LMI Stability Conditions for Takagi-Sugeno Uncertain Descriptors

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***Abstract*—In this paper, stability conditions for a wide class of Takagi-Sugeno uncertain Descriptors are proposed. These are based on a quadratic Lyapunov candidate function. In order to solve the stability conditions with classical convex optimization algorithms, matrices transformations have been used to write these conditions in term of linear matrix inequality (LMI). A designed example illustrates the efficiency of the proposed approach.**

I. Introduction

Takagi-Sugeno (T-S) fuzzy systems [1] have been widely used in the context of nonlinear models control. These consist on local linear time invariant (LTI) models blended together with normalized nonlinear membership functions. A lot of papers dealings with stability conditions for various class of classical state space representation of T-S fuzzy systems have been proposed, see e.g. [2][3][4] and references therein. An extended view of classical state space representation, called state space descriptor systems, has been first introduced by [5] as a wider class of systems. Fuzzy T-S descriptor systems have then been proposed by [6]. Since their introduction, the interest of nonlinear and/or fuzzy descriptors has increased for their ability to represent a large class of engineering systems such as singular systems [7], geometrically variable mechanical systems (i.e. with time varying inertia) [8][9], etc. For instance, they have already been used, for the joint torque estimation in human standing [8] and for modelling the pneumatic actuator nonlinearities of a two degree of freedom robot [9]. Since [6], few papers have been proposed in the literature dealing with the problem of stability of fuzzy descriptor systems. In fact, the stability of a particular class of uncertain fuzzy descriptor has been reported by [10][11]. Nevertheless, the above suggested stability conditions concern a restrictive class of descriptor systems that substantially reduce the range of their applicability. Moreover, redundancy of descriptor representation have been also used to reduce the number of LMI to be satisfied [19][20].

The aim of this paper is to propose stability conditions for a wider class of descriptor systems with parametric uncertainties via a quadratic Lyapunov function and a modified Paralleled Distributed Compensation (PDC) control law [6]. In order to solve the derived stability conditions by classical convex optimization algorithms [12], these conditions are put into Linear Matrix Inequalities (LMI) [13]. Then, less conservative stability conditions will be proposed using a relaxation scheme [14]. Finally, a numerical example illustrates the efficiency of the proposed approach.

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II. Useful notations, lemma and corollary

A. Notations

Let us consider the scalar functions $v_k(z)$ and $h_i(z)$ and the matrices E_k for $k \in \{1,\ldots,l\}$, Y_i for $i \in \{1,\ldots,r\}$ and T_{ik} for $k \in \{1,\ldots,l\}$ and $i \in \{1,\ldots,r\}$ with appropriate dimensions, we will denote:

$$E_v = \sum_{k=1}^{l} v_k(z) E_k \; , Y_h = \sum_{i=1}^{r} h_i(z) Y_i \; ,$$
$$T_{hv} = \sum_{k=1}^{l}\sum_{i=1}^{r} v_k(z) h_i(z) T_{ik} \; .$$

As usual a star $(*)$ indicates a transpose quantity in a symmetric matrix.

B. Useful lemma and corollary

The following matrix inequality lemma and corollary are needed to put and/or relax the further provided stability conditions into LMI.

Lemma 1 [15]: *for real matrices* A , B *and* $S = S^T > 0$ *with appropriate dimensions and a positive constant* τ *, we have:*

$$A^T B + B^T A \le \tau A^T A + \tau^{-1} B^T B \tag{1}$$
$$X^T Y + Y^T X \le X^T S^{-1} X + Y^T S Y \tag{2}$$

Corollary 1: *for real matrices* A, B, W, Y, Z and a regular positive matrix Q *with appropriate dimensions we have*

$$\begin{bmatrix} Y & W^T + B^T A^T \\ W + AB & Z \end{bmatrix} < 0 \Rightarrow \begin{bmatrix} Y + B^T Q^{-1} B & W^T \\ W & Z + AQA^T \end{bmatrix} < 0 \tag{3}$$

Lemma 2 [14]:
Consider the proposition "*For all combinations* $i, j = 1,2,...,r$ *and* $k = 1,2,...,l$ *we have* $\gamma_{ijk} < 0$".
This proposition is equivalent to: "*For all combinations* $i, j = 1,2,...,r$ *and* $k = 1,2,...,l$, $\gamma_{iik} < 0$ *and for*

