x(t))$ being bounded by 1 and

$\rho = \min(\eta_{a1}(x(t)))$, one can write:

$$\eta_{a1}(x(t)) = \underbrace{\frac{\eta_{a1}(x(t)) - \rho}{1 - \rho}}_{w_{11}}(1) + \underbrace{\frac{1 - \eta_{a1}(x(t))}{1 - \rho}}_{w_{12}}(\rho) \quad (24)$$

$$\eta_{a2}(x(t)) = \underbrace{\frac{\eta_{a2}(x(t)) - \rho}{1 - \rho}}_{w_{21}}(1) + \underbrace{\frac{1 - \eta_{a2}(x(t))}{1 - \rho}}_{w_{22}}(\rho) \quad (25)$$

Finally, the uncertain and disturbed T-S model of the planar two degrees of freedom robot is:

$$\sum_{k=1}^2 v_k(z(t)) E_k \dot{x}(t) = \sum_{i=1}^4 h_i(z(t)) [\tilde{A}_i + \Delta\tilde{A}(t)] x(t) + Bu(t) + \tilde{\phi}(t)$$

with $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & c \\ 0 & 0 & c & b \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a & -c \\ 0 & 0 & -c & b \end{bmatrix}$,

$$\tilde{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 \\ 0 & e & 0 & 0 \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 \\ 0 & e\rho & 0 & 0 \end{bmatrix},$$

$$\tilde{A}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\rho & 0 & 0 & 0 \\ 0 & e & 0 & 0 \end{bmatrix}, \tilde{A}_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ d\rho & 0 & 0 & 0 \\ 0 & e\rho & 0 & 0 \end{bmatrix},$$

To apply the LMI given by the theorem 2, one have to rewrite the uncertain matrix as

$$\Delta\tilde{A}(t) = H_a \Delta_a(t) N_a \text{ with } N_a = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c\beta \\ 0 & 0 & c\alpha & 0 \end{bmatrix},$$

$$H_a = I \text{ and } \Delta_a(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_1(t) \\ 0 & 0 & \Delta_2(t) & 0 \end{bmatrix}.$$

The solutions of the conditions given by theorem 2 are now derived for the planar robot using the MATLAB LMI Toolbox (Gahinet, *et al.*, 1995). The theoretical attenuation level is $\eta = 0.7501$ for a weighting matrix:

$$\tilde{Q} = 1.0e-3 * [14 \ 0 \ 0 \ 0; 0 \ 9.89 \ 0 \ 0; 0 \ 0 \ 9.82 \ 0; 0 \ 0 \ 0 \ 16];$$

The obtained gain matrices are given by:

$$K_{11} = \begin{bmatrix} 2403 & 522 \\ 3344.5 & 4595.5 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{12} = \begin{bmatrix} 1852 & 303 \\ 3350.8 & 5222 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{21} = \begin{bmatrix} 2403 & 522 \\ 3344.3 & 4595.3 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{22} = \begin{bmatrix} 1852 & 303 \\ 3350.6 & 5221.7 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{31} = \begin{bmatrix} 2393 & 522 \\ 3344.5 & 4595.5 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{32} = \begin{bmatrix} 1852 & 303 \\ 3350.8 & 5222 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

$$K_{41} = \begin{bmatrix} 2393 & 522 \\ 3344.3 & 4595.3 \\ 523 & 114 \\ 3456 & 4749 \end{bmatrix}^T, K_{42} = \begin{bmatrix} 1842 & 303 \\ 3350.6 & 5221.7 \\ 402 & 66 \\ 3462 & 5396 \end{bmatrix}^T,$$

The simulation of the non linear system (21) controlled via the obtained modified PDC control law was performed with the initial state conditions $x(0)=[\pi \ \pi \ 0 \ 0]^T$ (rad) and the external disturbances vector given by:

$$\tilde{\phi}(t)=[0 \ 0 \ 20\sin(7\pi t) \ 25\sin(7\pi t)]^T$$

Figure 2 illustrates the convergence of the state vector and the evolution of the control signals. The stability is achieved in less than 10 Sec. and the external disturbances are successfully balanced by the control signal.

