Modelling and simulation of two-link robot manipulators based on Takagi Sugeno fuzzy descriptor systems

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Abstract— **This paper presents the first steps towards a robust model-based controller design for two-link manipulators using a Takagi Sugeno fuzzy descriptor form. Due to the circumstances that this form is more similar to a given original nonlinear equation as a Takagi Sugeno fuzzy state-space system, it allows to reduce the conservatism of the controller design by using common matrix structures. The model-based control law here is equivalent to the well-known parallel distributed compensation scheme. The challenge of the investigated modelling and control problem in this case is the highly nonlinear dynamics of the dual-actuator drive powered by air-pressure that interacts with the dynamics of the robot manipulator.**

Keywords— **Descriptor systems, Control oriented models, Robotic manipulators, Fluid Power System**

I. Introduction

For several years Takagi Sugeno (TS) fuzzy systems [13] are widely used in a context of modeling for control design [1], [11], [14], [16]. Also the linear descriptor system, which differs from a state-space representation, has generated a great deal of interest in control system design. The descriptor system describes a wider class of systems including physical models and nondynamic constraints [6]. The interest to choose descriptor system rather than classical state space representation particular for mechanical systems based on

1. Reducing the number of rules → reducing conservatism, see [2]
2. Using the natural model structure due to the inertia matrix.

Then, the use of a descriptor representation is justified to model a robot if the inertia matrix contains nonlinear terms, i.e. if the structure of the considered robot is geometrically variable, that is the case of the SCARA robot.

There exist a large number of papers on the stability analysis of TS fuzzy systems based on the state-space representation and also some applications in the control design process e.g. [3], [5], [9], [10]. But in contrast, the stability analysis of TS fuzzy systems based on descriptor representation as weighted combinations of linear descriptor systems (from now called *TS fuzzy descriptor systems*) have not been investigated until [15] and only once applied in the context of observer design with applications in biomechanics [2]. The purpose of this work is to propose a robust fuzzy controller design based on a first derived descriptor form of a two-link robot manipulator powered by pneumatic actuators as a reasonable example of a highly nonlinear complex system.

This paper is organized as follows: First, for a class of serial manipulators, the so-called SCARA-type robot manipulators [12], a nonlinear state-space model is formulated by the Newton-Euler equation of the manipulator arm coupled with a reduced model of the actuators with two bounded uncertainties. These uncertainties are caused by the influence of the model reduction on the differential equations of the pressure evolution and the time-variable leakage air flows between the actuator chambers.

Then the state-space model combined with dynamic constraints is transformed into a Takagi Sugeno fuzzy descriptor form [15]. After this the complete dynamic model based on a combination of the actuator models in descriptor form and the robot arm dynamics in descriptor form is presented. It is shown by simulation results that this proposed model is capable to represent the coupled nonlinearities among the actuators and the manipulator arm, and the uncertainties of the plant caused by time-variable leakage air flows between the chambers. Finally, in a prospective way, a global controller is designed based on the previously derived model as a robust fuzzy gain-scheduler of state-feedback gains. Here, a systematic design procedure for a fuzzy controller is proposed through the parallel distributed compensation (PDC) [1].

II. Physical System Modelling

In the following the dynamic model of a SCARA-type robot manipulator is presented. The robot arm is powered by an actuator mechanism based on tangential feed as the main drive and an electric direct-drive as the actuator of the second axis that is shown in Fig. 2. The main drive consists of two rodless cylinders with pistons, two deflection rollers and a metal belt as a friction-locked connection

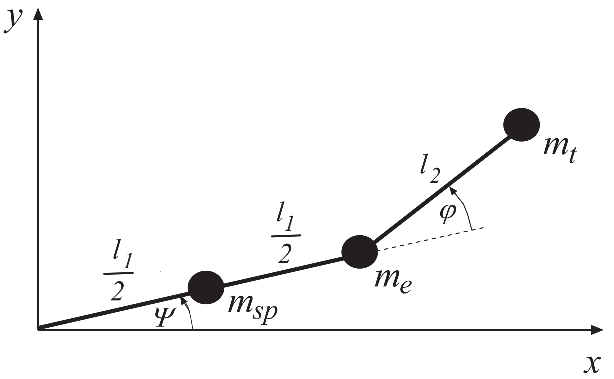

Fig. 1. Reduced mechanical system of the robot manipulator arm in the xy-plane.

between the rollers and the pistons (also called Belt-motor [8]). The forces on the pistons caused by opposed pressure differences in Actuator 1 and 2 are transmitted by the belt to the deflection rollers, where the linear motion of the pistons is converted to angular motion of the robot link 1. We assume that the normal strain of the belt is negligibly small (infinitely stiff) and the tangent stiffness is pure elastic.

