A Class of Non Linear Observers in Descriptor Form: LMI Based Design with Application in Biomechanics

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A CLASS OF NON LINEAR OBSERVERS IN DESCRIPTOR FORM: LMI BASED DESIGN WITH APPLICATION IN BIOMECHANICS

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Abstract: A class of nonlinear observers based on a state space descriptor form and a Takagi-Sugeno modeling are described. The conditions of convergence of the prediction error are investigated through a LMI based design. An example permits to show the interest of these observers. At last an application to biomechanics of human standing is proposed. An unknown input observer is designed to compute joint torques and velocities from position video based measurements.

Keywords: Descriptor Systems, Fuzzy Observers, Human Torque Estimation.

1. INTRODUCTION

For several years, Takagi Sugeno fuzzy models are widely used in a context of control or modeling (Guerra and Vermeiren 2001, Tanaka & al. 1998, Wang, *et al.*, 1996, Feng, *et al.*, 2001). Descriptors have been also used in control literature. A definition of fuzzy descriptors is presented in (Taniguchi, *et al.*, 1999, 2000). They also gave the first results for the stability and the stabilization using the well-known PDC (Parallel Distributed Compensation) control approach.

The purpose of this work is to propose a fuzzy observer based on a descriptor form. Its design is given part 2. For some nonlinear models (mechanical ones for example), its main interest is to keep a similar structure.

A Lyapunov approach based on a quadratic candidate function is then used to establish the conditions of convergence of the prediction error. A first result is given in part 3. The advantage of a similar structure close to the nonlinear model can allow reducing the conservatism of the results. A matrix transformation is then used to propose some new conditions that include the previous one. These results are also presented part 3. All the results are written in a LMI formulation using a relaxation scheme due to (Xiadong and Qinling 2003).

At last an illustrative example chosen in the field of biomechanics of human standing is discussed part 4.

Notations:

With $v_k(z)$ (resp. $h_i(z)$) scalar functions and E_k, $k \in \{1,\ldots,e\}$ (resp. Y_i, $i \in \{1,\ldots,r\}$) matrices of the same dimensions, we will denote:

$E_v = \sum_{k=1}^{e} v_k(z(t)) E_k$, $Y_h = \sum_{i=1}^{r} h_i(z(t)) Y_i$

and $Y_{hv} = \sum_{k=1}^{e}\sum_{i=1}^{r} v_k(z(t)) h_i(z(t)) Y_{ik}$.

Also for a function $\alpha(x) \in [\underline{\alpha} \quad \bar{\alpha}]$, $\underline{\alpha} = \min_x \alpha(x)$, $\bar{\alpha} = \max_x \alpha(x)$.

As usual, a star $(*)$ in a symmetric matrix indicates a transpose quantity.

2. OBSERVER DESIGN

Let us consider a fuzzy descriptor model as (Taniguchi, *et al.*, 2000):

$$\sum_{k=1}^{e} v_k(z(t)) E_k \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \big(A_i x(t) + B_i u(t)\big) \quad (1)$$

The functions $h_i(z(t))$, $i \in \{1,\ldots,r\}$, $v_k(z(t))$, $k \in \{1,\ldots,e\}$ are assumed to be positive, to use only measured variables $z(t)$ and to satisfy the convex sum property, i.e. $\sum_{k=1}^{e} v_k(z) = 1$ (resp. $\sum_{i=1}^{r} h_i(z) = 1$).

Using the previous notations (1) becomes:

$$\begin{cases} E_v \dot{x}(t) = A_h x(t) + B_h u(t) \\ y(t) = C_h x(t) \end{cases} \quad (2)$$

Defining $x^*(t) = \left[x^T(t), \dot{x}^T(t)\right]^T$, the system (2) can be written as:

$$\begin{cases} E^* \dot{x}^*(t) = A_{hv}^* x^*(t) + B_h^* u(t) \\ y(t) = C_h^* x^*(t) \end{cases} \quad (3)$$

where:

$E^* = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $A_{ik}^* = \begin{bmatrix} 0 & I \\ A_i & -E_k \end{bmatrix}$, $B_i^* = \begin{bmatrix} 0 \\ B_i \end{bmatrix}$ and $C_i^* = \begin{bmatrix} C_i & 0 \end{bmatrix}$.

