Single-antenna projection algorithm for Mode S based airport traffic surveillance

Emilio Piracci, Nicolas Petrochilos, Gaspare Galati

Single-antenna projection algorithm for Mode S based airport traffic surveillance

E. G. Piracci[1], N. Petrochilos[2], G. Galati[1]

1 Tor Vergata University of Rome, DISP, via del Politecnico 1, 00133 Rome Italy and Centro Ricerche Radar

2 University of Hawaii at Manoa, 2540 Dole Street, Honolulu, HI 96822

E-mail: piracci@disp.uniroma2.it petro@ieee.org g.galati@ieee.org

***Abstract*—Within the Surveillance function in the A-SMGCS frame, Multilateration (MLAT) systems are being more and more used for location and identification of aircraft and of cooperating vehicles [1] on the airport surface. This situation, with increasing traffic in the downlink SSR channel, may lead to a too high number of superimposed, i.e. garbled, signals.**

The aim of this work is the need for reliable, simple effective algorithms to separate overlapped SSR signals. We proposed earlier some solutions [2,3], as well others can be found in the literature [5,6]. All of them require an array antenna at the receiver stations, while today's receiver stations are equipped with only one omni-directional antenna.

In this paper, the sources separation algorithm based on array processing (PA: Projection Algorithm) is adapted to a single-antenna configuration, where the variation of carrier frequencies of different vehicular transponders [8] is exploited. Simulations with signals synthesized from real measurements give results that demonstrate the effectiveness of the algorithm in a frequency agility environment.

***Index Terms*—Secondary Surveillance Radar, Mode S, Multilateration system, Source separation, Projection algorithm**

I. Introduction

Location and identification of cooperating aircraft in the airport area (and beyond) may be implemented by multilateration (MLAT) systems. These systems are based on a network of receiving stations and use the Secondary Surveillance Radar (SSR) Mode S signals. The SSR Mode S signals are pulse position modulated finite-length at a carrier frequency of 1090 MHz (see Sect. II). The Multilateration systems rely on the estimation of time of arrival of the emitted signal from the target, to perform intersection of hyperbolic surfaces. Using non-avionic requirements transponders (i.e. "non transponder devices") it is possible to extend the surveillance to cooperating vehicles [1]. Long range distributed receive stations allows to perform a wide area multilateration (WAM) extending the surveillance area. The two enhancements, i.e. vehicular applications and WAM, increase the number of received replies per unit time. Most of these signals, spontaneously emitted from on-board Mode S transponders (and called "squitter"), arrive randomly at the receiving stations, with an arrival density that is expected to increase in the future. The use of omni-directional antennae at the receive stations increases the probability of overlapped Mode S signals in the time domain. When replies overlap, very often the reply message is corrupted and cannot be recovered, nor the aircraft can be located and identified.

Source separation by signals acquisition from an array antenna is well investigated [2,3]. However today MLAT receive stations are equipped with a single omni-directional antenna/receive channel. In the following, we describe our main contribution, i.e. the adaptation of the PA (projection algorithm) [2,4] for the case of a single antenna receiver. The PA allows us to separate superimposed Mode S replies by the use of spatial filtering, as the overlapped signals originate from different angles of arrival. We remind that for SSR signals there is a tolerance of ±1 MHz centred at 1090 MHz [9,10], and that propose to use frequency shifted signals for vehicles [8]. One antenna receiver cannot exploit the spatial diversity, therefore our modified algorithm uses the residual carrier frequency of the signals, and so the phase accumulation sample by sample as a "time diversity". In short, the novel idea is based on the re-shaping of the acquired data, and on the use of the frequency diversity of the overlapped signals to perform the source separation.

This paper deals with the problem of only two equal-length Mode S replies overlapping in time. We first present (Sect. II and III) the used data model and the PA algorithm for an array antenna system. In sect. IV we show the data model introduced to adapt the PA algorithm to the mono-antenna case. Finally (Sect. IV) we present a preliminary analysis on the algorithm performances and some study cases to show the source separation capability.