A. Mechanical model of the manipulator dynamics

Consider the two-link planar arm shown in Fig. 1 that represents a reduced mechanical system of the manipulator in Fig. 2. The robot arm has two vertical revolute joints, both joint axes are orthogonal to the xy-plane in Fig. 1. Here, the masses of the actuator cylinders and the mass of link 1 are combined to m_{sp}. Let m_e be the mass of the rotor of the electric drive, and m_t the mass of the tool and vertical drive in the tool center point (TCP, see Fig. 2), whereby the mass of link 2 is small in comparison to m_t. The distance of the centers of mass of m_{sp} and m_e to the Joint-axis 1 is defined by $l_1/2$, and l_1 as the length of link 1. The distance of the center of mass of m_t to the joint-axis 2 is defined by l_2 as the length of link 2. The equation of motion for this mechanical system can be written in the known matrix form [12] which represents the joint space dynamics model:

$$\boldsymbol{H}(\boldsymbol{q})\,\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q} = [\,\psi \;\; \varphi\,]^T$ is the vector of generalized coordinates with ψ as the angle of link 1 from the x-axis, and with φ as the angle between link 1 and link 2. The inertia matrix

$$\boldsymbol{H}(\boldsymbol{q}) = \begin{bmatrix} H_{11}(\boldsymbol{q}) & H_{12}(\boldsymbol{q}) \\ H_{21}(\boldsymbol{q}) & H_{22} \end{bmatrix} \tag{2}$$

depends on the current arm configuration with

$$\begin{aligned} H_{11}(\boldsymbol{q}) = \; & I_{L1} + m_{sp}\left(\frac{l_1}{2}\right)^2 + m_e\, l_1^2 + I_{L2} \\ & + m_t\left(l_1^2 + l_2^2 + 2\, l_1\, l_2 \cos\varphi\right) \;, \end{aligned}$$

$$H_{12}(\boldsymbol{q}) = H_{21}(\boldsymbol{q}) = I_{L2} + m_t\, l_2\,(l_2 + l_1 \cos\varphi) \;,$$

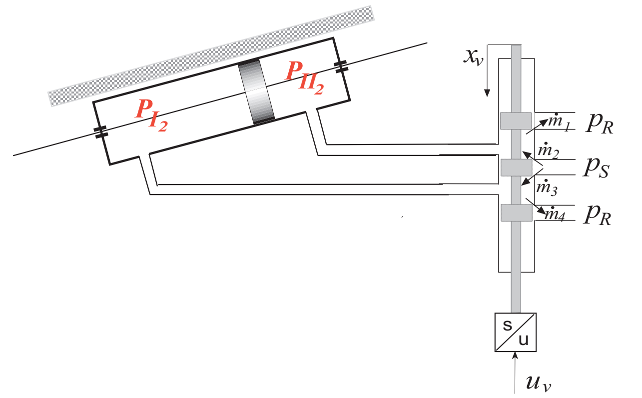

Fig. 3. One cylinder of the dual-actuator with the accessory servo-valve

$$H_{22} = I_{L2} + m_t\, l_2^2 \;.$$

Note that for this application the assumption holds (the links are made of carbon-fiber-reinforced plastic) that the moments of inertias relative to the centers of mass of link 1 and link 2 as I_{L1} and I_{L1} can be neglected against the other terms, such as $\left(m_e\, l_1^2\right)$. So we set in the following considerations $I_{L1} = I_{L1} = 0$. The actuation torques in (2) are represented by $\boldsymbol{\tau} = [\,\tau_1 \;\; \tau_2\,]^T$. Finally, the vector

$$\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} h_1(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ h_2(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix} \tag{3}$$

in (1) represents the centrifugal effects and Coriolis effects with

$$\begin{aligned} h_1(\boldsymbol{q},\dot{\boldsymbol{q}}) &= m_t\, l_1\, l_2 \sin\varphi\,(-\dot{\varphi}^2 - 2\,\dot{\psi}\,\dot{\varphi}) \;, \\ h_2(\boldsymbol{q},\dot{\boldsymbol{q}}) &= m_t\, l_1\, l_2 \sin\varphi\,\dot{\psi}^2 \;. \end{aligned}$$

