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1 Introduction

In the past decades, considerable efforts have been devoted to the enhancement of nonlinear systems and their stability and nonlinear phenomena. Although the stability of these systems can be improved by Lyapunov analysis. Because linear superposition is no longer available, explicit formula are difficult to come by, numerical approximations are not always sufficiently accurate [1]. The advent of powerful computers has fomented a veritable revolution in our understanding of nonlinear systems. Indeed, many of the most important modern analytical techniques drew their inspiration from early computer-aided investigations of nonlinear systems [2]. Most systems are nonlinear with characteristics that change with time, since that, in a dynamic operating mode, we cannot guaranteed the strong performance of the linearized model-based controllers of nonlinear systems. Therefore, the nonlinearities of nonlinear systems are required for an intelligent controller. To improve the behavior of nonlinear controllers, many techniques have been proposed for their design, such as fuzzy logic [3], adaptive antiswing control strategy for crane systems [4], artificial neural networks [5, 6], adaptive switched non-strict-feedback nonlinear systems [7], metaheuristic algorithms [8], nonlinear designs using variable structure control (sliding mode, synergetic control), design of an adaptive fuzzy control for stochastic nonlinear systems with unmeasured states [9, 10], controlling underactuated systems [11], stabilizing control strategy for underactuated ship-mounted crane systems [12], and many other nonlinear control techniques [13, 14]. The advantage of fuzzy logic control methods is that we can obtain and adjust online the parameters of the system. In order to make the design more adaptable, the combination between fuzzy logic and other control methods such as synergetic control is made [15]. To reduce the algorithm complexity, simplified

models of nonlinear systems are used. Furthermore, reduced computational burdens for realistic models are required to design a robust controller under different operating conditions. An adaptive nonlinear fuzzy control technique has obtained a great attention based on the universal approximation theorem, however, most results are limited to continuous nonlinear systems[2], which cannot be extended to discrete-time nonlinear systems directly. The implementation of a fuzzy control algorithms in digital calculators involve the loss of some advantages of continuous time controllers. However, it should be mentioned that the used controller is a discrete-time systems [16]. In [17], an indirect fuzzy adaptive controller algorithm for uncertain nonlinear systems was developed where Takagi-Sugeno (T-S) fuzzy input output model was used to approximate the nonlinear system dynamics, an adaptive feedback controller model was designed [18], [19] using the (T-S) fuzzy models. Authors in [20] had developed a solution framework and control design for adaptive control discrete-time input-output multiple-delay T-S fuzzy systems. We have proposed a new adaptive fuzzy synergetic control scheme for uncertain nonlinear discrete-time systems.
So we have organized our work as follows. Section 2 presents the basics in the design of a synergetic control, in Section 3, the nonlinear systems described by discrete-time equations for which the design of a synergetic controller is performed. Fuzzy IF-THEN rules are presented in section 4 which show how an adaptive synergetic controller can estimates the nonlinear dynamics of the nonlinear systems based on the intelligent approach of the fuzzy logic. In section 5, through the Lyapunov stability analysis, we demonstrate the stability of the nonlinear discrete-time system and derive the adaptation law. The effectiveness of the proposed adaptive synergetic fuzzy controller (ASFC) is illustrated by the design for a real world example in section 6. In Section 7, conclusion and some perspectives are given.

2 Basics in Synergetic Control Design

The synergetic control is a novel nonlinear control technique that take into account the nonlinearities of system in the control design. A systematic design procedure which yields control laws suitable for digital implementation are offered [21, 22]. Moreover, synergetic control not only gives constant switching frequency operation, but also provides asymptotic stability with respect to the required operating conditions, and robustness in parameters variation of the system [1, 15].
The designer can select the characteristics of the macro-variable according to performances and control specifications (overshoot, control signal limits,

etc...) which make the design more robust. Thus the control law, will not cause chattering phenomena as in the sliding mode control approach.
The parameters were optimized and the implementation is easy to realize due to use of measurable variables in the control law.
Let's consider an nth order dynamic nonlinear systems described as :

$$x(k+1) = f(x(k), u(k), k) \tag{1}$$

Where $x(k)$ represents the system vector, $u(k)$ the control input vector and f is an nonlinear function. The synthesis of a synergetic controller starts with the selection of a function of the system state variables, which is called the macro-variable and depend on the state variables.