II. DATA MODEL

We use a model for Mode S reply. A mode S reply contains 56 (short) or 112 (long) binary symbols $\mathbf{b}[n]$. The symbol period is 1 μs, and each symbol is made up by two 0,5 μs chips. The bit are encoded as the Manchester scheme, that is the b_n=0 is coded as [0,1], and the b_n=1 is coded as [1,0]. A reply is formed by a preamble, $\mathbf{p}=[1,0,1,0,0,0,0,1,0,1]$, followed by the encoded data, $\mathbf{b}=[\mathbf{p},0,0,0,0,0,0,\mathbf{b}_1,\mathbf{b}_2,\ldots,\mathbf{b}_n]$. The Mode S reply emitted by the transponder has the form:

$$b(t)=\sum_{n=0}^{127/239}\mathbf{b}[n]\,p(t-nT)$$

where *p(t)* is a rectangular pulse of width T= 0,5 μs. For transmission the signal is up-converted to the nominal frequency f_n=1090 MHz, with a ±1 MHz of tolerance permitted to the airborne transponder. At the ground receiver, after the down-conversion to base band, a residual frequency f_r remains, adding a phase rotation to the transmitted symbols. The received base band signal is: $s[n]=b[n]\exp(j\,2\pi\,n\,f_r\,T)=b[n]\phi^n$, where:

$$\phi=\exp(j\,2\pi\,f_r\,T) \qquad \textbf{(2.1)}$$

is the phase shift due to the residual carrier frequency over a sampling period.

We consider an *m* elements array antenna and *d* independent source signals. The base band signals are sampled at frequency f_s and stacked in vectors $\mathbf{x}[n]$ (size *m*). After collecting *N* samples, the observation model is:

$$\mathbf{X}=\mathbf{M}\,\mathbf{S}+\mathbf{N},$$

where $\mathbf{X}=[\mathbf{x}[1],\ldots,\mathbf{x}[N]]$ is the $m\times N$ received signal matrix. $\mathbf{S}=[\mathbf{s}[1],\ldots,\mathbf{s}[N]]$ is the $d\times N$ source matrix, where $\mathbf{s}[n]=[s_1[n],\ldots,s_d[n]]^T$ is a stacking of the *d* source signals. $\mathbf{N}$ is the $m\times N$ noise matrix, whose elements are temporally and spatially white. $\mathbf{M}$ is the $m\times d$ mixing matrix that contains the array signatures and the complex gains of the sources. We assume that the replies are independent (so uncorrelated) and $\mathbf{M}$ is full column rank. Finally, we assume $d<m$.

III. PA PROJECTION ALGORITHM

In this work we consider the case of two sources, i.e. two overlapping Mode S signals of equal length. (see Fig. 1).

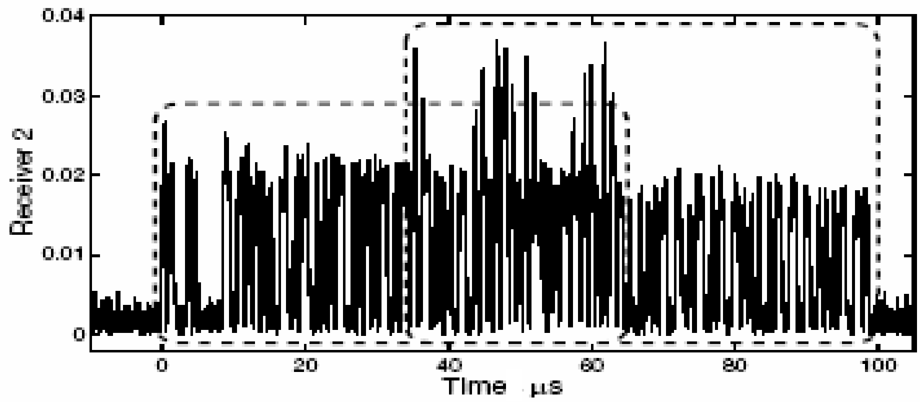

Fig. 1: A record of two Mode S overlapped replies (case w5)

If the two Mode S replies have significantly different times of arrival, t_1 and t_2, the time support of the two sources is partly overlapping. Then at the beginning of the data record (t_1 till t_2) there is only one source present; and at the end of the data (t_3 till t_4), only the second source is present.