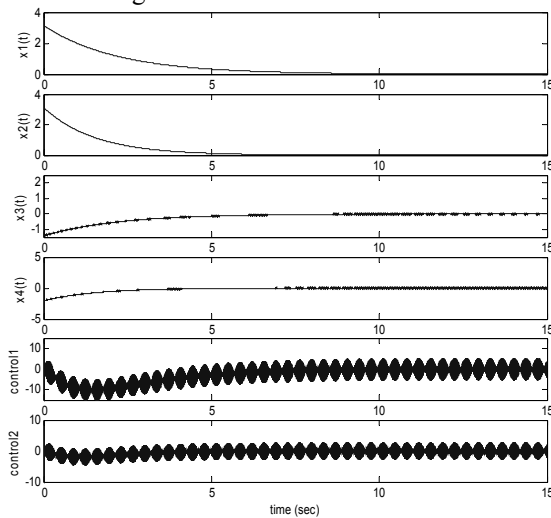


Fig. 2. Simulation results

CONCLUSION

In this paper, a fuzzy H_∞ controller design for a general class of uncertain fuzzy descriptor with external disturbances is proposed. The stability conditions have been obtained via a quadratic Lyapunov function and the modified PDC control law. The LMI formulation has been provided and a relaxation scheme due to (Tuan, *et al.*, 2001) has been applied. At last, a practical example has illustrated the efficiency of the proposed approach.

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REFERENCES

- Dai, L. (1989). Singular control systems. *Lecture Notes in Cont. and Information Sc.*, pp.12-45.
- Gahinet, P., A. Nemirovski, A.J. Laub and M. Chilali (1995). *LMI Control Toolbox for Use with MATLAB*, The Mathworks Partner Series.
- Guelton, K., S. Delprat and T.M. Guerra (2006). Joint torques estimation in standing human by Mean of a fuzzy descriptor unknown inputs observer, in *Proc. 6th IFAC Symposium on Modelling and Control in Biomedical Systems, Reims, France, Elsevier*, pp. 405-410.
- Liu, X, and Q. Zhang (2003). New approaches to H-infinity controller design based on fuzzy observers for fuzzy TS systems via LMI, *Automatica*, Vol. 12(9), pp.1571-1582.
- Ma, B.P., and J. Sun (2004). Robust stabilization of uncertain T-S fuzzy descriptor systems, in *Proc. 3rd IEEE International Conference on Machine Learning and Cybernetics*, Shanghai, China.
- Sala, A., Guerra, T.M., Babuska, R., (2005). Perspectives of fuzzy systems and control, *Fuzzy Sets & Systems*, Vol. 153 (3), 2005, pp. 432-444.
- Schulte, H., K. Guelton (2006). Modelling and simulation of two-link robot manipulators based on Takagi Sugeno fuzzy descriptor systems, in *Proc. IEEE International Conference on Industrial Technology, Mumbai, India*.
- Takagi, T. and M. Sugeno (1985). fuzzy identification of systems and its applications to modelling and control, *IEEE Trans. On System Man and Cybernetics*, Vol. 15 (1), pp. 116-132.
- Tanaka, K., T. Ikeda, and H.O. Wang (1998). Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs, *IEEE Trans. Fuzzy Syst.*, Vol. 6(2), pp. 250-265.
- Tanaka, K., and H.O. Wang (2001). *fuzzy control systems design and analysis. A linear matrix inequality approach*, Wiley, New York.
- Taniguchi, T., K. Tanaka and H.O. Wang (1999). Fuzzy descriptor systems and nonlinear model following control, *IEEE Trans. on Fuzzy Systems*, Vol. 8 (4), pp.442-452.
- Tuan, H.D., P. Apkarian, T. Narikiyo, and Y. Yamamoto (2001). Parametrized linear matrix inequality techniques in fuzzy control design," *IEEE Trans. Fuzzy Syst.*, Vol. 9(2), pp.324-332.
- Yoneyama, J., and A. Ichikawa (1999). H_∞ -control for Takagi-Sugeno fuzzy descriptor systems", *In Proc. IEEE Conference on Systems, Man and Cybernetics*, pp.III28-III33, Tokyo.
- Yue, D., and J. Lam (2004). Suboptimal robust mixed H_2/H_∞ controller design for uncertain descriptor systems with distributed delays, *Computers and Mathematics With Applications*, Vol. 47, 1041- 1055.
- Zhou, K., and P.P. Khargonekar (1988). Robust stabilization of linear systems with norm-bounded time-varying uncertainty," *Syst. Control Letters*, Vol. 10, pp. 17-20.