Consider the following observer:

$$\begin{cases} E_v \dot{\hat{x}}(t) = A_h \hat{x}(t) + B_h u(t) + K_{hv}\big(y(t) - \hat{y}(t)\big) \\ \hat{y}(t) = C_h \hat{x}(t) \end{cases} \quad (4)$$

or with $\hat{x}^*(t)=\left[\hat{x}^T(t),\dot{\hat{x}}^T(t)\right]^T$:

$$\begin{cases} E^*\dot{\hat{x}}^*(t)=A^*_{hv}\hat{x}^*(t)+B^*_h u(t)+K^*_{hv}\left(y(t)-\hat{y}(t)\right) \\ \hat{y}(t)=C^*_h\hat{x}^*(t)\end{cases} \quad (5)$$

Let us define the augmented prediction error: $\tilde{x}^*(t)=\left[\left(x(t)-\hat{x}(t)\right)^T,\left(\dot{x}(t)-\dot{\hat{x}}(t)\right)^T\right]^T$, we can write its derivative as:

$$E^*\dot{\tilde{x}}^*(t)=\left(A^*_{hv}-K^*_{hv}C^*_h\right)\tilde{x}^*(t) \quad (6)$$

3. STABILITY CONDITIONS

According to the work of (Taniguchi, *et al.*, 2000) the following theorem conditions ensure the convergence of this prediction error.

Theorem 1: The fuzzy descriptor model (6) is quadratically stable if there exists a common matrix X such that:

$$E^*X=X^TE^*\geq 0 \quad (7)$$

$$\left(A^*_{hv}-K^*_{hv}C^*_h\right)^T X+X^T\left(A^*_{hv}-K^*_{hv}C^*_h\right)<0 \quad (8)$$

Proof: It is straightforward considering the following Lyapunov candidate function: $V\left(\tilde{x}^*(t)\right)=\tilde{x}^{*T}(t)E^{*T}X\tilde{x}^*(t)$.

The goal is now to propose LMI conditions for ensuring to find X and the gains K_{ik}, $i\in\{1,\dots,r\}$, $k\in\{1,\dots,e\}$. Let us define $X=\begin{bmatrix}P_1 & P_2\\ P_3 & P_4\end{bmatrix}$, condition (7) implies: $P_1=P_1^T\geq 0$ and $P_2=0$. Then condition (8) can be written as:

$$\begin{bmatrix}0 & A_h^T-C_h^TK_{hv}^T\\ I & -E_v^T\end{bmatrix}\begin{bmatrix}P_1 & 0\\ P_3 & P_4\end{bmatrix}+\begin{bmatrix}P_1 & P_3^T\\ 0 & P_4^T\end{bmatrix}\begin{bmatrix}0 & I\\ A_h-K_{hv}C_h & -E_v\end{bmatrix}<0 \quad (9)$$

3.1 First approach.

Let us rewrite (9) as:

$$\begin{bmatrix}A_h^TP_3+P_3^TA_h-C_h^TK_{hv}^TP_3-P_3^TK_{hv}C_h & (*)\\ P_1-E_v^TP_3+P_4^TA_h-P_4^TK_{hv}C_h & -E_v^TP_4-P_4^TE_v\end{bmatrix}<0 \quad (10)$$

Due to the products of unknown variables $P_3^TK_{ik}$ and $P_4^TK_{ik}$, a way to obtain LMI conditions is then to choose: $P_3=P_4$ (with P_3 non-singular) and to use the following change of variables: $M_{ik}=P_3^TK_{ik}$. Then, let us define:

$$\Upsilon_{ij}^k=\begin{bmatrix}A_i^TP_3+P_3^TA_i-C_i^TM_{jk}^T-M_{jk}C_i & (*)\\ P_1-E_k^TP_3+P_3^TA_i-M_{jk}C_i & -E_k^TP_3-P_3^TE_k\end{bmatrix} \quad (11)$$

The following theorem gives the result. Condition (14) corresponds to a relaxation scheme due to (Xiadong and Qinling, 2003) based on a previous one from (Kim and Lee, 2000).