$$\psi = \psi(x(k), k) \tag{2}$$

The control objective is to force the system state to operate on the manifold $\psi = 0$. The designer can select the characteristics of the macro-variable according to performances and control specifications (overshoot, control signal limits, etc...).
In continuous-time synergetic approach, to control theory (SACT) procedure, $T\dot{\psi} + \psi = 0$ defines the speed and trajectory of convergence to the invariant manifold [23]. Considering sampling period T_s, the discrete counterpart is derived as follows :

$$T\Big[\frac{\psi(k+1) - \psi(k)}{T_s}\Big] + \psi(k) = 0 \tag{3}$$

Where T is a design parameter that specifies the convergence speed to the manifold.
Equation (3) can be rewritten as :

$$\frac{T}{T_s}\Big(\frac{T}{T_s - T}\Big)\psi(k+1) + \psi(k) = 0 \tag{4}$$

3 Design of Synergetic Controller

Consider a discrete-time nonlinear systems which have the state space representation :

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = x_3(k) \\ \quad\vdots \\ x_{n-1}(k+1) = x_n(k) \\ x_n(k+1) = f(x(k)) + u(k) + d(k) \\ y(k) = x_1(k) \end{cases} \tag{5}$$

where $f(x(k))$ is a nonlinear function, $x(k) = [x_1(k), x_2(k), ..., x_n(k)]^T \in \mathbb{R}^n$ is the state vector of the systems which is assumed to be available for measurement, $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$ are respectively, the input and the output of the system, and $d(k)$ is the external disturbance which is assumed to be bounded.
Define the tracking errors as :

$$\begin{aligned} e_1(k) &= x_1(k) - y_d(k) \\ e_2(k) &= x_2(k) - y_d(k+1) \\ &\vdots \\ e_n(k) &= x_n(k) - y_d(k+n-1) \end{aligned}$$

where $y_d(k)$ denote the reference trajectory. The tracking error equation is given as :

$$\begin{aligned} e(k+1) = Ae(k) + B\Big[&f(x(k)) + u(k) \\ &- y_d(k+n) + d(k)\Big] \end{aligned} \tag{6}$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

The objective of the control is to design a controller for the state $x_1(k)$ to track a desired reference signal $y_d(k)$ in the presence of external disturbance and uncertainties $d(k)$. The control $u(k)$ is calculated according to (4) and (6), which gives a control signal that ensures a specified properties. Let's define the macro-variable as :

$$\psi(k) = \kappa_1 e_1(k) + e_2(k) = \sum_{i=1}^{n-1} \kappa_i e_i(k) + e_n(k) \tag{8}$$

$$\psi(k+1) = \kappa_1 e_1(k+1) + e_2(k+1) \tag{9}$$

Where κ_1 is the SACT controller parameter.
Where :

$$e_1(k+1) = x_1(k+1) - y_d(k+1) \tag{10}$$

$$e_2(k+1) = x_2(k+1) - y_d(k) \tag{11}$$

$$
\begin{aligned}
\psi(k+1) = \kappa_1 x_1(k+1) - \kappa_1 y_d(k+1) \\
+x_2(k+1) - y_d(k)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
\psi(k+1) = \kappa_1 x_2(k) - \kappa_1 y_d(k+1) + f(x(k)) \\
+ u(k) + d(k) - y_d(k)
\end{aligned} \tag{13}
$$

The incremental change in $\psi(k)$ can be expressed by :

$$
\begin{aligned}
\Delta\psi(k+1) &= \psi(k+1) - \psi(k) \\
&= \sum_{i=1}^{n-1} \kappa_i e_i(k+1) + e_n(k+1) \\
&\quad - \sum_{i=1}^{n-1} \kappa_i e_i(k) - e_n(k) \\
\Delta\psi(k+1) &= \sum_{i=1}^{n-1} \kappa_i e_i(k+1) + x_n(k+1) \\
&\quad - y_d(k+n) - \sum_{i=1}^{n-1} \kappa_i e_i(k) - e_n(k) \\
\Delta\psi(k+1) &= \sum_{i=1}^{n-1} \kappa_i e_i(k+1) + f(x(k)) \\
&\quad - y_d(k+n) + u_{eq}(k) \\
&\quad - \sum_{i=1}^{n-1} \kappa_i e_i(k) - e_n(k)
\end{aligned} \tag{14}
$$