$1 \le i \ne j \le r$, *we have* $\frac{1}{r-1}\gamma_{iik} + \frac{1}{2}\left(\gamma_{ijk} + \gamma_{jik}\right) < 0$ *".*

III. Class of Descriptor Systems

A. Uncertain T-S descriptor models

Let us consider the class of uncertain T-S descriptor system described by:

$$\sum_{k=1}^{l} v_k\left(z(t)\right)\left(E_k + \Delta E_k(t)\right)\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{\left(A_i + \Delta A_i(t)\right)x(t) + \left(B_i + \Delta B_i(t)\right)u(t)\right\} \quad (4)$$

where: l and r represent respectively the number of fuzzy rules for the left and the right part of the state equation. $x(t) \in \Re^n$, $u(t) \in \Re^m$ represent respectively the state and the input vectors, $v_k\left(z(t)\right)$ and $h_i\left(z(t)\right)$ are the positive membership functions associated to the fuzzy rules satisfying the convex sum proprieties $\sum_{i=1}^{r} h_i\left(z(t)\right) = 1$ and $\sum_{k=1}^{l} v_k\left(z(t)\right) = 1$. $E_k \in \Re^{n\times n}$, $A_i \in \Re^{n\times n}$ and $B_i \in \Re^{n\times m}$ are real state matrices. $\Delta E_k(t) \in \Re^{n\times n}$, $\Delta A_i(t) \in \Re^{n\times n}$ and $\Delta B_i(t) \in \Re^{n\times m}$ contain the bounded uncertain terms which can be rewritten as:

$$\Delta A_i(t) = H_{ai}\Delta_{ai}(t)N_{ai},\ \Delta B_i(t) = H_{bi}\Delta_{bi}(t)N_{bi},$$
$$\text{and } \Delta E_k(t) = H_{ek}\Delta_{ek}(t)N_{ek} \quad (5)$$

with H_{ai}, H_{bi}, H_{ek}, N_{ai}, N_{bi}, and N_{ek} are known constant matrices and, $\Delta_{ai}(t)$, $\Delta_{bi}(t)$ and $\Delta_{ek}(t)$ are unknown matrices functions bounded as:

For all index $\xi = a, b$ *or* e and $\theta = i$ *or* k, $i = 1,...,r$ and $k = 1,...l$, one has $\Delta_{\xi\theta}^{T}(t)\Delta_{\xi\theta}(t) \le I$ (6)

Comment: Let us point out that, in our opinion, LMI stability conditions for global uncertain T-S descriptors define by (4) do not exist in the previous literature. Indeed, the available studies consider the class of uncertain descriptor where the left part (i.e. $E\dot{x}(t) = ...$) is linear time invariant and not uncertain [10][11], that is to say the class of systems defined by:

$$E\dot{x}(t) = \sum_{i=1}^{r} h_i\left(z(t)\right)\left\{\left(A_i + \Delta A_i(t)\right)x(t) + \left(B_i + \Delta B_i(t)\right)u(t)\right\} \quad (7)$$

That leads to solutions that are not useful for a large class of systems. For instance, as stated in [8][9], geometrically variable mechanical systems (i.e. with time varying inertia) should naturally be modelled as a descriptor with a nonlinear state dependant left part ($E\left(x(t)\right)\dot{x}(t) = ...$) leading to less conservative LMI based design in the case of T-S modelling. Moreover, the interest of taking into account uncertainties in the left member should be used in the case of mechanical systems, to model various complex systems where inertia are unknown or difficult to estimate. Note also that, in that way, the class studied in this paper (presented by equation (4)) are more general than the ones proposed in the literature.

IV. Stability Conditions and LMI Formulation

A. Sufficient stability conditions

Let consider the modified PDC control law [6]:

$$u(t) = -\sum_{i=1}^{r}\sum_{k=1}^{l} h_i\left(z(t)\right)v_k\left(z(t)\right)K_{ik}x(t) \quad (8)$$

where $K_{ik} \in \Re^{m\times n}$ are the local feedback gains.