Theorem 2: Let us consider the fuzzy descriptor model (6), the Υ_{ij}^k defined in (11). The convergence of the prediction error is ensured if there exists: $P_1=P_1^T>0$, P_3 regular, M_{ik}, $Q_{ii}^k=\left(Q_{ii}^k\right)^T>0$ and $Q_{ij}^k=\left(Q_{ji}^k\right)^T$, $k\in\{1,\dots,e\}$, $i,j\in\{1,\dots,r\}$, $j>i$, such that: :

$$\Upsilon_{ii}^k+Q_{ii}^k<0 \quad (12)$$

$$\Upsilon_{ij}^k+\Upsilon_{ji}^k+Q_{ij}^k+Q_{ji}^k<0 \quad (13)$$

$$Q^k=\begin{bmatrix}Q_{11}^k & Q_{12}^k & \cdots & Q_{1r}^k\\ Q_{21}^k & Q_{22}^k & & \\ \vdots & & \ddots & \vdots\\ Q_{r1}^k & & \cdots & Q_{rr}^k\end{bmatrix}>0 \quad (14)$$

Proof: Obviously, (10) can be written as:

$$\sum_{k=1}^{e}v_k\left(\sum_{i=1}^{r}h_i^2\Upsilon_{ii}^k+\sum_{i=1}^{r}\sum_{j>i}^{r}h_ih_j\left(\Upsilon_{ij}^k+\Upsilon_{ji}^k\right)\right)<0 \quad (15)$$

According to equations (12) and (13), condition (15) is satisfied if:

$$\sum_{k=1}^{e}v_k(z)\left(\sum_{i=1}^{r}h_i^2(z)Q_{ii}^k+2\sum_{i=1}^{r}\sum_{j>i}^{r}h_i(z)h_j(z)Q_{ij}^k\right)=\sum_{k=1}^{e}v_k(z)\Xi^k>0 \quad (16)$$

According to condition (14),

$$\forall s\neq 0,\quad s^T(t)\Xi^ks(t)=\begin{bmatrix}h_1(z)s\\ h_2(z)s\\ \vdots\\ h_r(z)s\end{bmatrix}^T Q^k\begin{bmatrix}h_1(z)s\\ h_2(z)s\\ \vdots\\ h_r(z)s\end{bmatrix}>0$$

and the proof is complete.

Example: It is constructed in a way such that classical conditions for fuzzy observer design (Xiadong and Qinling, 2003) cannot be fulfilled. Consider the following model:

$$E_v=\begin{bmatrix}\dfrac{1}{1+x_1^2} & 1\\ -1 & \dfrac{1}{1+x_2^2}\end{bmatrix},\ A=\begin{bmatrix}-4.7 & -4.7\\ 0.2 & 1.5\end{bmatrix},$$

$C_h=\left[\cos(x_2)+3 \quad 1\right]$, and $x=\left[x_1 \quad x_2\right]^T$.

The observer in a descriptor form will have 4 rules for E_v due to the nonlinear terms $\dfrac{1}{1+x_1^2}$ and $\dfrac{1}{1+x_1^2}$. and 2 rules for C_h due to $\cos(x_2)$. Then 8 rules are necessary to describe the nonlinear model and the number of LMI will be without relaxation scheme: $e\cdot r(r+1)/2=12$. Let us notice that a "classical" TS observer will be obtained using E_v^{-1} that leads to a more complicated form and a greater number of rules. In fact it can be shown that 16 rules are necessary, which leads to $r(r+1)/2=136$. It explains that classical conditions fail to prove the error convergence.

3.2 Matrix transformation.

The goal is to relax the conditions obtained in the theorem 2. We will use the following lemma. It is a slightly modified version of the property given in (Peaucelle, *et al.*, 2000) and also used in the context of Takagi-Sugeno fuzzy models stabilization (Guerra, *et al.*, 2003).

Lemma 1: Let P, Γ, Φ and Ψ be matrices of appropriate dimensions.

$$\text{Find, } P, \Phi, \Psi, \begin{bmatrix} \Gamma^T\Phi+\Phi^T\Gamma & (*) \\ P-\Phi+\Psi^T\Gamma & -\Psi-\Psi^T \end{bmatrix}<0 \tag{17}$$

$$\Leftrightarrow \quad \text{Find } P, \Gamma^T P+P^T\Gamma<0$$

Proof:

$(\Rightarrow)$ Define $\Pi=\begin{bmatrix} I & \Gamma^T \end{bmatrix}$ which is row full rank, then the result is obtained multiplying left by Π and right by Π^T.