Let's take

$$
\alpha = \frac{T}{T_s}\Big(\frac{T}{T_s - T}\Big) \tag{15}
$$

Combining equations (12) and (4) yields :

$$
\begin{aligned}
\alpha\Big[\kappa_1 x_2(k) - \kappa_1 y_d(k+1) + f(x(k)) \\
+ u(k) + d(k) - y_d(k)\Big] + \psi(k) = 0
\end{aligned} \tag{16}
$$

The synergetic control law is then deduced and given as (17) :

$$
\begin{aligned}
u(k) = -f(x(k)) - \kappa_1 x_2(k) + \kappa_1 y_d(k) \\
+ y_d(k) - d(k) - \frac{1}{\alpha}\psi(k)
\end{aligned} \tag{17}
$$

If $f(x(k))$ is known, we can easily construct the synergetic control law (17). However, this controller contains a constraint : knowledge of $f(x(k))$ which is not always possible making the implementation impossible.
Therefore, to overcome this obvious problem, an adaptive synergetic fuzzy controller using fuzzy logic system is proposed in the next section.

4 Adaptive Synergetic Fuzzy Controller

Since $f(x(k))$ is unknown, we cannot implement the ideal controller (17), we suppose that a fuzzy system can approximate $f(x(k))$. A fuzzy system is a collection of IF-THEN rules in the form :

$$R^{(l)} : \text{IF } x_1 \text{ is } F_1^l \text{ and ... and } x_n \text{ is } F_n^l \text{ THEN } y \text{ is } G^l \tag{18}$$

Where $x = (x_1, ..., x_n)^T$ is the input of the fuzzy systems, and y is it's output, F_i^l and G^l are fuzzy sets, for $l = 1, ..., m$.
Using singleton fuzzification method, product inference, and center-average defuzzification, $y(x)$ is given by [24, 25, 26, 27] :

$$y(x) = \frac{\sum_{j=1}^{m} y^j \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)} \tag{19}$$

Where $\mu_{F_i^j}(x_i)$ is the membership function of the linguistic variable x_i and y^j is the point at which the membership function of G^l achieves its maximum value.
Introducing the concept of fuzzy basis function vector $\xi(x)$, $y(x)$ is given by (19) and can be rewritten as :

$$y(x) = \theta^T \xi(x) = \xi(x)^T \theta \tag{20}$$

Where $\theta = [y^1, ..., y^m]^T$, $\quad \xi(x) = [\xi^1(x), ..., \xi^m(x)]^T$ are the fuzzy basis functions given by :

$$\xi^j(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)} \tag{21}$$

Using the universal approximation theorem, we can use $\hat{f}(x(k)|\theta_f) = \theta_f^T \xi_f(x(k))$ in the form of (20) in order to approximate the function $f(x(k))$.
Therefore, a new control law is obtained :

$$\begin{aligned} u_c(k) = & -\hat{f}(x(k)) - \kappa x_2(k) + \kappa y_d(k) \\ & + y_d(k) - d(k) - \frac{1}{\alpha}\psi(k) \end{aligned} \tag{22}$$

5 Stability Analysis

Theorem :
Taking in account the control problem of the nonlinear system (5), with the robust adaptive fuzzy synergetic controller given by (22), $\hat{f}$ is used and the parameter vector $\theta_f(k)$ can be tuned by the adaptive law given in (23), the signals in the closed-loop are ultimately bounded and the tracking error converges to neighborhood of origin [28, 29].

$$\Delta\theta_f(k) = \gamma\xi(k)\psi(k) \tag{23}$$

Where γ is a real constant determining the adaptation rate. Let us defining the optimal parameter vector :

$$\theta_f^* = \arg \min_{\theta_f \in \Omega_{\theta_f}} \left\{ \sup_{x \in \Omega_x} \left| f(x(k)) - f(x(k)|\theta_f) \right| \right\} \tag{24}$$

Where Ω_{θ_f} and Ω_x are constraint sets for θ_f and x respectively.
The minimum approximation error is as follows :

$$\varepsilon(k) = f(x(k)) - f(x(k)|\theta_f^*) + d(k) \tag{25}$$

The fuzzy control $u(k)$, is chosen in the closed-loop system as :

$$u(k) = u_c(k) + u_r(k) + u_\psi(k) = u_c(k) + u_r(k) - \tau\psi(k) \tag{26}$$

Where the term $u_r(k)$ is a robust controller which is used to attenuate the external disturbance.

$$u_r(k) = -\frac{1}{2}\Big[-\beta(k) + \Big(\beta(k)^2 - 4A_0(k)\Big)^{1/2} \Big]\psi(k)$$