Theorem 1: *The T-S uncertain descriptor system* (4) *is quadratically stable via the modified PDC control law* (8) *if there exists free matrices* $z_1 = z_1^T > 0$, z_3, z_4 *and the gain matrices* K_{jk} *such that the following conditions are satisfied:*

For $i, j = 1,2,...,r$ *and* $k = 1,2,...,l$,

$$\begin{bmatrix} -z_3^T - z_3 & (*) \\ \begin{pmatrix} z_4^T + A_i z_1 + E_k z_3 - B_i K_{jk} z_1 \\ +\Delta A_i(t) z_1 + \Delta B_i(t) K_{jk} z_1 \\ +\Delta E_k(t) z_3 \end{pmatrix} & \begin{pmatrix} -z_4^T E_k^T - E_k z_4 \\ -z_4^T \Delta E_k^T(t) \\ -\Delta E_k(t) z_4 \end{pmatrix} \end{bmatrix} < 0 \quad (9)$$

Proof: Let $x^*(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) \end{bmatrix}^T$, then, the uncertain fuzzy descriptor (4) can be rewritten with the notation given in section 2 as:

$$E_v^*\dot{x}^*(t) = A_{hv}^*x^*(t) + B_h^*u(t) \quad (10)$$

with $E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $A_{hv}^* = \begin{bmatrix} 0 & I \\ A_h + \Delta A_h & -E_v - \Delta E_v \end{bmatrix}$, $B_h^* = \begin{bmatrix} 0 \\ B_h + \Delta B_h \end{bmatrix}$.

According to the notations defined below, the modified PDC control law (8) can also be rewritten as:

$$u(t) = -K_{hv}^*x^*(t) \quad (11)$$

where $K_{hv}^* = \begin{bmatrix} K_{hv} & 0 \end{bmatrix}$.

Substituting (11) into (10), the closed-loop system becomes:

$$E_v^* \dot{x}^*(t) = \left[A_{hv}^* - B_h^* K_{hv}^* \right] x^*(t) \tag{12}$$

Now, let us consider the following candidate Lyapunov function:

$$V\left(x^*(t)\right) = x^{*T}(t) E^{*T} P x^*(t) \tag{13}$$

with the needed symmetric condition $E^{*T} P = P^T E^* \geq 0$ to be considered as a quadratic function. We consider $P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix}$, where P_1, P_2, P_3, P_4 $\in \Re^{n \times n}$ are constant matrices. The symmetric condition leads to:

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} = \begin{bmatrix} P_1^T & P_3^T \\ P_2^T & P_4^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \tag{14}$$

and then $P_1 = P_1^T \geq 0$, $P_2 = 0$, P_3 and P_4 are free matrices.

(12) is stable if the Lyapunov function decreases, that is to say if:

$$\dot{V}\left(x^*(t)\right) = \dot{x}^{*T}(t) E^{*T} P x^*(t) + x^{*T}(t) E^{*T} P \dot{x}^*(t) < 0 \tag{15}$$

By substituting (12) into (15), one obtains:

$$\dot{V}\left(x^*(t)\right) = x^{*T}(t) \Psi_{hv} x^*(t) < 0 \tag{16}$$

with $\Psi_{hv} = \left(A_{hv}^* - B_h^* K_{hv}^*\right)^T P + P^T \left(A_{hv}^* - B_h^* K_{hv}^*\right)$

We consider the following change of variables

$$X = P^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -P_4^{-1} P_3 P_1^{-1} & P_4^{-1} \end{bmatrix} = \begin{bmatrix} z_1 & 0 \\ -z_3 & z_4 \end{bmatrix} \tag{17}$$

And after multiplying $\Psi_{hv} < 0$ left by $X^T = P^{-T}$ and right by $X = P^{-1}$ we obtains:

$$X^T A_{hv}^{*\,T} - X^T K_{hv}^{*\,T} B_h^{*T} + A_{hv}^* X - B_h^* K_{hv}^* X < 0 \tag{18}$$

(18) can be developed, with the matrices defined in (10) and (11)defined below, as follows:

$$\begin{bmatrix} -z_3^T - z_3 & (*) \\ \begin{pmatrix} z_4^T + A_h z_1 + E_v z_3 - B_h K_{hv} z_1 \\ +\Delta A_h(t) z_1 + \Delta B_h(t) K_{hv} z_1 \\ +\Delta E_v(t) z_3 \end{pmatrix} & \begin{pmatrix} -z_4^T E_v^T - E_v z_4 \\ -z_4^T \Delta E_v^T(t) \\ -\Delta E_v(t) z_4 \end{pmatrix} \end{bmatrix} < 0 \tag{19}$$

which is obviously satisfied if conditions (9) hold. ∎

B. LMI stability conditions

The conditions provided by Theorem 1 are note easily solvable by classical convex optimization algorithms because of their BMI (Bilinear Matrix Inequality) structure and of the time dependant terms included in the uncertain part. The following theorem provides solvable LMI conditions.