The first step of the algorithm consists in the detection of the t_i's. The data resulting from signal sampling at f_s=50 Ms/s is sliced by time slots of 200 samples (4 μs), on each time slot is performed a whiteness test based on the Singular Value Decomposition (SVD), see Fig. 2, of the matrix **X**.

This allows us to estimate the presence of the sources as a function of time, and to isolate the two time support when each source is single.

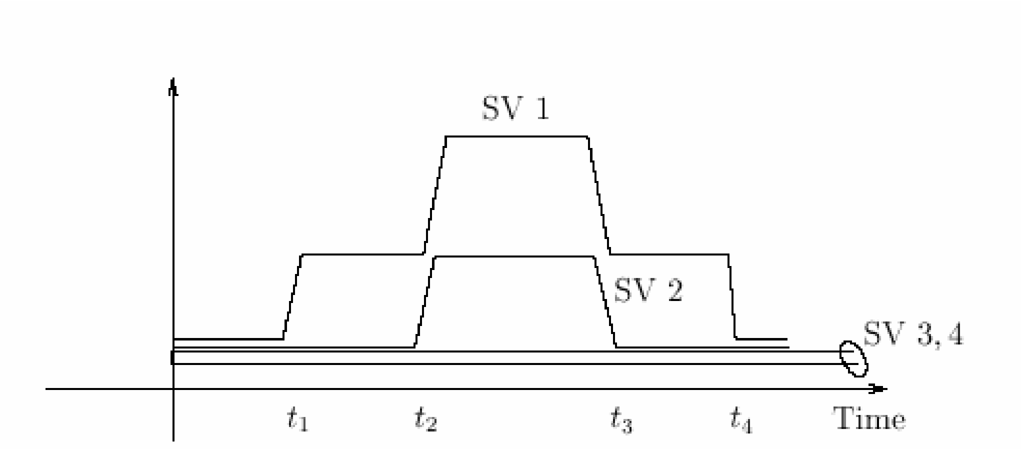

Fig. 2: The singular values as function of time

By the notation $(.)^{(1)}$ we indicate the matrix collecting the subset of the columns related to the time interval t_1 till t_2 (selection of the columns). Similarly, we define the notation $(.)^{(2)}$ for the subset of the columns related to t_3 till t_4. Hence, we have the following relationship:

$$\mathbf{X}^{(1)} = \mathbf{M}\,\mathbf{S}^{(1)} + \mathbf{N}^{(1)}$$
$$\mathbf{X}^{(2)} = \mathbf{M}\,\mathbf{S}^{(2)} + \mathbf{N}^{(2)}$$

where the matrix $\mathbf{S}^{(1)}$ is the sub-matrix of **S** containing the samples in $[t_1, t_2]$. Therefore $\mathbf{X}^{(1)}$ contains only the first source and can be simplified as (resp. for $\mathbf{X}^{(2)}$):

$$\mathbf{X}^{(1)} = \mathbf{m}_1\mathbf{s}_1^{(1)} + \mathbf{N}^{(1)}$$
$$\mathbf{X}^{(2)} = \mathbf{m}_2\mathbf{s}_2^{(2)} + \mathbf{N}^{(2)}$$

where the $\mathbf{m}_i$'s are the columns of **M**, and the $\mathbf{s}_i$'s the rows of **S**.

Note that $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are rank-one matrices in the noiseless case. By SVD on $\mathbf{X}^{(i)}$, we can estimate the main vector $\hat{\mathbf{m}}_i$, which is the vector corresponding to the highest singular value (i=1, 2).

Once the space signatures $\hat{\mathbf{m}}_1$ and $\hat{\mathbf{m}}_2$ have been identified, the matrix $\hat{\mathbf{M}}$ is thus estimated. We finally multiply **X** by the Moore-Penrose pseudo-inverse of $\hat{\mathbf{M}}$ ($\mathbf{M}^{\dagger} = \left(\mathbf{M}^{H}\mathbf{M}\right)^{-1}\mathbf{M}^{H}$), and recover the estimated sources:

$$\hat{\mathbf{S}} = \mathbf{M}^{\dagger}\mathbf{X}.$$

IV. MONO ANTENNA ADAPTATION

The adaptation of the PA to the single-antenna case is achieved by a rearrangement of the received data: from a time series originated by a single antenna/receiver consisting of *N* samples, we construct a matrix of size $m \times l$, where the first column of this matrix contains the first *m* data samples, the second column the next *m* samples, and so forth, *l* being to the rounded part of *N/m*. An example can be seen in Fig. 3, where the principle is shown for a single reply.