B. Dual-Actuator model

The physical description of the main drive (see Fig.2) based on the continuity and enthalpy balance equation of the gas flow through the actuator and the constitutive relations of the air. Whereby the most important aspect of the actuator dynamics that interact with the robot arm is described by the pressure evolution of each chamber. Assuming isentropic behavior of the gas flow, the pressure evolution in the chambers of actuator $j, j = 1,2$ (see Fig.2) is modeled by the equations

$$\begin{aligned} & \dot{p}_{I_j}\,\left(V_{0I} + A_K\,(x_K - x_{K0})\right) = \\ & \qquad \kappa\,\left[R_g\,T_I\,\left(\dot{m}_2 - \dot{m}_1\right) - p_I\,A_K\,\dot{x}_K\right] \end{aligned} \tag{4a}$$

and

$$\begin{aligned} & \dot{p}_{II_j}\,\left(V_{0II} - A_K\,(x_K - x_{K0})\right) = \\ & \qquad \kappa\,\left[-R_g\,T_{II}\,\left(\dot{m}_4 - \dot{m}_3\right) + p_{II}\,A_K\,\dot{x}_K\right] \;. \end{aligned} \tag{4b}$$

The *variables* used in the above equations are:

- p_I, p_{II} as the pressure in actuator chambers I, II [N/m^2],
- x_K as the actuator piston displacement [m],

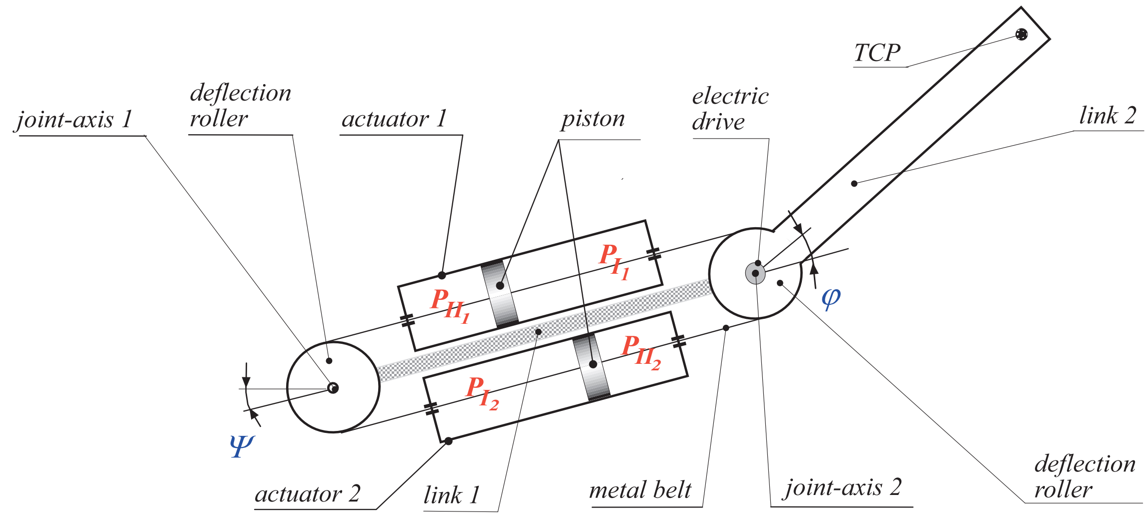

Fig. 2. Two-link manipulator with a dual servo-pneumatic actuator as main drive (ψ) and an electric direct-drive (φ)

$\dot{m}_n$ as the mass flow over the servovalve control edges $n = 1, 2, 3, 4$ [kg/s] (see Fig. 3),

T_I, T_{II} as the temperature in the chamber I, II [K], and

T_S as the temperature associated with the system pressures p_S and p_R [K].

The *model parameters* are:

x_{K0} as the start position of the actuator piston [m],

$V_{0I,0II}$ as the start volume of the actuator chamber I, II [m^3],

A_K as the actuator piston area [m^3],

R_g as the gas constant [$\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$], and

κ as the adiabatic exponent of the gas [$-$].