Theorem 2: *The fuzzy uncertain descriptor system* (4) *is quadratically stable via the modified PDC control law* (8) *if there exists the matrices* $z_1 = z_1^T > 0$, z_3, z_4, M_{jk} *and scalars* τ_1, τ_2, τ_3 *and* τ_4^{-1} *such that the following conditions are satisfied:*

For $i, j = 1,2,...,r$ *and* $k = 1,2,...,l$,

$$\begin{bmatrix} -z_3^T - z_3 & (*) & (*) & (*) & (*) & 0 \\ N_{ai} z_1 & -\tau_1 I & 0 & 0 & 0 & 0 \\ N_{bi} M_{jk} & 0 & -\tau_2 I & 0 & 0 & 0 \\ N_{ek} z_3 & 0 & 0 & -\tau_3 I & 0 & 0 \\ \Omega_{ijk} & 0 & 0 & 0 & \Pi_{ik} & (*) \\ 0 & 0 & 0 & 0 & N_{ek} z_4 & -\tau_4^{-1} I \end{bmatrix} < 0 \tag{20}$$

with $\Omega_{ijk} = \begin{pmatrix} z_4^T + A_i z_1 \\ + E_k z_3 \\ - B_i M_{jk} \end{pmatrix}$ and $\Pi_{ik} = \begin{pmatrix} -z_4^T E_k^T - E_k z_4 \\ +\tau_2 H_{bi} H_{bi}^T + \tau_1 H_{ai} H_{ai}^T \\ +\tau_3 H_{ek} H_{ek}^T + \tau_4^{-1} H_{ek} H_{ek}^T \end{pmatrix}$

Proof: Starting from the conditions provided by the theorem 1. With the bijective change of variables $M_{hv} = K_{hv} z_1$, one obtains:

$$\begin{bmatrix} 0 & (*) \\ \Delta A_h(t) z_1 + \Delta E_v(t) z_3 + \Delta B_h(t) M_{hv} & -z_4^T \Delta E_v(t)^T - \Delta E_v(t) z_4 \end{bmatrix} + \begin{bmatrix} -z_3^T - z_3 & (*) \\ z_4^T + A_h z_1 + E_v z_3 - B_h M_{hv} & -z_4^T E_v^T - E_v z_4 \end{bmatrix} < 0 \tag{21}$$

Then, the uncertain terms are assumed to be bounded as described in (5), then (21) becomes

$$\begin{bmatrix} 0 & (*) \\ \begin{pmatrix} H_{ah} \Delta_{ah}(t) N_{ah} z_1 \\ +H_{ev} \Delta_{ev}(t) N_{ev} z_3 \\ +H_{bh} \Delta_{bh}(t) N_{bh} M_{hv} \end{pmatrix} & \begin{pmatrix} -z_4^T \left(H_{ev} \Delta_{ev}(t) N_{ev}\right)^T \\ -H_{ev} \Delta_{ev}(t) N_{ev} z_4 \end{pmatrix} \end{bmatrix} + \begin{bmatrix} -z_3^T - z_3 & (*) \\ \left(z_4^T + A_h z_1 + E_v z_3 - B_h M_{hv}\right) & \left(-z_4^T E_h^T - E_v z_4\right) \end{bmatrix} < 0 \tag{22}$$