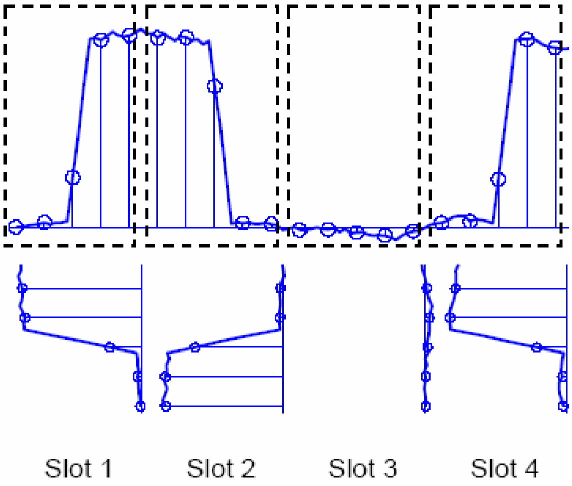

Fig. 3: How to perform the transformation x[n] to X

By doing so, we aim at recovering our previous model: $\mathbf{X} = \mathbf{M}\,\mathbf{S} + \mathbf{N}$. Meanwhile, there are two major differences: first, as shown in example from Fig. 3, it is obvious that this signal will need two components to be described: the one from slot 1 and the one from slot 2, slot 3 being only noise, and slot 4 being equivalent to slot 1. It appears then that a source is not anymore described by a rank-one matrix. In [4], we have seen that the rank of this matrix for a single reply is equal to m (the number of lines), with a notable exception: when m is set equal to number of samples in a pulse (i.e. in half a symbol), then a special case arises and two components only are needed. As in the example from Figure 3, the rank of matrix $\boldsymbol{M}$ is 2 (in the noiseless case) instead of m. We define the rows of the matrix $\boldsymbol{M}$ as the signature vectors (in conventional array processing, they are called array signature vectors). By extension it appears that for $m = 25 \times p$ (corresponding to time slices integer multiple of 0.5 μs.), the number of component needed is $2p$.

Second, if we look more carefully at the matrix $\boldsymbol{M}$, we note that a column is not originated from space diversity anymore, but by frequency diversity, Indeed, if the initial sampling would be synchronized with a pulse, a vector m would in fact contains the accumulated phase shift ϕ, described in eq. **2.1**: $m = [1\ \phi^2 \ldots\ \phi^{m-1}]^T$. Given that the frequency shift with respect to the nominal value is a kind of signature for each reply, and frequency shifts are variable from one reply to the other one, our technique generates, with several sources, a matrix $\boldsymbol{M}$ which is full column rank. Nevertheless the conditioning number directly depends on the choice of m and, of course, on the difference between the frequency shift. It is easily understood that small values of m causes high conditioning numbers. The consequence of a high conditioning number is that the pseudo-inverse is not robust, i.e. the noise contribution increases dramatically. Indeed, we observe that to separate two overlapped pulses, the needed conditioning number of 10, when the two frequencies have a difference of 150 kHz, the length m has to be over 35 [4].

Next to the generation of the model, out technique propose a projection over the subspace spanned by the matrix $\mathbf{M}$, thus reducing the noise by removing the noise subspace and separating the contributions of the two sources into two matrices $\mathbf{X_1}$ and $\mathbf{X_2}$, the last step is to apply the reverse rearrangement, in order to produce the time series s_1 and s_2.

Next, in this simulated example two overlapping Mode S replies, with the standard length of 64 μs, an amplitude ratio equal to 0.6 (i.e. the leading reply has an amplitude 60 % of the trailing one) and a time delay of 33 μs are received on a **single antenna.** The SNR is 85 dB, and the residual frequency shifts are equal to -100 and +50 kHz respectively. Figure 4 shows the absolute values of the matrix **X** displayed column-wise.