The two variable volume chambers in each actuator cylinder are connected to a four-way servovalve. The configuration for one actuator cylinder in combination with the servovalve is shown in Fig. 3. In this case the servovalve controls the air-mass-flow $\dot{m}_1, ..., \dot{m}_4$ by adjusting four orifice areas between the constant pressure supply p_s, the exhaust pressure p_R, and the chamber pressures $p_{I,II}$. The four orifice areas are continuously controlled by the servovalve voltage u_v. The equations for the mass flows are

$$\dot{m}_1 = -\alpha_{D1}\, A_1(x_v)\, \psi\left(\frac{p_R}{p_{II}}\right) p_I \sqrt{\frac{2}{R_g\, T_{II}}} \tag{5a}$$

$$\dot{m}_2 = \alpha_{D2}\, A_2(x_v)\, \psi\left(\frac{p_{II}}{p_S}\right) p_S \sqrt{\frac{2}{R_g\, T_S}} \tag{5b}$$

$$\dot{m}_3 = -\alpha_{D3}\, A_3(x_v)\, \psi\left(\frac{p_I}{p_S}\right) p_S \sqrt{\frac{2}{R_g\, T_S}} \tag{5c}$$

$$\dot{m}_4 = \alpha_{D4}\, A_4(x_v)\, \psi\left(\frac{p_R}{p_I}\right) p_I \sqrt{\frac{2}{R_g\, T_I}} \tag{5d}$$

with the servovalve displacement x_v and the orifice areas of the control edges for $n = 2, 4$

$$A_n(x_v) = \begin{cases} x_v\, \pi\, d_n & \text{for } x_v \geq x_{\ddot{u}_n} \\ 0 & \text{for } x_v < x_{\ddot{u}_n} \end{cases} \tag{6a}$$

and for $n = 1, 3$

$$A_n(x_v) = \begin{cases} x_v\, \pi\, d_n & \text{for } x_v \leq -x_{\ddot{u}_n} \\ 0 & \text{for } x_v > -x_{\ddot{u}_n} \end{cases} , \tag{6b}$$

and the flow function

$$\psi\left(\frac{p_a}{p_b}\right) = \begin{cases} \psi_0 \sqrt{1 - \left(\frac{\frac{p_a}{p_b} - p_{krit}}{1 - p_{krit}}\right)^2} & \text{for } \frac{p_a}{p_b} \geq p_{krit} \\ \psi_0 & \text{for } \frac{p_a}{p_b} < p_{krit} \end{cases} . \tag{6c}$$

The not yet defined model parameters are

α_{D_n} as the flow coefficient of the control edge $n = 1, ..., 4$,

d_n as the effective diameter of the control edge n [m],

R_g as the gas constant [$\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$],

κ as the adiabatic exponent of the gas [$-$],

p_{krit} as the critical pressure ratio [$-$],

ψ_0 as the maximum value of the flow function ψ, and

$x_{\ddot{u}_n}$ as the valve overlap of the control edge n [m].

III. Reduced physical model in TS fuzzy descriptor form

A. Reduced actuator model in descriptor form

The detailed physical description of the servo-pneumatic actuators is reduced by simplification of the mass flow relations (5) and by linearization of the switching functions for zero valve overlap ($x_{\ddot{u}_n} = 0$ for $n = 1, ..., 4$). The influence of the model reductions on the differential equation of the chamber pressures p_I, p_{II} is condensed in two bounded uncertainties as $\Delta\boldsymbol{B}$ and $\Delta\boldsymbol{a}$. The remaining nonlinear terms are transfered into TS fuzzy relations using the lemma of Morère [7]. It leads to the following descriptor form

$$\sum_{j=1}^{2} \nu_j(x_K)\, \boldsymbol{E}_j^P\, \dot{\boldsymbol{x}}^P = \sum_{i=1}^{8} h_i(\dot{x}_K, p_I, p_{II}) \left[\boldsymbol{A}_i^P\, \boldsymbol{x}^P + \boldsymbol{B}_i^P\, x_v \right] + \Delta\boldsymbol{B}\, x_v + \Delta\boldsymbol{a} \tag{7}$$

with the state vector

$$\boldsymbol{x}^P = [\, p_I \,,\, p_{II} \,]^T \tag{8a}$$

and

$$\boldsymbol{E}_j^P = \begin{bmatrix} \frac{V_{0I} + A_K(*_1 - x_{K0})}{\kappa} & 0 \\ 0 & \frac{V_{0II} - A_K(*_1 - x_{K0})}{\kappa} \end{bmatrix}, \quad *_1 \in \{\underline{x}_K, \bar{x}_K\} \tag{8b}$$

$$\boldsymbol{A}_i^P = \begin{bmatrix} A_K *_2 & 0 \\ 0 & A_K *_2 \end{bmatrix} \quad , \quad *_2 \in \{\underline{\dot{x}}_K, \bar{\dot{x}}_K\} \quad , \tag{8c}$$