With $A_0(k)$ and $\beta(k)$ to be defined later.
Then substituting (22)-(25) into (12), after simple manipulation, we obtain :

$$\Delta\psi(k+1) = u_\psi(k) + u_r(k) + \tilde{\theta}_f^T(k)\xi(k) + \varepsilon(k) \tag{27}$$

Let $\tilde{\theta}_f(k) = \theta_f^* - \theta_f(k)$.
Now consider the candidate Lyapunov function :

$$V(k) = \frac{1}{2}\Big(\psi^2(k) + \frac{1}{\gamma}\tilde{\theta}_f^T(k-1)\tilde{\theta}_f(k-1)\Big) \tag{28}$$

$\Delta V(k+1)$ can be calculated as :

$$\begin{aligned}\Delta V(k+1) &= V(k+1) - V(k) \\ &= \frac{1}{2}\psi^2(k+1) - \frac{1}{2}\psi^2(k) \\ &\quad + \frac{1}{2\gamma}\tilde{\theta}_f^T(k)\tilde{\theta}_f(k) \\ &\quad - \frac{1}{2\gamma}\tilde{\theta}_f^T(k-1)\tilde{\theta}_f(k-1)\end{aligned} \tag{29}$$

Let

$$\Delta\theta_{tf} = \frac{1}{2\gamma}\tilde{\theta}_f^T(k)\tilde{\theta}_f(k) - \frac{1}{2\gamma}\tilde{\theta}_f^T(k-1)\tilde{\theta}_f(k-1) \tag{30}$$

By using (30), we can rewrite (29) as :

$$\begin{aligned}\Delta V(k+1) &= \frac{1}{2}\psi^2(k+1) - \frac{1}{2}\psi^2(k) + \Delta\theta_{tf} \\ &= \frac{1}{2}(\Delta\psi(k+1) + \psi(k))^2 - \frac{1}{2}\psi^2(k) + \Delta\theta_{tf} \\ &= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k)\Delta\psi(k+1) + \Delta\theta_{tf} \\ &= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k)\Big[u_\psi(k) + u_r(k) \\ &\quad + \theta_f^T(k)\xi(k)\Big] + \Delta\theta_{tf} \\ \Delta V(k+1) &= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k)u_\psi(k) + \psi(k)u_r(k) \\ &\quad + \psi(k)\theta_f^T(k)\xi(k) + \Delta\theta_{tf}\end{aligned} \tag{31}$$

From (30), $\Delta\theta_{tf}$ can be expressed as :

$$\begin{aligned}\Delta\theta_{tf} &= \frac{1}{2\gamma}\Big(\tilde{\theta}_f^T(k)\tilde{\theta}_f(k) - \Big[\tilde{\theta}_f(k) - \Delta\tilde{\theta}_f(k)\Big]^T \\ &\quad \times \Big[\tilde{\theta}_f(k) - \Delta\tilde{\theta}_f(k)\Big]\Big) \\ \Delta\theta_{tf} &= \frac{1}{\gamma}\theta_f^T(k)\Delta\tilde{\theta}(k) - \frac{1}{2\gamma}\Delta\tilde{\theta}_f^T(k)\Delta\tilde{\theta}_f(k)\end{aligned} \tag{32}$$

Substituting (32) into (31), yields :

$$\begin{aligned}
\Delta V(k+1) &= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k)u_\psi(k) + \psi(k)u_r(k) \\
&+ \varepsilon(k)\psi(k) + \Delta\tilde{\theta}_f^T(k)\psi(k) + \Delta\theta_{tf} \\
&= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k)u_\psi(k) + \psi(k)u_r(k) \\
&+ \varepsilon(k)\psi(k) + \theta_f^T(k)\Big[\xi(k)\psi(k) \\
&- \frac{1}{\gamma}\Delta\tilde{\theta}_f(k)\Big] - \frac{1}{2\gamma}\Delta\tilde{\theta}_f^T(k)\Delta\tilde{\theta}_f(k)
\end{aligned} \tag{33}$$

Using the adaptive law (23), we get :

$$\begin{aligned}
\Delta V(k+1) &= \frac{1}{2}\Delta\psi^2(k+1) + \psi(k).u_\psi(k) + \psi(k).u_r(k) \\
&+ \varepsilon(k)\psi(k) - \frac{1}{2\gamma}\Delta\tilde{\theta}_f^T(k)\Delta\tilde{\theta}_f(k)
\end{aligned} \tag{34}$$