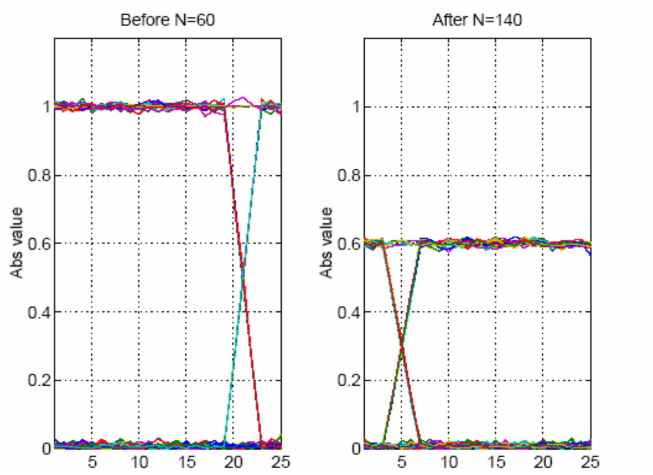

Fig. 4: Plot of the columns of submatrix of X, left: before *n*= 60 (i.e. 30 *μs*), right: after *n*= 140 (i.e. 70 *μs*) Where *n* is the column number, and *m*= 25

We observe that we need only 2 or 3 signature vectors to represent the pulses of each reply (and not *m*). In this example, 5 signature vectors only are needed for both sources (3+2) to create a projection matrix that reach a very acceptable separation, (see Fig. 5). The conditioning number of the estimated **M** was 36.

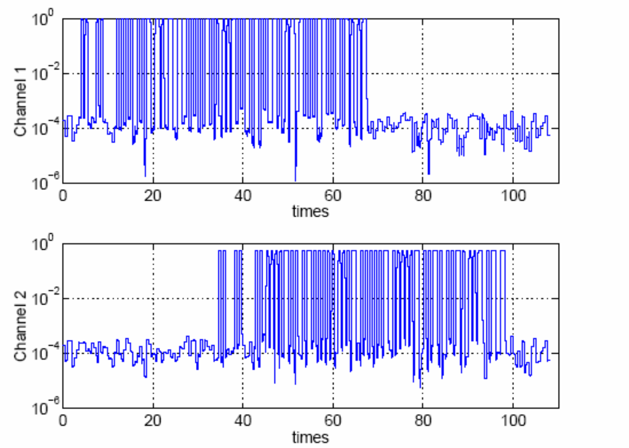

Fig. 5: Result of the separation of the synthesized case in logarithmic scale

V. EXPERIMENTAL PERFORMANCES

In this section we present the results of performance evaluation tests with semi-synthesized signals. The analysis is done using real-world SSR signals, received and recorded (at 50 Mega samples per second) by means of an ad-hoc system implemented by the Technical University of Delft (CAS/IRCTR). This system is made up by a four-elements receive array connected to a wide band acquisition system, but of course in this study we used one channel only. We mean by the term “semi-synthesized” a real signal modified after the recording, in order to change the carrier frequency and the SNR of each source. Choosing a time delay we apply a coherent sum of the two sources to obtain an overlapping case very close to a real case. The carrier frequency variation becomes a residual frequency shift after the base-band translation.

We generated the overlapped signals in order to appreciate the algorithm behavior versus the frequencies shifts and the value of *m*, equal to {25, 50, 75}. The evaluations were performed in the following conditions. First, we selected replies allowing to obtain a signal-to-noise ratio (SNR) of at least 20 dB. Second, the delay of the trailing reply with respect to the leading reply was set to 30+ε *μs* where ε is a random variable uniformly distributed in [-0.5, +0.5] *μs*. Third, the frequency shift between both replies was set between 0.1 and 1 MHz with 10 steps by 0.1 MHz each. Overall, we got about 1560 cases.