$$\boldsymbol{B}_i^P = \begin{bmatrix} R_g\, T_I\, \alpha_D\, \pi\, d\, (\, \tilde{a}_{02} + \tilde{b}_{02} *_3 \,) \\ -R_g\, T_{II}\, \alpha_D\, \pi\, d\, (\, \tilde{a}_{03} + \tilde{b}_{03} *_4 \,) \end{bmatrix}, \quad *_3 \in \{\underline{p}_I, \bar{p}_I\} \,, \quad *_4 \in \{\underline{p}_{II}, \bar{p}_{II}\} \,. \tag{8d}$$

and the uncertainty terms

$$\Delta\boldsymbol{a} = \begin{bmatrix} \Delta p_I \\ \Delta p_{II} \end{bmatrix} \quad , \tag{8e}$$

$$\Delta\boldsymbol{B} = \begin{bmatrix} R_g\, T_I\, \alpha_D\, \pi\, d\, \Delta\tilde{a}_2 \\ -R_g\, T_{II}\, \alpha_D\, \pi\, d\, \Delta\tilde{a}_3 \end{bmatrix} \quad . \tag{8f}$$

B. Model of the manipulator dynamics in TS fuzzy descriptor form

The equations (1),(2) can be directly written in the following "exact" TS fuzzy descriptor form

$$\sum_{j=1}^{2} \nu_j(\varphi)\, \boldsymbol{E}_j^R\, \dot{\boldsymbol{x}}^R = \sum_{i=1}^{4} h_i(\varphi, \dot{\varphi}, \dot{\psi})\, \boldsymbol{A}_i^R\, \boldsymbol{x}^R + \boldsymbol{B}^R \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{9}$$

with

$$\boldsymbol{x}^R = \left[\, \psi \,,\, \varphi \,,\, \dot{\psi} \,,\, \dot{\varphi} \,\right]^T \,, \tag{10a}$$

$$\boldsymbol{E}_j^R = \begin{bmatrix} \boldsymbol{I}_{2\times2} & \boldsymbol{0}_{2\times2} \\ \boldsymbol{0}_{2\times2} & \begin{bmatrix} a + 2\, b *_5 & c + b *_5 \\ c + b *_5 & c \end{bmatrix} \end{bmatrix}, \quad *_5 \in \{\underline{f}_1, \bar{f}_1\} \,, \tag{10b}$$

$$\boldsymbol{A}_i^R = \begin{bmatrix} \boldsymbol{0}_{2\times2} & \boldsymbol{I}_{2\times2} \\ \boldsymbol{0}_{2\times2} & \begin{bmatrix} 2\, b *_6 & b *_6 \\ -b *_7 & 0 \end{bmatrix} \end{bmatrix}, \quad *_6 \in \{\underline{g}_1, \bar{g}_1\} \,, \quad *_7 \in \{\underline{g}_2, \bar{g}_2\} \,, \tag{10c}$$

and

$$\boldsymbol{B}^R = \begin{bmatrix} \boldsymbol{0}_{2\times2} \\ \boldsymbol{I}_{2\times2} \end{bmatrix} \quad , \tag{10d}$$

whereby

$$\begin{aligned} f_1 &= \cos\varphi \quad , \\ g_1 &= \dot{\varphi}\, \sin\varphi \quad , \\ g_2 &= \dot{\psi}\, \sin\varphi \quad , \\ a &:= m_{sp} \left(\frac{l_1}{2} \right)^2 + m_e\, l_1^2 + m_t\, (\, l_1^2 + l_2^2 \,) \quad , \\ b &:= m_t\; l_1\; l_2 \quad , \\ c &:= m_t\; l_2^2 \quad . \end{aligned}$$

Based on the assumptions that, first, the piston area of the actuators are equal $A_{K_1} = A_{K_2} =: A_K$, and second, the mass flow control of each actuator caused by opposed pressure differences $p_{L_j} = p_{I_j} - p_{II_j}$, $j = 1, 2$ (see Fig. 2) the magnitudes are

$$p_{L_1} = p_{L_2} =: p_L \,, \tag{11}$$

the driving torque of joint 1 can write as

$$\tau_1 = p_{L_1}\, A_{K_1}\, \frac{d_r}{2} + p_{L_2}\, A_{K_2}\, \frac{d_r}{2} = p_L\, A_K\; d_r \,, \tag{12}$$

with d_r as the roller diameter. In this consideration it is assumed that the electric actuator of joint 2, see Fig. 2, behaves as a ideal torque-controlled generator, which gives a proportional relation between the torque τ_2 and the control voltage u_m established by the motor constant k_m.