$(\Leftarrow)$ As $\Gamma^T P+P^T\Gamma<0$, it always exists an enough small ε^2 such that:

$$\Gamma^T P+P^T\Gamma+\frac{\varepsilon^2}{2}\Gamma^T\Gamma<0 \tag{18}$$

Using the Schur complement (18) is equivalent to:

$$\begin{bmatrix} \Gamma^T P+P^T\Gamma & \varepsilon^2\Gamma^T \\ \varepsilon^2\Gamma & -2\varepsilon^2 I \end{bmatrix}<0 \tag{19}$$

If we choose $\Phi=P$ and $\Psi=\varepsilon^2 I$, then the first term inequality of (17) is verified.

The following subsection corresponds to the application of this lemma to obtain new relaxed stabilization conditions that outperform those of the *approach 1*.

3.3 Second approach.

Applying this lemma to the first block of equation (10) gives:

$$\begin{bmatrix} \Theta_{11} & (*) & (*) \\ \Theta_{21} & -E_v^T P_4-P_4^T E_v & 0 \\ P_3-\Phi_3+\Psi_3 A_h & 0 & -\Psi_3-\Psi_3^T \end{bmatrix}<0 \tag{20}$$

with: $$\Theta_{11}=A_h^T\Phi_3+\Phi_3^T A_h-C_h^T K_{hv}^T P_3-P_3^T K_{hv} C_h$$
$$\Theta_{21}=P_1-E_v^T P_3+P_4^T A_h-P_4^T K_{hv} C_h$$

(20) $\Rightarrow$ (10) is obtained multiplying right by $\Pi=\begin{bmatrix} I & 0 & A_h^T \\ 0 & I & 0 \end{bmatrix}^T$ and left by Π^T. As Φ_3 and Ψ_3 are unspecified matrices, we can replace them by: Φ_{3hv} and Ψ_{3hv}. To obtain a LMI formulation, a bijective change of variables is necessary. We choose: $P_3^T K_{hv}=M_{hv}$ and $P_3=P_4$. Let us define:

$$\Upsilon_{ij}^k=\begin{bmatrix} \Upsilon_{ij}^k(1,1) & (*) & (*) \\ \Upsilon_{ij}^k(2,1) & \Upsilon_{ij}^k(2,2) & 0 \\ \Upsilon_{ij}^k(3,1) & 0 & \Upsilon_{ij}^k(3,3) \end{bmatrix} \tag{21}$$

with:
$$\Upsilon_{ij}^k(1,1)=A_i^T\Phi_{3jk}+\Phi_{3jk}^T A_i-C_i^T M_{jk}^T-M_{jk}C_i$$
$$\Upsilon_{ij}^k(2,1)=P_1-E_k^T P_3+P_3^T A_i-M_{jk}C_i$$
$$\Upsilon_{ij}^k(3,1)=P_3-\Phi_{3jk}+\Psi_{3jk}A_i$$
$$\Upsilon_{ij}^k(2,2)=-E_k^T P_3-P_3^T E_k$$
$$\Upsilon_{ij}^k(3,3)=-\Psi_{3jk}-\Psi_{3jk}^T$$

Theorem 3: Let us consider the fuzzy descriptor model (6) and the Υ_{ij}^k defined in (21). The convergence of the prediction error is ensured if there exists: $P_1=P_1^T>0$, P_3, Φ_{3ik}, Ψ_{3ik}, M_{ik}, $Q_{ii}^k>0$ and $Q_{ij}^k=\left(Q_{ji}^k\right)^T$ $k\in\{1,\dots,e\}$, $i,j\in\{1,\dots,r\}$, $j>i$ such that: (12), (13) and (14) are satisfied.

Lemma 2:
The *approach 2* always includes *approach 1*.