Let us now apply the Lemma 1 and its corollary 1 to major the uncertainties. Thus, the inequality (22) leads to:

$$\begin{bmatrix} \delta_{hhv} & (*) \\ \begin{pmatrix} z_4^T + A_h z_1 + \\ E_v z_3 - B_h M_{hv} \end{pmatrix} & \mu_{hv} \end{bmatrix} < 0 \tag{23}$$

with $\mu_{hv} = \begin{pmatrix} -z_4^T E_v^T - E_v z_4 + \tau_4 z_4^T N_{ev}^T N_{ev} z_4 + \tau_1 H_{ah} H_{ah}^T \\ +\tau_2 H_{bh} H_{bh}^T + \tau_3 H_{ev} H_{ev}^T + \tau_4^{-1} H_{ev} H_{ev}^T \end{pmatrix}$ and

$$\delta_{hhv} = \begin{pmatrix} -z_3^T - z_3 + \tau_1^{-1} z_1^T N_{ah}^T N_{ah} z_1 \\ + \tau_2^{-1} M_{hv}^T N_{bh}^T N_{bh} M_{hv} + \tau_3^{-1} z_3^T N_{ev}^T N_{ev} z_3 \end{pmatrix}.$$

Then, applying the Schur complement [13] on $\tau_1^{-1} z_1^T N_{ah}^T N_{ah} z_1$, $\tau_2^{-1} M_{hv}^T N_{bh}^T N_{bh} M_{hv}$, $\tau_3^{-1} z_3^T N_{ev}^T N_{ev} z_3$ and $\tau_4 z_4^T N_{ev}^T N_{ev} z_4$,and according to the notations defined in section 2, the conditions of theorem 2 hold. ■

C. Relaxed LMI stability conditions

The conditions provided by the theorem 1 and 2 are obtained considering that (19) holds, that is to say if:

$$\Psi_{hhv} = \sum_{k=1}^{l} \sum_{i=1}^{r} \sum_{j=1}^{r} v_k(z) h_j(z) h_i(z) \Psi_{ik} < 0 \tag{24}$$

This means that for each $i, j = 1,...,r$ and $k = 1,...,l$, $\Psi_{ijk} < 0$. Obviously, Ψ_{hv} resulting of a blended sum of Ψ_{ijk}, it is possible to find a combination of Ψ_{ijk} where some are positive and leading to $\Psi_{hv} < 0$. Consequently, the provided conditions are conservative. In order to relax these conditions, many scheme should be employed [14][16][17]. In our case, we choose to apply the relaxation scheme given in [14] and summarized by the lemma 2. Applying this lemma on the conditions given by theorem 2, we obtain the relaxed stability conditions given by the following theorem.

Theorem 3: *The fuzzy uncertain descriptor system* (4) *is quadratically stable via the modified PDC control law* (8) *if there exists the matrices* $z_1 = z_1^T > 0$, z_3, z_4, M_{jk} *and scalars* τ_1, τ_2, τ_3 *and* τ_4^{-1} *such that the following conditions are satisfied:*

- for $i = 1,2,...,r$ and $k = 1,2,...,l$, $\Upsilon_{iik} < 0$,
- for $i = 1,2,...,r$, $1 \le i \ne j \le r$ and $k = 1,2,...,l$,

$$\frac{1}{r-1}\Upsilon_{iik} + \frac{1}{2}\left(\Upsilon_{ijk} + \Upsilon_{jik}\right) < 0$$

with $$\Upsilon_{ijk} = \begin{bmatrix} -z_3^T - z_3 & (*) & (*) & (*) & (*) & 0 \\ N_{ai} z_1 & -\tau_1 I & 0 & 0 & 0 & 0 \\ N_{bi} M_{jk} & 0 & -\tau_2 I & 0 & 0 & 0 \\ N_{ek} z_3 & 0 & 0 & -\tau_3 I & 0 & 0 \\ \Omega_{ijk} & 0 & 0 & 0 & \Pi_{ik} & (*) \\ 0 & 0 & 0 & 0 & N_{ek} z_4 & -\tau_4^{-1} I \end{bmatrix},$$

$\Omega_{ijk} = \begin{pmatrix} z_4^T + A_i z_1 + \\ E_k z_3 - B_i M_{jk} \end{pmatrix}$ and $\Pi_{ik} = \begin{pmatrix} -z_4^T E_k^T - E_k z_4 + \tau_2 H_{bi} H_{bi}^T \\ +\tau_1 H_{ai} H_{ai}^T + \tau_3 H_{ek} H_{ek}^T \\ +\tau_4^{-1} H_{ek} H_{ek}^T \end{pmatrix}$