$$\tau_2 = k_m\, u_m \tag{13}$$

C. Complete dynamic model in descriptor form

The complete dynamic model based on an combination of the actuator model in descriptor form (7) and the robot arm dynamics in descriptor form (9). It was derived by using the following algebraic constraints between the translational motion of the actuator piston and the rotational motion of the deflection roller:

$$x_K = \psi\, \frac{d_r}{2} \qquad , \qquad \dot{x}_K = \dot{\psi}\, \frac{d_r}{2} \,. \tag{14}$$

With the assumption that the piston areas are $A_K = A_{K_1} = A_{K_2}$ and the mass flow control of each actuator caused by opposed pressure difference in both actuators the chamber pressures are

$$p_I = p_{I_1} = p_{I_2} \quad , \quad p_{II} = p_{II_1} = p_{II_2} \quad ,$$

the driving torque in joint 1 is

$$\begin{aligned} \tau_1 &= (\, p_{I_1} - p_{II_1}\,)\, A_{K_1} \frac{d_r}{2} + (\, p_{I_2} - p_{II_2}\,)\, A_{K_2} \frac{d_r}{2} \\ &= (\, p_I - p_{II}\,)\, A_K\, d_r \quad . \end{aligned} \tag{15}$$

So the complete dynamic model can be presented as

$$\sum_{k=1}^{4} \tilde{\nu}_k(\,\psi,\varphi\,)\, \boldsymbol{E}_k\, \dot{\boldsymbol{x}} = \sum_{l=1}^{32} h_l\, [\, \boldsymbol{A}_l\, \boldsymbol{x} + (\, \boldsymbol{B}_l + \Delta\boldsymbol{B}\,)\, \boldsymbol{u}\,] + \Delta\tilde{\boldsymbol{a}} \tag{16}$$

with $\boldsymbol{E}_k \in \mathbb{R}^{6\times6}$, $\boldsymbol{A}_l \in \mathbb{R}^{6\times6}$, $\boldsymbol{B}_l \in \mathbb{R}^{6\times2}$, $\Delta\tilde{\boldsymbol{a}} \in \mathbb{R}^{6\times1}$, the right hand side weighting function

$$h_l = h_l(\varphi, \dot{\phi}, \psi, \dot{\psi}, p_I, p_{II}) \quad , \tag{17a}$$

the input vector

$$\boldsymbol{u} = [\, x_v\,,\, \tau_1\,]^T \quad , \tag{17b}$$

and the state vector

$$\boldsymbol{x} = \left[\psi\,,\, \varphi\,,\, \dot{\psi}\,,\, \dot{\varphi}\,,\, p_I\,,\, p_{II}\right]^T \quad . \tag{17c}$$

In detail the matrices are

$$\boldsymbol{E}_1 = \begin{bmatrix} \boldsymbol{E}_1^R & \boldsymbol{0}_{4\times2} \\ \boldsymbol{0}_{2\times4} & \boldsymbol{E}_1^P \end{bmatrix} \quad , \quad \boldsymbol{E}_2 = \begin{bmatrix} \boldsymbol{E}_2^R & \boldsymbol{0}_{4\times2} \\ \boldsymbol{0}_{2\times4} & \boldsymbol{E}_1^P \end{bmatrix} \quad , \tag{18a}$$

$$\boldsymbol{E}_3 = \begin{bmatrix} \boldsymbol{E}_1^R & \boldsymbol{0}_{4\times2} \\ \boldsymbol{0}_{2\times4} & \boldsymbol{E}_2^P \end{bmatrix} \quad , \quad \boldsymbol{E}_4 = \begin{bmatrix} \boldsymbol{E}_2^R & \boldsymbol{0}_{4\times2} \\ \boldsymbol{0}_{2\times4} & \boldsymbol{E}_2^P \end{bmatrix} \quad , \tag{18b}$$

$$\boldsymbol{A}_l = \begin{bmatrix} \boldsymbol{0}_{2\times2} & \boldsymbol{I}_{2\times2} & \boldsymbol{0}_{2\times2} \\ \boldsymbol{0}_{2\times2} & \begin{bmatrix} 2\,b\,*_6 & b\,*_6 \\ -b\,*_7 & 0 \end{bmatrix} & \begin{bmatrix} A_K\, d_r & -A_K\, d_r \\ 0 & 0 \end{bmatrix} \\ \boldsymbol{0}_{2\times2} & \boldsymbol{0}_{2\times2} & \begin{bmatrix} A_K\, *_8 & 0 \\ 0 & A_K\, *_8 \end{bmatrix} \end{bmatrix} , \tag{18c}$$