Proof: In the proof, the exponent $^{(1)}$ stands for *approach 1*, the exponent $^{(2)}$ stands for *approach 2*. Suppose that the conditions of *approach 1* (theorem 2) are satisfied. Then there exists $P_1=P_1^T>0$, P_3 regular, M_{ik}, $Q_{ii}^{k(1)}>0$ and $Q_{ij}^{k(1)}=\left(Q_{ji}^{k(1)}\right)^T$ satisfying (12)~(14). The goal is to prove that these matrices are also solution of *approach 2*. Choose $Q_{ii}^{k(2)}=\begin{bmatrix} Q_{ii}^{k(2)} & 0 \\ 0 & \varepsilon^2 I \end{bmatrix}$, $Q_{ij}^{k(2)}=\begin{bmatrix} Q_{ij}^{k(1)} & 0 \\ 0 & 0 \end{bmatrix}$. It follows directly that the condition $Q^{k(2)}>0$ is true if and only if: $\begin{bmatrix} Q^{k(1)} & 0 \\ 0 & \varepsilon^2 I_{n\times r} \end{bmatrix}>0$ which is clearly satisfied as $Q^{k(1)}>0$. Choose also: $\Phi_{3ik}=P_3$, $\Psi_{3ik}=\varepsilon^2 I$ $k\in\{1,\dots,e\}$, $i\in\{1,\dots,r\}$ and then (21) becomes:

$$\Upsilon_{ij}^{k(2)}=\begin{bmatrix} \Upsilon_{ij(1,1)}^{k(2)} & (*) & (*) \\ \Upsilon_{ij(2,1)}^{k(2)} & -E_k^T P_3-P_3^T E_k & 0 \\ \varepsilon^2 A_i & 0 & -2\varepsilon^2 I_n \end{bmatrix} \tag{22}$$

with: $$\Upsilon_{ij(1,1)}^{k(2)}=A_i^T P_3+P_3^T A_i-C_i^T M_{jk}^T-M_{jk}C_i$$
$$\Upsilon_{ij(2,1)}^{k(2)}=P_1-E_k^T P_3+P_3^T A_i-M_{jk}C_i$$

Define: $U_i=\begin{bmatrix} A_i & 0 \end{bmatrix}$, then (22) becomes:

$$\Upsilon_{ij}^{k(2)}=\begin{bmatrix} \Upsilon_{ij}^{k(1)} & 0 \\ \varepsilon^2 U_i & -2\varepsilon^2 I_n \end{bmatrix} \tag{23}$$

$$\Upsilon_{ij}^{k(2)}+\Upsilon_{ji}^{k(2)}+Q_{ij}^{k(2)}+Q_{ji}^{k(2)} =\begin{bmatrix} \Upsilon_{ij}^{k(1)}+\Upsilon_{ji}^{k(1)}+Q_{ij}^{k(1)}+Q_{ji}^{k(1)} & (*) \\ \varepsilon^2\left(U_i+U_j\right) & -4\varepsilon^2 I_n \end{bmatrix} \tag{24}$$

And with the Schur's complement:

$$(23) \Leftrightarrow \quad \Upsilon_{ii}^{k(1)}+Q_{ii}^{k(1)}+\varepsilon^2 U_i^T U_i<0 \tag{25}$$

$$(24) \Leftrightarrow \quad \begin{aligned} &\Upsilon_{ij}^{k(1)} + \Upsilon_{ji}^{k(1)} + Q_{ij}^{k(1)} + Q_{ji}^{k(1)} \\ &+ \frac{\varepsilon^2}{4}\left(U_i + U_j\right)^T \left(U_i + U_j\right) < 0 \end{aligned} \tag{26}$$

As *approach 1* is verified, $\Upsilon_{ii}^{k(1)} + Q_{ii}^{k(1)} < 0$ and $\Upsilon_{ij}^{k(1)} + \Upsilon_{ji}^{k(1)} + Q_{ij}^{k(1)} + Q_{ji}^{k(1)} < 0$ are both true. Then it exists always an enough small ε^2 such that (25) and (26) hold.