A. Analysis of the conditioning numbers

Figure 6 shows the measured probability density of the conditioning number's of the **M** matrix for each semi-synthesized signal as a function of *m*. As we did expect, there is an improvement for the conditioning of **M** when *m* increases. The "rule of the thumb" for the conditioning number says that to obtain a good separation the conditioning number must be less than 10 for SNR equal or above 20 dB. Figure 6 shows that over 1560 superimposed signals, only approximately 200 signals present a conditioning number for the matrix **M** less than 10 (using *m*=75). Table 1 presents, for each *m*, the statistical properties of the conditioning number's (mean and median) and the percentage of conditioning number's less than 10.

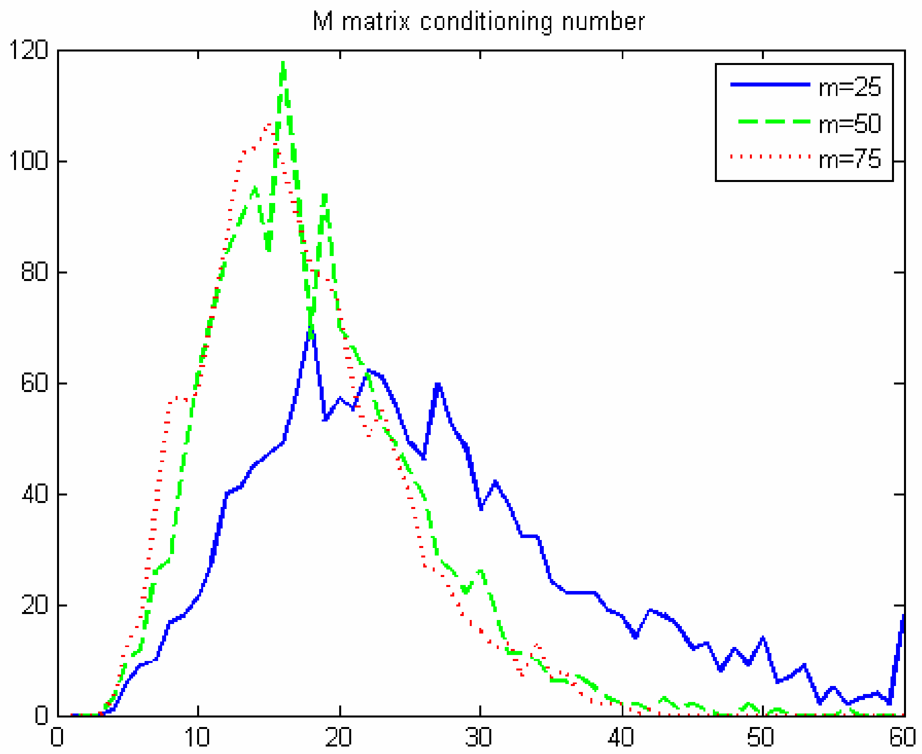

Fig. 6: Conditioning number density of M for *m* = {25, 50, 75}

	m=25	m=50	m=75
mean	26	18	17
median	24	17	16
% of cond(**M**)<10	4 %	8 %	11.7 %

Table 1: Conditioning number of M for *m* = {25, 50, 75}: statistics

B. Effect of frequency shift

We compare the PA with the conventional decoding algorithm presently used in SSR receivers. Fig. 7 shows the failure rate (i.e. the fraction of cases when reply decoding failed, [3]) as a function of the frequency shift between the overlapped replies: at the left, shows the failure rate for detection of the leading reply, at the right shows the failure rate for detection of the trailing reply. In the case of the trailing reply, the failure rate is equal to one for the conventional decoder which, in absence of a sources separation function upstream, cannot work because of the garble of the preamble due to the leading reply. We evaluated the effect of the *m* value on the source separation and decoding. Figure 7 shows that the failure rate decrease as *m* increases. The results with *m* equal to 50 or 75 are quite similar but there is a lot of difference using *m* equal to 25. In any case we found a much better performance of the PA with respect to the conventional decoder.