with $*_8 \in \{\underline{\dot{x}}_K, \bar{\dot{x}}_K\}$ where $\underline{\dot{x}}_K = \underline{\dot{\psi}}\, \frac{d_r}{2}$, $\bar{\dot{x}}_K = \bar{\dot{\psi}}\, \frac{d_r}{2}$ and

$$\boldsymbol{B}_l = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ b_5(*_3) & 0 \\ b_6(*_4) & 0 \end{bmatrix} \quad , \quad \Delta\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0}_{4\times2} & \\ \Delta b_5 & 0 \\ \Delta b_6 & 0 \end{bmatrix} \tag{18d}$$

with

$$\begin{aligned} b_5(*_3) &= R_g\, T_I\, \alpha_D\, \pi\, d\, (\, \tilde{a}_{02} + \tilde{b}_{02}\, *_3\,) \quad , \\ b_6(*_4) &= -R_g\, T_{II}\, \alpha_D\, \pi\, d\, (\, \tilde{a}_{03} + \tilde{b}_{03}\, *_4\,) \quad , \\ \Delta b_5 &= R_g\, T_I\, \alpha_D\, \pi\, d\, \Delta\tilde{a}_2 \\ \Delta b_6 &= -R_g\, T_{II}\, \alpha_D\, \pi\, d\, \Delta\tilde{a}_3 \end{aligned}$$

and finally

$$\Delta\tilde{\boldsymbol{a}} = \begin{bmatrix} \boldsymbol{0}_{4\times1} \\ \Delta p_I \\ \Delta p_{II} \end{bmatrix} \quad . \tag{18e}$$

IV. Model-based controller design concept

The basic idea behind the investigated controller concept is that the model base of the *feedback loop controllers* will be increased so a less accurate knowledge of the system model is required in the *feedforward compensation.* In this paper we consider the special case of a structure without a common feedforward compensation as an appropriate starting point.

By defining $\tilde{\boldsymbol{x}} = [\, \boldsymbol{x}^T\ \ \dot{\boldsymbol{x}}^T\,]$, the fuzzy descriptor system can be rewritten as

$$\tilde{\boldsymbol{E}}\, \dot{\tilde{\boldsymbol{x}}} = \sum_{l=1}^{32} \sum_{k=1}^{4} h_l\, \tilde{\nu}_k\, \left[\tilde{\boldsymbol{A}}_{lk}\, \tilde{\boldsymbol{x}} + (\, \tilde{\boldsymbol{B}}_l + \Delta\tilde{\boldsymbol{B}}\,)\, \boldsymbol{u} \right] + \Delta\tilde{\tilde{\boldsymbol{a}}} \tag{19}$$

with $h_l = h_l(\varphi, \dot{\phi}, \psi, \dot{\psi}, p_I, p_{II})$ and $\tilde{\nu}_k = \tilde{\nu}_k(\,\psi,\varphi\,)$ where

$$\tilde{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} , \quad \tilde{\boldsymbol{A}}_{lk} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{A}_l & -\boldsymbol{E}_k \end{bmatrix} ,$$

$$\tilde{\boldsymbol{B}}_l = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{B}_l \end{bmatrix} , \quad \Delta\tilde{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{0} \\ \Delta\boldsymbol{B} \end{bmatrix} , \quad \Delta\tilde{\tilde{\boldsymbol{a}}} = \begin{bmatrix} \boldsymbol{0} \\ \Delta\tilde{\boldsymbol{a}} \end{bmatrix} .$$

We propose a modified parallel distributed compensation scheme (PDC)

$$\boldsymbol{u} = \sum_{l=1}^{32} \sum_{k=1}^{4} h_l\, \tilde{\nu}_k\, \tilde{\boldsymbol{F}}_{lk}\, \dot{\tilde{\boldsymbol{x}}} \tag{20}$$

with $\tilde{\boldsymbol{F}}_{lk} = [\, \boldsymbol{F}_{lk}\ \boldsymbol{0}\,]$ to stabilize the fuzzy descriptor system (19). The fuzzy controller design problem here is to determine the local feedback gains using the local models $\{\tilde{\boldsymbol{A}}_{lk}, \tilde{\boldsymbol{B}}_l, \Delta\tilde{\boldsymbol{B}}, \Delta\tilde{\tilde{\boldsymbol{a}}}\}$ that globally stabilize the fuzzy descriptor system with uncertainties by (20). We currently solve this problem using a novel LMI (Linear Matrix Inequalities) condition derived in [4].