4. APPLICATION TO THE BIOMECHANICS OF HUMAN STANDING

4.1 Human standing descriptor model to be considered.

When an individual is quietly standing in the sagittal plane, knee and neck movements are slower than ankle and hip movements. Thus, depending on the environment and certain pathological assumptions, two sway strategies can be defined: the "ankle" strategy and the "hip" strategy (Nashner and McCollum, 1985). Then, a planar double inverted pendulum has been chosen to model the human body in the sagittal plane, figure 1. The lower pendulum oscillates around a point A located at the ankle, and the upper pendulum around a point H located at the hip. This model is based on two assumptions:

1) both inverted pendulum segments are rigid and connected by hinge joints
2) the neuromuscular system, i.e. the muscle actuators and the central nervous system, provides the necessary joint torques at the ankle Γ_1 and hip Γ_2 to stabilize quiet standing.

These torques are the inputs of the double inverted pendulum model, and the associated angular positions are its outputs. In figure 1, segment 1 represents the lower limbs, not including the feet; segment 2 represents the trunk, head and upper limbs. Estimates of the inertial characteristics m_i, I_i and K for each segment can be made using anthropometric tables (Winter, 1990). θ_i is the generalized angular coordinate of segment i with respect to the vertical axis.

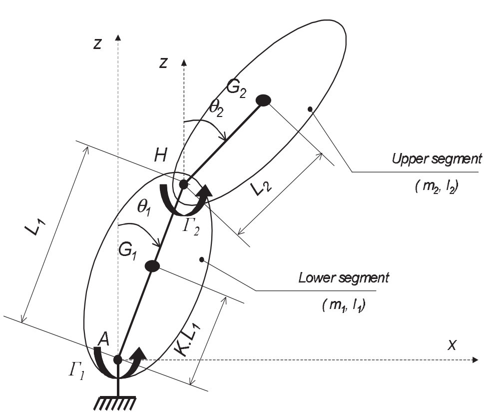

Fig. 1. A double inverted pendulum model of human standing.

The double inverted pendulum model is derived using Lagrange method and is given by:

$$M(y)\cdot \ddot{y} + S(y,\dot{y})\cdot \dot{y} + G(y)\cdot y = Ru \tag{27}$$

with:

$u(t) = \left[\Gamma_1(t) \quad \Gamma_2(t)\right]^T$ the input vector,

$y(t) = \left[\theta_1(t) \quad \theta_2(t)\right]^T$ the output vector,

$M(y) = \begin{bmatrix} a & c\cos(\theta_1-\theta_2) \\ c\cos(\theta_1-\theta_2) & b \end{bmatrix}$ the inertia matrix,

$S(y,\dot{y}) = \begin{bmatrix} 0 & c\dot{\theta}_2 \sin(\theta_1-\theta_2) \\ -c\dot{\theta}_1 \sin(\theta_1-\theta_2) & 0 \end{bmatrix}$ the anti-symmetric matrix,

$G(y) = \begin{bmatrix} d\dfrac{\sin\theta_1}{\theta_1} & 0 \\ 0 & e\dfrac{\sin\theta_2}{\theta_2} \end{bmatrix}$ the gravitational matrix,

$R = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ the matrix linking joint torque to generalized torque and:

$a = I_1 + m_1 K^2 L_1^2 + m_2 L_1^2$, $b = I_2 + m_2 L_2^2$,

$c = m_2 L_1 L_2$, $d = (m_2 + m_1 K) g L_1$, $e = m_2 g L_2$

Considering $x(t) = \left[y^T(t) \quad \dot{y}^T(t)\right]^T$ the state vector of the system, (27) can be written as the following descriptor form:

$$\begin{cases} E(y(t))\dot{x}(t) = A(x(t))x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{28}$$

with:

$E(y(t)) = \begin{bmatrix} I & 0 \\ 0 & M(\theta_1,\theta_2) \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ R \end{bmatrix}$, $C = \begin{bmatrix} I & 0 \end{bmatrix}$

and $A(y(t)) = \begin{bmatrix} 0 & 0 \\ G(\theta_1,\theta_2) & -S(\theta_1,\theta_2,\dot{\theta}_1,\dot{\theta}_2) \end{bmatrix}$.