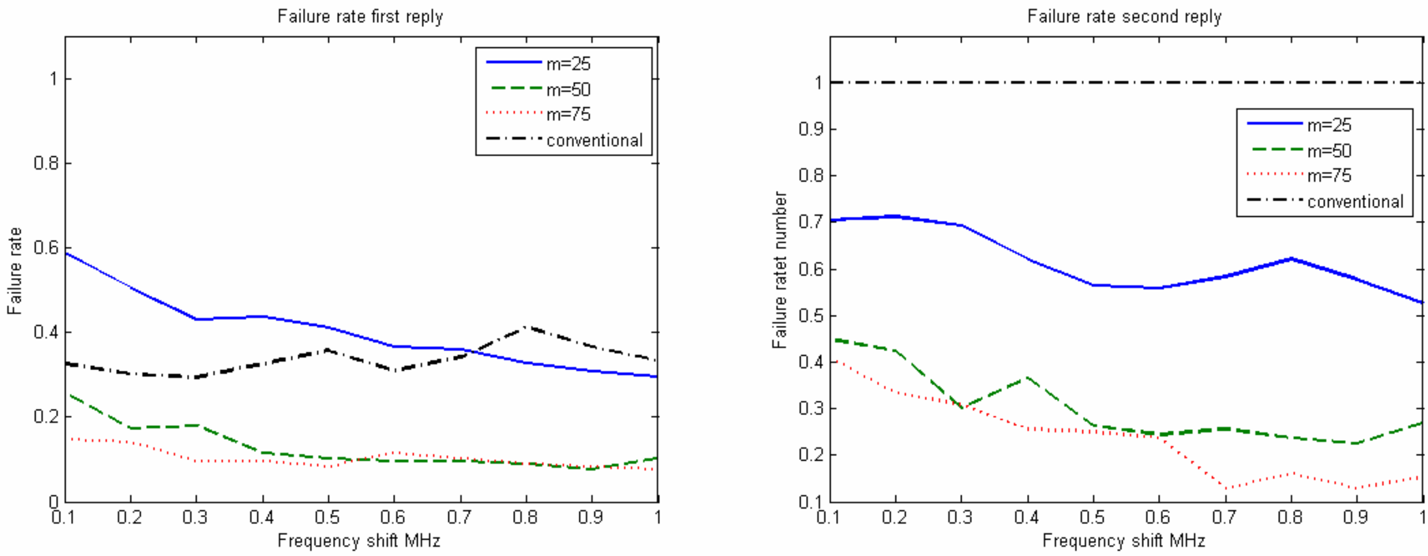

Fig. 7: Failure rate as a function of the frequency shift

Figure 8 present the average error bit number for reply [3], comparing PA with conventional algorithm: as in the previous case the trailing reply is not detectable using the conventional decoder. The results are presented as a function of the frequency shift and for *m* equal to {25, 50, 75.