V. Simulation results

We will now consider the results of the simulation with the global decriptor model of the plant (19), that is operated in closed loop. For the model validation purpose a fixed linear state-space controller for the whole operating space is used. It is worth to note that this controller is not be able to handle the dominat nonlinearities in the plant. The validation process is exemplified by a reference trajectory shown in Fig. 5 that defines the desired tool center

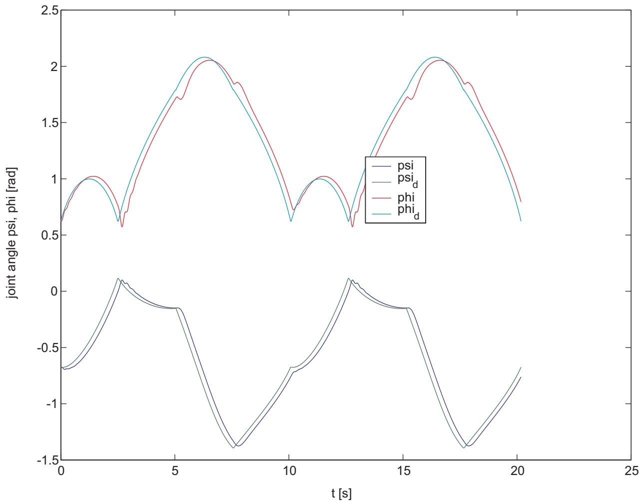

Fig. 4. Time history of the reference joint positions $\psi_d(t), \varphi_d(t)$ and simulated joint positions $\psi(t), \varphi(t)$ relate to Fig. 5.

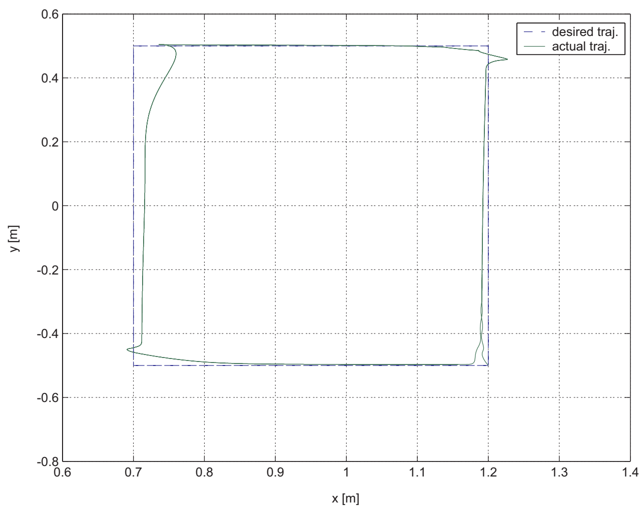

Fig. 5. The reference (desired) rectangle trajectory of the TCP, see Fig. 2, and the actual simulated TCP-position. Starting position: $x = 1.2\,\mathrm{m}$, $y = -0.5\,\mathrm{m}$

position (TCP), see Fig. 2, in the xy-plane of the working space. The corresponding joint angles in Fig. 4 have been calculated by the well-known inverse kinematics of a two-link planar arm [12]. The simulation model of the dual actuators based on the descriptor form (7), but in the first step without uncertainties. The influence of the air-pressure evolution in the actuator cylinders that caused a weakly damped dynamics is recognized in Fig. 5 where the desired trajectory distinctly differ from the actual path. The large tracking error at the corners of the rectangle is an indication that the linear controller is not able to handle the dynamics of the dual-actuator pneumatic drive that interacts with the dynamics of the robot arm. This fact is also a motivation for a nonlinear control design, for instance based on (20).

VI. Conclusion

In this paper the possibilities of the use of a TS fuzzy descriptor form to describe the dynamics of a two-link manipulators with dual-actuators were analysed. A complete manipulator model using the combination of the actuator models in descriptor form and the robot arm dynamics in descriptor form was derived for a model-based control concept. We must insist on the fact that this paper is a first step dealing with TS modelization and a further paper will present the TS control. We think this is an appropriate starting point for a robot control concept that systematically take into consideration the influnce of model reduction by structured uncertainties.

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