4.2 Observer design based on the human standing descriptor model.

In human studies, it is quite difficult to measure joint torques with external apparatus. To overcome this problem, an unknown inputs observer is designed. The following hypothesis is made: the dynamic behavior of the input signals are slow in comparison with the system dynamics, thus we can write $\dot{u}(t) \approx 0$. Now we can define a new extended observer state vector: $x^e(t) = \left[x^T(t) \quad u^T(t)\right]^T$. According to this vector and the model defined equation (28), the following extended observer, can be designed:

$$\begin{cases} E^e \dot{\hat{x}}^e = A^e \hat{x}^e + K^e\left(y - C^e \hat{x}^e\right) \\ y = C^e x^e \end{cases} \tag{29}$$

with: $E^e = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$, $A^e = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$, $C^e = \begin{bmatrix} C & 0 \end{bmatrix}$.

The nonlinear functions to be considered in the TS fuzzy observer formulation are:

$\omega = \cos(\theta_1 - \theta_2) \in [\underline{\omega} \quad \overline{\omega}]$ appearing in E^e and $\eta_1 = \dfrac{\sin(\theta_1)}{\theta_1} \in [\underline{\eta}_1 \quad \overline{\eta}_1]$, $\quad \eta_2 = \dfrac{\sin(\theta_2)}{\theta_2} \in [\underline{\eta}_2 \quad \overline{\eta}_2]$ appearing in A^e.

A^e contains also the functions $\eta_3 = \dot{\theta}_1 \sin(\theta_1 - \theta_2)$ and $\eta_4 = \dot{\theta}_2 \sin(\theta_1 - \theta_2)$. As the angular velocity cannot be measured with a motion capture apparatus, these functions are removed. The TS model is then obtained by splitting the nonlinear function as (Morère, 2001):

$$f = \frac{\overline{f} - f}{\overline{f} - \underline{f}} \cdot \underline{f} + \frac{f - \underline{f}}{\overline{f} - \underline{f}} \cdot \overline{f} \quad \text{with } f \in \{\omega, \eta_1, \eta_2\},$$

$$v_1 = \frac{\overline{\omega} - \omega}{\overline{\omega} - \underline{\omega}} = 1 - v_2\,, \quad h_1 = \frac{\overline{\eta}_1 - \eta_1}{\overline{\eta}_1 - \underline{\eta}_1} \cdot \frac{\overline{\eta}_2 - \eta_2}{\overline{\eta}_2 - \underline{\eta}_2}\,, \ \ldots$$

$$E_1^e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & c\underline{\omega} & 0 & 0 \\ 0 & 0 & c\underline{\omega} & b & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_1^e = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ d\underline{\eta}_1 & 0 & 0 & 0 & 1 & -1 \\ 0 & e\underline{\eta}_2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \ldots$$

yield the simplified fuzzy observer with the proposed notations:

$$\begin{cases} E_v^e \dot{\hat{x}}^e = A_h^e \hat{x}^e + K_{hv}^e \left(y - C_h^e \hat{x}^e \right) \\ y = C_h^e x^e \end{cases} \tag{30}$$

4.3 Results and discussion

Simulation

The validity of the fuzzy observer was tested in simulation with the non-linear descriptor (28) including the neglected velocities. K_{hv}^e was chosen via pole placement with two poles fixed at -18 and four auxiliary poles set at -50. The set points are two steps allowing the model moving from the erect position $(\theta_1 = 0, \theta_2 = 0)$ to the angular positions $(\theta_1 = -\pi/6, \theta_2 = \pi/3)$ with a realistic dynamics regarding to physiological constraints. Figure 2 shows the comparison between the joint torques obtained by simulation of the nonlinear model and the observer based estimation (OE).

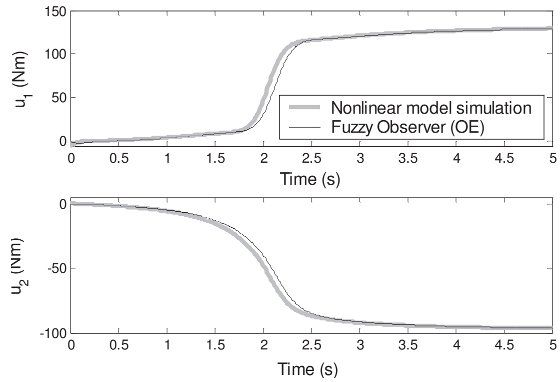

Fig. 2. Joint torques obtained by the simulation of the non linear model and observer estimation.

