Fractional-Order Adaptive Fuzzy Backstepping Algorithm for Automated Control of Blood Glucose in Diabetes Mellitus Type 1 Patients

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Abstract: This study proposes a fractional-order control for the automatic stabilization of the blood glucose level in Diabetes Mellitus Type-1 patients. The control scheme under study, which is applied to a fractional Extended Bergman's minimal model (EBMM), combines the feedback form of backstepping technique with three fractional Takagi–Sugeno fuzzy systems. This allows getting rid of the requirements of the patient parameters knowledge and aims to attenuate the effect of disturbances to the blood glucose level during meal intake. A mathematical analysis has been done to prove the asymptotic stability using a fractional Lyapunov function. To demonstrate the effectiveness of the fractional proposed approach, a comparative study with other methods is presented.

Keywords: Fractional-order calculus, Backstepping, Adaptive Takagi-Sugeno fuzzy logic system, Lyapunov, Diabetes mellitus type 1.

1. INTRODUCTION

Diabetes is a serious threat to global health which increasing at a very high rate. This metabolic disease, which is characterized by hyperglycemia resulting from defects in insulin secretion, insulin action or both, can lead to frequent hospital admissions and premature death (Vettoretti et al. (2017), DiMeglio et al. (2018), Association et al. (2014)).

According to the current classification there are two major types: type 1 and type 2 diabetes. The distinction between the two types is mainly based, among other symptoms, on degree of loss of β cell function and the degree of insulin resistance (Vettoretti et al. (2017), DiMeglio et al. (2018), Association et al. (2014)). Treatment for the first type is focused on infusing insulin to stabilize the blood-glucose level, while the second type is treated via an appropriate medications. Regarding the insulin, it needs to be taken by injection or another delivery means such as by infusion via an insulin pump for which a controller is designed using the measurements from continuous glucose monitoring (CGM).

Thus, several methods including the proportional–integral Derivative (PID) controllers (Farman et al. (2018), Patra and Rout (2015)), Linear Quadratic Gaussian (LQG) control algorithm (Patra and Rout (2015)) and combined PI -Fuzzy techniques (Beneyto and Vehi (2018), Soylu and Danisman (2018)) are adopted into the control process. In terms of convergence time and steady state errors in steady, these linear methods uses some simplifying assumptions with partial model approximation supposed to reflect a non-linear glucose-insulin dynamics.

Reason why nonlinear approach is more appropriate to deal with complex practical scenarios and reach the needed level of performance. Among them the Sliding Mode Control (SMC) (Abu-Rmileh et al. (2010), Menani et al. (2017)) , super twisting SMC (Ahmad et al. (2017), Djouima et al. (2018)) , Terminal fuzzy-SMC (Jajarm et al. (2012)), $H\infty$ Controller (Duangpim and Assawinchaichote (2016)), and the backstepping approach (Babar et al. (2019), Ahmad et al. (2019)). This latest is a recursive method for designing stabilizing control laws in its strict feedback form.

In addition, the fractional calculus deals with differentiation and integration operators in an appropriate order not necessarily integer. Its application in the design of the controllers provides a more accurate description of complex dynamic systems and improves the control performances from different aspects (Krishnan and Jayakumar (2018), Ortigueira (2011), Delavari et al. (2018)). Thus, the fuzzy logic based backstepping techniques are proved to be successful in controlling various processes: such as wind energy conversion systems. In (Aounallah et al. (2018)) , the combination of fractional robust algorithm with the universal approximator for wind generators improves the control performance and overcome the constraint of model parameters knowledge. These facts motivated us to extend and adapt such approach to the medical field requirements and specificities.

In this work, we propose a fractional adaptive Takagi–Sugeno fuzzy logic backstepping algorithm for automated control of blood glucose in Type 1 Diabetic Patients. The control scheme is designed for an Extended Bergman's minimal model (EBMM) (Bergman et al. (1981)). Fuzzy systems can overcome the constraint of the model parameters knowledge and the adaptive approach allows to attenuate the effect of both fuzzy logic approximation errors and the external disturbances (Essounbouli

et al. (2002a)). Adaptive fuzzy systems allow to overcome the constraint of the model parameters knowledge (Aounallah et al. (2018), Mendel (2001)). The global stability is obtained by a fractional Lyapunov function ((Chen et al., 2014)).
Our main objective is to exploit the performance of the fractional-order theory to obtain good stabilization of blood glucose level in presence of external disturbances (meal intake). Simulation results, via matlab /Simulink environment, demonstrate the efficiency of the fractional proposed method.
The paper is organised as follows: Section 2 describes the fractional Extended Bergman's minimal model. Section 3 is dedicated to the conventional backstepping control. Section 4 concerns the proposed controller design. Section 5 shows the simulation results and the comparative study. Finally, the conclusion is given in section 6.