From (27), we have :

$$\begin{aligned}
|\Delta\psi(k+1)| &\leq |u_\psi(k)| + |u_r(k)| + |\tilde{\theta}_f^T(k)\xi(k)| + |\varepsilon(k)| \\
&\leq s_\psi + s_f\|\xi(k)\| + |u_r(k)| + s_\varepsilon \\
&\leq |A_0(k)| + |u_r(k)|
\end{aligned} \tag{35}$$

With $A_0(k) = s_\psi + s_f\|\xi(k)\| + s_\varepsilon$
Taking the square from both sides of (35), we get :

$$\begin{aligned}
|\Delta\psi(k+1)|^2 &\leq |u_r(k)|^2 + 2|A_0(k)||u_r(k)| \\
&\quad + 2|\psi(k)||u_r(k)| + |A_0(k)|^2 \\
&\quad - 2\psi(k)|u_r(k)| \\
&\leq \Big[-\beta(k) + \Big(\beta(k)^2 - 4A_0(k)\Big)^{1/2}\Big]|\psi(k)|
\end{aligned} \tag{36}$$

Where $\beta(k) = 2(A_0(k) - |\psi(k)|)$.
Therefore, (34) becomes as follows :

$$\Delta V(k+1) = \varepsilon(k)\psi(k) + \psi(k)u_\psi(k) - \frac{1}{2\gamma}\Delta\tilde{\theta}_f^T(k)\Delta\tilde{\theta}_f(k) \tag{37}$$

Because $\psi(k)u_\psi(k) < 0$, $\Delta\tilde{\theta}_f^T(k)\Delta\tilde{\theta}_f(k) > 0$, and based on the universal approximation theorem, the term $\varepsilon(k)\psi(k)$ is very small.
So we have :

$$\Delta V(k+1) \leq 0$$

6 Results and discussions

A simulation is carried out for a real nonlinear discrete-time system model [30], given in the canonical form of (5).

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = f(x(k)) + (K/T)u(k) + d(k) \\ y(k) = x_1(k) \end{cases} \tag{38}$$

Where $f(x(k)) = -[a_1 x_2 + a_2 x_2^3(k)]/T$ represents the system dynamics. $y_d(k) = \sin(k\pi/20)$ is the trajectory of reference.

$$d(k) = \begin{cases} 0, & \text{if } k \leq 500 \\ 0.1 \tanh(0.5k), & \text{if } k > 500 \end{cases} \tag{39}$$

represents the external disturbance, the initial conditions are chosen as : $[x_1(0), x_2(0)]^T = [0, 0]$, $a_1 = 0.5$, $a_2 = 30$, $K = 0.5$, $T = 64$. The membership functions for the system states x_i, $i = 1, 2$ are selected as : $\mu(x_i) = e^{(-0.5(x_i+6-2(j+1))^2)}$, $j = 1, ..., 5$, $\gamma = 10^{-3}$, the sampling period is $T_s = 0.02$s.

The simulation is carried out where the time evolutions of the variable $x_1(k)$ and the trajectory reference $y_d(k)$ is shown on figure 1. From this figure, a good tracking performance is obtained, where the total sampling number is 3000. Figure 2 indicates that the proposed adaptive Synergetic Fuzzy Controller $u(k)$ is bounded. The Trajectory of the macro-variable $\psi(k)$ is illustrated in figure 3 which converges to zero as k increases. Figure 4 indicates the time evolutions of the dynamics of $f(k)$ and it's estimate $\hat{f}(k)$, it is shown that $\theta_f^T \xi_f(x(k))$ approaches $f(k)$ as k increases, and the estimation error converges to neighborhood of zero. Figure 5 shows the norm of adaptive parameter vector $\theta_f(k)$ which is bounded.

Figure 6 illustrate that if we change the dynamic of the system by amont of 10% of its actual value, the tracking performance is upheld. Comparing our results with those of [30], we can see that better tracking performance is obtained in this paper. The form of the curves in control input of our study is smoother than in [30]. Thus, in conclusion, all signals in the closed-loop system are bounded, illustrating the effectiveness of the proposed technique.