Experimental results

One 28 year-old male subject, weighing $69.8\,Kg$ and measuring $1.85\,m$ was asked to perform a combined trunk flexion/extension that disturbed the subject's equilibrium, provoking ankle and hip oscillation in the sagittal plane. Movements were captured at a frequency of $120\,Hz$ using an AMTI force-plate synchronised with a VICON 612 motion capture device (Figure 3). The 3-dimensional reconstruction error was evaluated at $0.6\,mm$. Reflective markers define 14 segments of the subject's body. Based on the measured positions of these 14 body segments, and through the use of anthropometric tables, the centre of mass and the inertia of both parts of the double inverted pendulum in the sagittal plane were determined (Winter, 1990).

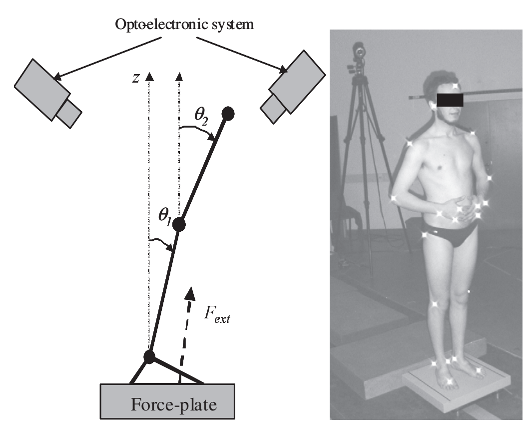

Fig. 3. Experimental devices for measuring angular position and external force

To have indications on the accuracy of the observer based estimations (OE), the ankle and hip torques estimations were compared to those computed by the well-known bottom-up inverse dynamics (ID) technique that is classically used in biomechanics studies (Winter, 1990). Figure 4 shows that the OE Joint torques estimations follow the same path as the ID ones. That is confirmed by the high values of the correlation coefficients given table 1 for both the ankle and hip joint torques.

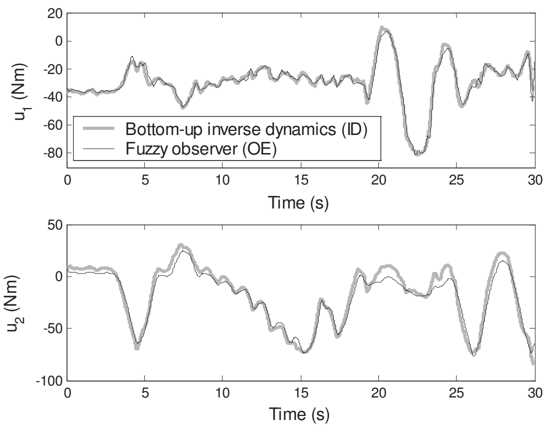

Fig. 4. ID and OE Joint torques obtained from experimental data.

The standard error computed on both the ankle torques estimation is small $(2.83\,Nm)$. Nevertheless, at the average of time $22s$, a difference between the two paths of the hip torques estimations can be noticed. This one is confirmed by a higher standard error ($6.29Nm$) than those obtained at the ankle torques (Table 1).

OE vs ID	Correlation coefficient	Standard error (Nm)
Ankle	0,98	2,83
Hip	0,98	6,29

Table 1. Quantitative comparison of the OE and ID joint torques estimations.

These estimation errors can be due to the fact that the "bottom-up" inverse dynamics require the use of two different apparatus (Force-plate and motion capture device) that bring on the uncertainties (McCaw and De Vita, 1995). Moreover, it is also well known that inverse dynamics is subject to increasing uncertainties in proportion to the number of the considered links (Hatze, 2001). Consequently, unless using an internal measurement apparatus that is ethically outlaw, we do not have any measurement reference available to claim that using an unknown input observer is a better method to compute joint torques than using inverse dynamics. Nevertheless, when a dynamical model can be easily obtained and due to the fact that it only requires the segments positions measurements, the use of an unknown inputs observer is an alternative to the use of inverse dynamics techniques in biomechanical studies.

5. CONCLUSION

This paper has presented a nonlinear observer based on a descriptor form and a Takagi-Sugeno representation. Stability conditions of the prediction error based on LMI design were investigated. Two approaches were proposed, the last one including the first one. An example to show the interest of such observer was also given. At last the results were applied successfully on an unknown inputs observer designed for biomechanical study of human standing.

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