V. Numerical Example and Simulation

To illustrate the proposed approach, let us consider the example given by the following nonlinear descriptor:

$$\begin{cases} E(x(t))\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) \\ y(t) = C(x(t))x(t) \end{cases} \tag{25}$$

with: $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $E(x(t)) = \begin{bmatrix} 1 & 1 \\ -1 & \cos^2(x_2(t)) \end{bmatrix}$,

$$A(x(t)) = \begin{bmatrix} 0 & \cos^2(x_2(t)) - \dfrac{1}{1+x_1^2(t)} \\ -1.5 & -3 + b\left(1 + \dfrac{1}{1+x_1^2(t)}\right)\dfrac{\sin(x_2(t))}{x_2(t)} \end{bmatrix}$$

and $B(x(t)) = \begin{bmatrix} 1 + \dfrac{1}{1+x_1^2(t)} \\ a\cos^2(x_2(t)) - 2 \end{bmatrix}$.

A. Uncertain T-S descriptor modeling.

Let us recall that there is a systematic way to go from the nonlinear descriptor (25) to one of its fuzzy T-S representation. This way is called the sector non linearity approach [2]. Thus, a T-S descriptor should match exactly the nonlinear model in a compact set of the state variables. Note also that infinity of T-S models can represent a single nonlinear model [18]. The number of LTI models constituting the nonlinear T-S fuzzy descriptor is $n = l \times r = 2^{nl} \times 2^{nr}$ where nl and nr are respectively the number of nonlinearity to be treated in the left and right part of the descriptor (25). $E(x(t))$, for the left part, contains the nonlinearity $\cos^2(x_2(t))$ that leads to $l = 2$. In the same way, $A(x(t))$ and $B(x(t))$, for the right part, contain the nonlinearities $\cos^2(x_2(t))$, $\dfrac{\sin(x_2(t))}{x_2(t)}$ and $\dfrac{1}{1+x_1^2(t)}$ that leads to $r = 8$. Hecne, the number of conditions to be solved in the LMI problems is $l.r(r+1)/2 = 72$, which could leads to very conservative and make impossible the resolution with actual solvers. Thus it is of high interest to obtain a T-S fuzzy representation of nonlinear models with a reduced number of rules. In order to rewrite (25) as a nonlinear uncertain descriptor, we propose to rewrite the terms $\cos^2(x_2(t))$ and $\dfrac{\sin(x_2(t))}{x_2(t)}$ as fellows:

$$\cos^2(x_2(t)) = \frac{1}{2} + \frac{1}{2}\Delta_1(t) \tag{26}$$

$$\frac{\sin(x_2(t))}{x_2(t)} = \frac{1-\rho}{2} + \frac{1+\rho}{2}\Delta_2(t) \tag{27}$$

where $\rho \approx -0.2172$ is the minimum value of $\frac{\sin(x_2(t))}{x_2(t)}$.

The functions $\Delta_1(t) = 2\cos^2(x_2(t)) - 1$ and $\Delta_2(t) = \frac{1}{1+\rho}\left(2\frac{\sin(x_2(t))}{x_2(t)} - 1 + \rho\right)$, included in (26) and (27), are bounded on $\Re$ such as $\forall t \in \Re, \Delta_1^2(t) \le 1\, et\, \Delta_2^2(t) \le 1$. These functions are then considered as uncertainties to obtain an uncertain representation of (25) given by:

$$\begin{aligned}&\left[\tilde{E} + \Delta\tilde{E}(t)\right]\dot{x}(t)\\&= \left[\tilde{A}(x(t)) + \Delta\tilde{A}(x(t))\right]x(t) + \left[\tilde{B}(x(t)) + \Delta\tilde{B}(t)\right]u(t)\end{aligned} \tag{28}$$

with $\tilde{E} = \begin{bmatrix} 1 & 1 \\ -1 & \frac{1}{2} \end{bmatrix}$, $\Delta\tilde{E}(t) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{2}\Delta_1(t) \end{bmatrix}$,

$\tilde{A}(x(t)) = \begin{bmatrix} 0 & \frac{1}{2} - f(x_1(t)) \\ -\frac{3}{2} & -3 + b(1 + f(x_1(t)))\frac{1-\rho}{2} \end{bmatrix}$,

$\Delta\tilde{A}(x(t)) = \begin{bmatrix} 0 & \frac{1}{2}\Delta_1(t) \\ 0 & b(1 + f(x_1(t)))\frac{1+\rho}{2}\Delta_2(t) \end{bmatrix}$, $\Delta\tilde{B}(t) = \begin{bmatrix} 0 \\ \frac{a}{2}\Delta_1(t) \end{bmatrix}$,

$\tilde{B}(x(t)) = \begin{bmatrix} 1 + f(x_1(t)) \\ \frac{a}{2} - 2 \end{bmatrix}$, and $f(x_1(t)) = \frac{1}{1 + x_1^2(t)}$.