7 Conclusion

The work considered in this paper consists of a robust synergetic based adaptive fuzzy controller designed for a class of nonlinear discrete-time systems. Stability analysis is based on Lyapunov theory, we have shown that all the signals in the closed-loop system are bounded and the tracking error is very small. As future work, we will engage to realize a state observer in the case, where the system states are not all available, and we will use the same algorithm for chaotic nonlinear systems such as Henon chaos system.

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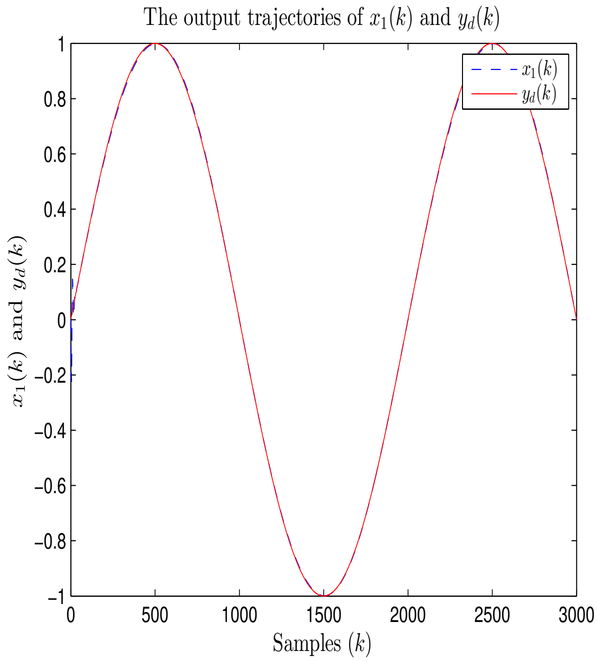

Figure 1: Evolution of output trajectories of $x_1(k)$ and $y_d(k)$.

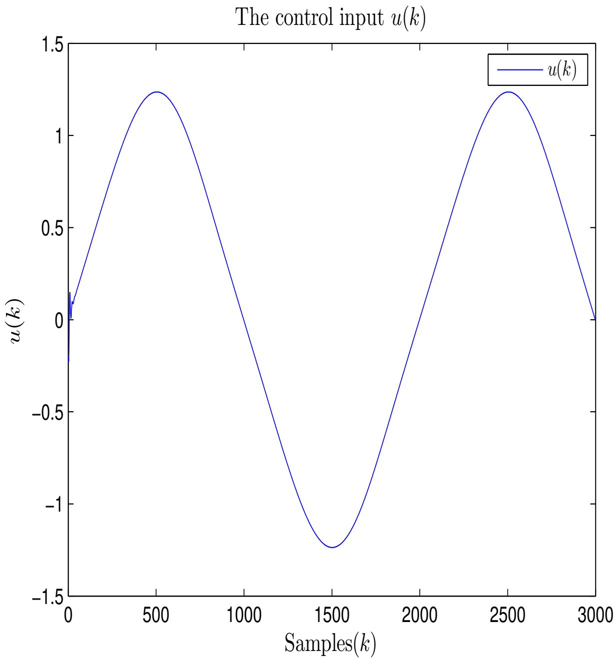

Figure 2: Control input evolution $u(k)$.

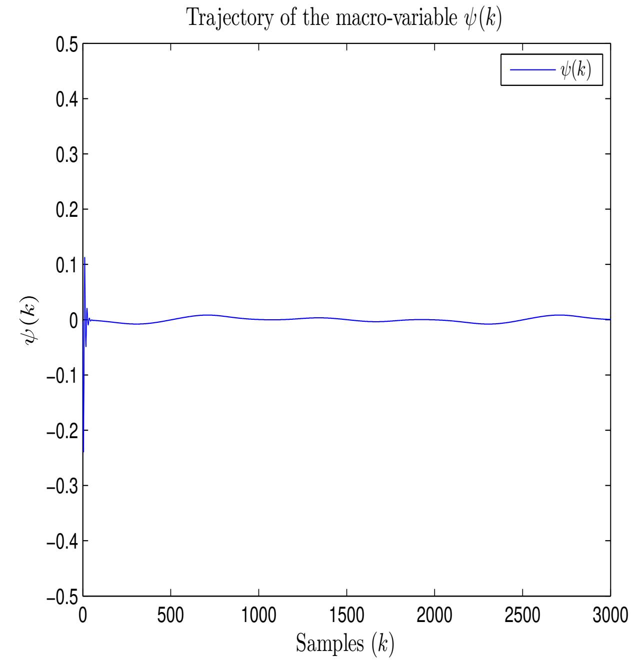

Figure 3: Trajectory of the Macro-variable $\psi(k)$.