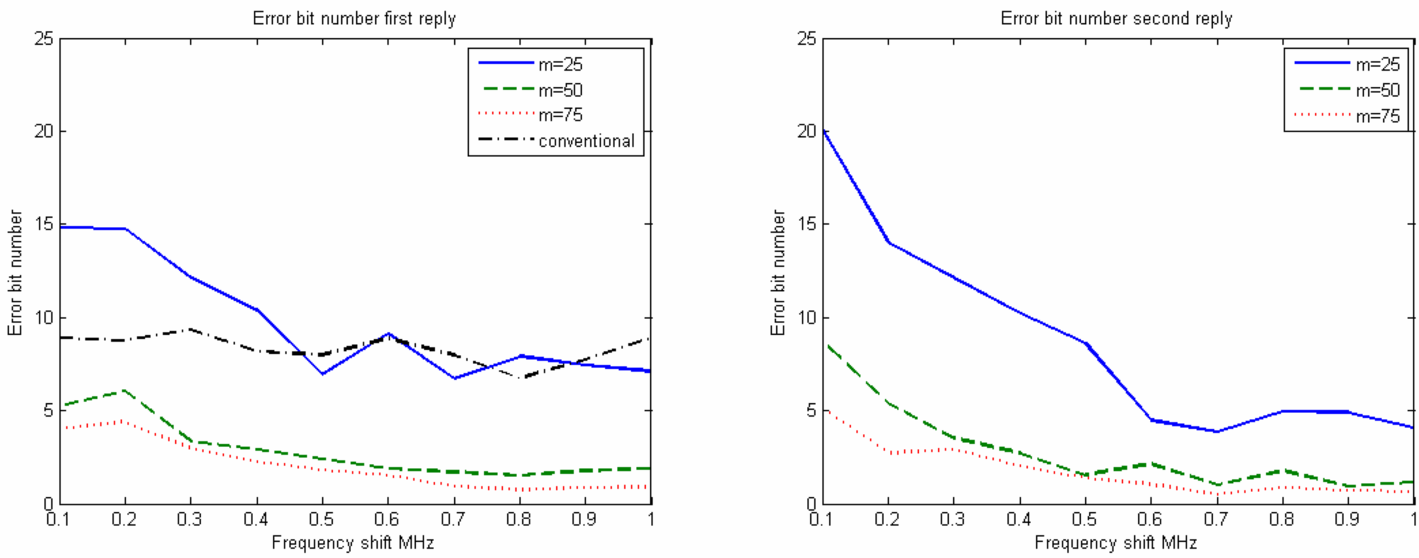

Fig. 8: Average error bit number as a function of the frequency shift

The error bit number per reply decreases as the frequency shift increases. As for the failure rate analysis, setting *m* equal to 50 or 75 gives quite similar results, but there is a significant performance degradation using *m* equal to 25.

VI. CONCLUSION

In this article, we presented an adaptation of the PA for mono-antenna receiver, by the application to the data of a “reshaping” into a matrix form. In which case, the frequency diversity between the replies is naturally exploited, and we advocate to force this diversity to the cooperating vehicles found on the airport grounds. With our preliminary study, we demonstrate the improvement to overcome the inevitable garbling by using mono-antenna-PA. The results evidence that to obtain a satisfactory separation it is necessary set m at least equal to 50, while it is not clear if the improvement by m equal to 75 is worth the computational cost. Future studies will allow to evaluate the performances of our algorithm under various operating conditions.

ACKNOWLEDGMENT

The contribution by the Radar Research Centre (CRR: Centro Ricerche Radar) is gratefully acknowledged.

REFERENCES

[1] G. Galati, P. Magarò, E.G. Piracci, L. Cicciotti, “Multilateration applied to airport vehicles management systems: the Agile Trasponder”, in *Proc. of EuMW 2006 - EuRAD*, (Manchester, GB), 12-15 September 2006.

[2] N. Petrochilos, G. Galati, E. G. Piracci, “Projection techniques for separation of multiple secondary surveillance radar sources in a real environment”, in *Proc. of IEEE SAM 2006*, (Waltham, MA), 12-14 July 2006.

[3] N. Petrochilos, G. Galati, L. Mené, E. Piracci, “Separation of multiple Secondary Surveillance Radar sources in a real environment by a novel projection algorithm”, *in Proc. of IEEE ISSPIT 2005*, (Athens, Greece), 17-21 December 2005.

[4] N. Petrochilos, “SSR receiver: A real implementation”, research report, Volterra Center, University of Rome “Tor Vergata”, February 2006

[5] N. Petrochilos, A.J. Van der Veen, “Algorithms to separate overlapping secondary surveillance radar”, in *Proc. of IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP 2004*, (Montreal Canada), pp. II.49-53, 17-21 May 2004.

[6] E. Chaumette, P. Comon, D. Muller, “An ICA-Based Technique for Radiating Sources Estimation; Application to Airport Surveillance", *IEE Proceedings - Part F*, vol. 140, pp. 395-401, December 1993.

[7] John G. Proakis, Dimits G. Manolakis, “Digital Signal Processing”, Prentice-Hall International Inc. 1996

[8] G. Galati “High–capacity location and identification system for cooperating mobiles with frequency agile and time division transponder device on board” Assignee: Università degli Studi di Roma "Tor Vergata"; International Patent Application PCT/IB2005/05 3343, 11 October 2005.

[9] M.C. Stevens “Secondary Surveillance Radar”, Chapt. 12, Artech House, 1988

[10] International Civil Aviation Organization “Annex 10 – Aeronautical Telecommunication”