Hence, the uncertain T-S fuzzy representation of (28) can be obtaining by splitting the nonlinearity $f(x_1(t))$ such that:

$$f(x_1(t)) = h_1(z(t)) \times 0 + h_2(z(t)) \times 1 \tag{29}$$

with $h_1(z(t)) = 1 - f(x_1(t))$, $h_2(z(t)) = f(x_1(t))$ and $z(t) \equiv x_1(t)$.

Then, the uncertain fuzzy descriptor with two rules is given by :

Rule 1: ***If*** $x_1(t)$ *is* $h_1(z(t))$ ***Then***

$$\left(\tilde{E} + \Delta\tilde{E}(t)\right)\dot{x}(t) = \left(\tilde{A}_1 + \Delta\tilde{A}_1(t)\right)x(t) + \left(\tilde{B}_1 + \Delta\tilde{B}(t)\right)u(t)$$

Rule2: ***If*** $x_1(t)$ *is* $h_2(z(t))$ ***Then***

$$\left(\tilde{E} + \Delta\tilde{E}(t)\right)\dot{x}(t) = \left(\tilde{A}_2 + \Delta\tilde{A}_2(t)\right)x(t) + \left(\tilde{B}_2 + \Delta\tilde{B}(t)\right)u(t)$$

with $\tilde{A}_1 = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{3}{2} & -3 + b\frac{1-\rho}{2} \end{bmatrix}$, $\tilde{A}_2 = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{3}{2} & -3 + b(1-\rho) \end{bmatrix}$,

$\Delta\tilde{A}_1(t) = \begin{bmatrix} 0 & \frac{1}{2}\Delta_1(t) \\ 0 & b\frac{1+\rho}{2}\Delta_2(t) \end{bmatrix}$, $\Delta\tilde{A}_2(t) = \begin{bmatrix} 0 & \frac{1}{2}\Delta_1(t) \\ 0 & b(1+\rho)\Delta_2(t) \end{bmatrix}$,

$\tilde{B}_1 = \begin{bmatrix} 1 \\ \frac{a}{2} - 2 \end{bmatrix}$ et $\tilde{B}_2 = \begin{bmatrix} 2 \\ \frac{a}{2} - 2 \end{bmatrix}$.

That is to say in its compact formulation:

$$\left[\tilde{E} + \Delta\tilde{E}(t)\right]\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))\left\{\begin{aligned}&\left[\tilde{A}_i + \Delta\tilde{A}_i(t)\right]x(t)\\&+\left[\tilde{B}_i + \Delta\tilde{B}(t)\right]u(t)\end{aligned}\right\} \tag{30}$$

To compute the conditions given in theorem 2 and 3, the uncertainties matrices have to be rewritten as:

$\Delta E(t) = H_e\Delta_e(t)N_e$, $\Delta A_1(t) = H_{a1}\Delta_{a1}(t)N_{a1}$,
$\Delta A_2(t) = H_{a2}\Delta_{a2}(t)N_{a2}$ and $\Delta B(t) = H_b\Delta_b(t)N_b$

with $H_e = H_{a1} = H_{a2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $H_b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $N_e = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$,

$\Delta_e(t) = \Delta_b(t) = \Delta_1(t)$, $\Delta_{a1}(t) = \Delta_{a2}(t) = \begin{bmatrix} \Delta_1(t) & 0 \\ 0 & \Delta_2(t) \end{bmatrix}$,

$N_{a1} = \begin{bmatrix} 0 & 0.5 \\ 0 & \frac{1+\rho}{2}b \end{bmatrix}$, $N_{a2} = \begin{bmatrix} 0 & 0.5 \\ 0 & (1+\rho)b \end{bmatrix}$ and $N_b = 0.5a$.

B. Simulation and results

In order to illustrate the efficiency of theorems 3 compared to theorem 2, the feasibility field, solution of each theorem using the MATLAB LMI Toolbox [12], with the parameters $a \in [-2.5 \quad 1.5]$ and $b \in [-9 \quad 6]$ are presented in figure 1.