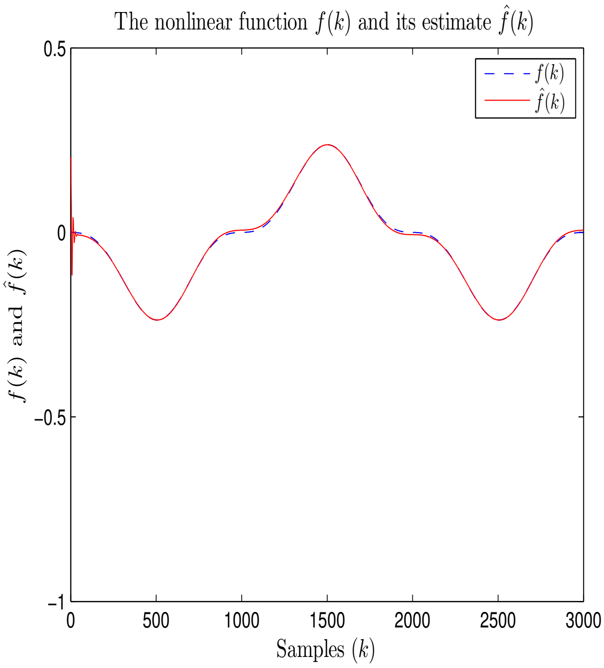

Figure 4: The nonlinear function $f(k)$ and it's estimation $\hat{f}(k)$.

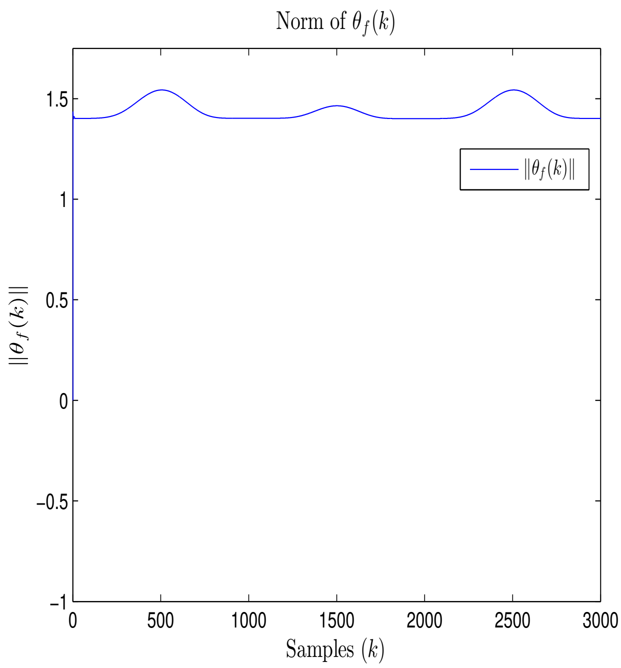

Figure 5: Norm of $\theta_f(k)$.

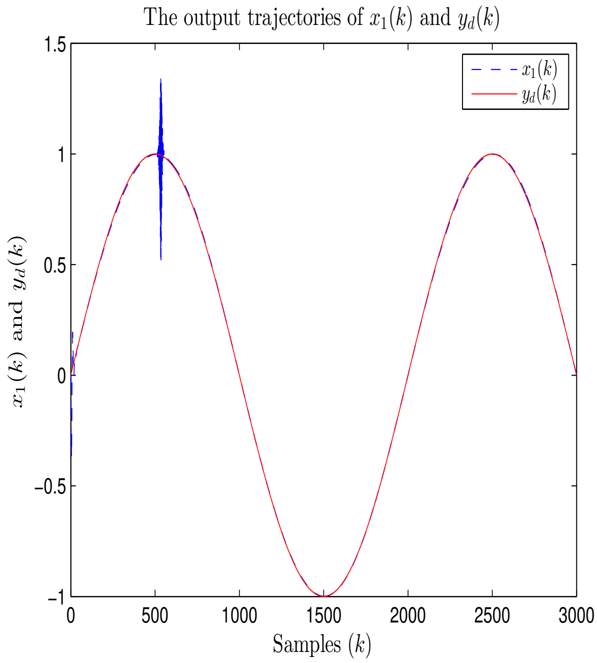

Figure 6: parametric uncertainties $f(k) + \Delta f(k)$.