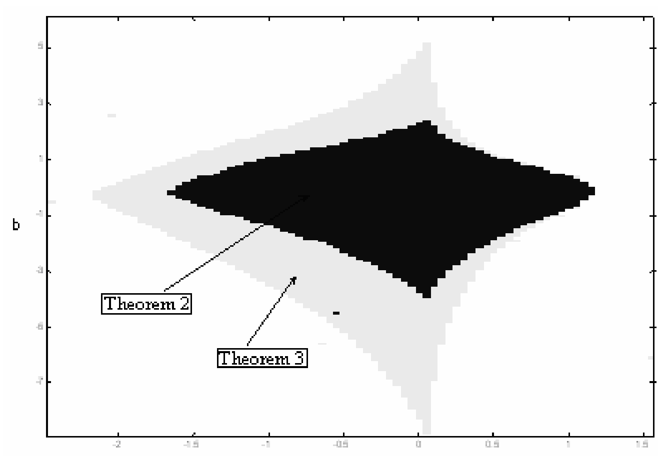

Fig1: Feasibility fields

As an example, the solution of theorem 3, for $a=-1.5$ and $b=-1$, is given by the following set of scalars and matrices:
$K_{11}=\begin{bmatrix}0.9119 & -1.2475\end{bmatrix}$, $K_{21}=\begin{bmatrix}1.1639 & -0.9451\end{bmatrix}$,
$\tau_1=21.1611$, $\tau_2=65.0254$, $\tau_3=11.5796$, $\tau_4=33.7578$,
$z_1=\begin{bmatrix}118.3405 & -4.5515\\ -4.5515 & 23.6827\end{bmatrix}$, $z_3=\begin{bmatrix}126.0740 & -37.1366\\ -10.5783 & 20.0279\end{bmatrix}$ and
$z_4=\begin{bmatrix}107.4726 & -81.9021\\ -22.4079 & 121.5464\end{bmatrix}$.

Figure 2 illustrates the state variables and the input signal convergences when simulating the nonlinear system (25) stabilized by the modified PDC control law (8) (reduced to $u(t)=-\sum_{i=1}^{2}h_i(z(t))K_{i1}x(t)$) and the initial state $x(0)^T=\begin{bmatrix}10 & 20\end{bmatrix}^T$. One notices some fluctuations during the transient of the nonlinear system. Let us recall that the global nonlinear system (25) is stabilized via a control law synthesized with all the terms including the state variable $x_2(t)$ as uncertain. Nevertheless, we claim the efficiency of the proposed approach since the stability is achieved after 2s.

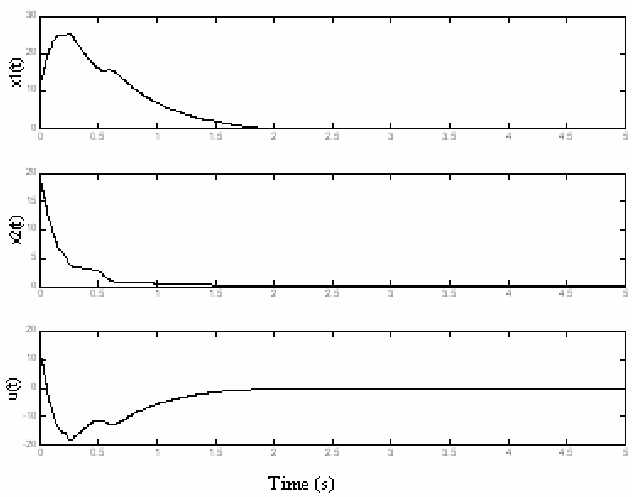

Fig2: simulation results

VI. Conclusion

In this paper, stability conditions for a wide class of T-S uncertain descriptor have been proposed. These are based on a quadratic Lyapunov candidate function and a modified PDC control law. In order to solve the stability conditions with classical convex optimization algorithm, matrices transformations have been used to write these conditions in term of linear matrix inequality (LMI).These are given by theorem 2 which constitutes the main contribution of this study. A less conservative stability condition has been investigated and proposed in theorem 3 with the use of the relaxation scheme proposed by [14]. A design example has illustrated the efficiency of the proposed approach for T-S Uncertain Descriptor.

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