2. FRACTIONAL EXTENDED BERGMAN'S MINIMAL MODEL (FEBMM)

2.1 Fractional method

The mathematical expression of the fractional-order differential integral operator is given as follows:

$$ {}_{t0}D_t^m = \begin{cases} \frac{d^m}{dt^m} & \Re(m) > 0 \\ 1 & \Re(m) = 0 \\ \int\limits_{t_0}^{t} (dt)^{-m} & \Re(m) < 0 \end{cases} \quad (1)$$

Whith:

- $m \epsilon \mathbb{C}$:is a fractional order.
- $\mathbb{R}(m)$: is the real part of m.
- t_0 and t : is the fractional operator domain limits.

The mathematical Riemann-liouville definitions of fractional derivatives and integrals are given by the following equations:

a) Fractional-order integral:

$$ {}_{t_0}^{RL}I_t^m f(t) = \frac{1}{\Gamma(m)} \int\limits_{t_0}^{t} (t-\tau)^{m-1} f(\tau) d\tau \quad (2)$$

With $\Gamma(.)$: is the Euler's gamma function.

$$ \Gamma(x) = \int\limits_{0}^{\infty} y^{X-1} e^{-y} dy \ , \ x > 0 \quad (3)$$

b) Fractional-order derivative:

$$ {}_{t_0}^{RL}D_t^m f(t) = \frac{1}{\Gamma(n-m)} \frac{d^n}{dt^n} \int\limits_{t_0}^{t} (t-\tau)^{n-m-1} f(\tau) d\tau \quad (4)$$

Where n is an the integer number such that : $(n-1) < m < n$.

2.2 FEBMM description

The non-linear extended model for type 1 diabetic patients has been proposed in (Bergman et al. (1981)). Meal disturbance to the blood glucose level which was considered as fixed value in BMM are defined as a fourth state of the system given as:

$$ \begin{cases} D^m x_1 = -p_1 (x_1 - G_b) - x_1.x_2 + x_4 \\ D^m x_2 = -p_2 x_2 + p_3 (x_3 - I_b) \\ D^m x_3 = -p_4 (x_3 - I_b) + U(t) \\ D^m x_4 = -p_5 x_4 \end{cases} \quad (5)$$

where:
- $x1$, $x2$, $x3$ and $x4$: are glucose concentration, remote insulin concentration , plasma insulin concentration and Fisher's Meal disturbance respectively.
-G_b : is the basal plasma glucose .
-I_b: is the basal plasma insulin .
-p_1: is the glucose effectiveness factor .
-p_2: is the delay in insulin action .
-p_3: is the patient paramettter .
-p_4: is the insulin degradation rate .
-p_5: is the meal disturbance .

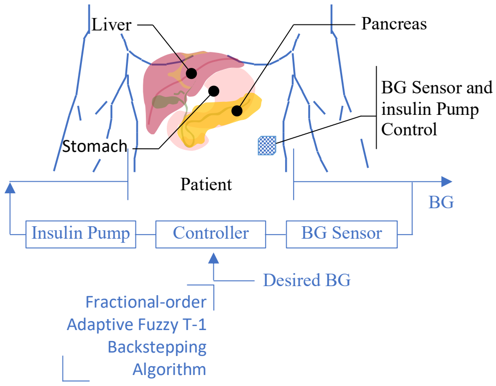

Fig. 1. Overall scheme of Automated Blood Glucose Control.

3. FRACTIONAL BACKSTEPPING CONTROL METHOD FOR EBMM

This section aims to find the appropriate external insulin infusion rate U(t), which can guarantee the asymptotic tracking of the desired blood glucose level. The proposed approach is based on a fractional order mathematical model where the control laws are deduced from the fractional Lyapunov stability analysis. The mathematical demonstration of the proposed control method is given according to the following steps.

3.1 First virtual control law

The synthesis of the first virtual control law is obtained via the expression of the following error:

$$ e_1 = x_1 - Y_{ref} \quad (6)$$

The Lyapunov function used for stability analysis is given by:

$$ V_1 = \frac{1}{2} e_1^T e_1 \quad (7)$$

Its fractional time derivative is :

$$D^m V_1 = e_1^T D^m e_1 \tag{8}$$

The virtual control law based on the desired state values is written as follows:

$$\begin{aligned} x_{2d} = x_1^{-1} & \left(-p_1 \left(x_1 - G_b\right) - D^m Y_{ref}\right. \\ & \left. + c_1 e_1 + k_1 sign(e_1)\right) \end{aligned} \tag{9}$$

A second tracking error, resulting from the expression of the first virtual control law, is added to the control process:

$$x_2 = x_{2d} + e_2 \tag{10}$$

Thus, the new fractional time derivative of the Lyapunov function is taken as:

$$D^m V_1 = -c_1 e_1^T e_1 - e_1^T x_1 e_2 - e_1^T \left(k_1 sign\left(e_1\right) - x_4\right) \tag{11}$$

The stability condition $D^m V_1 < 0$ is ensured as long as $c_1 > 0$ and $|x_4| < k_1$, with the residual term $e_1^T x_1 e_2$, must be compensated in the second step.

3.2 Second virtual control law

The first intermediate control law introduces a second error e_2 whose the fractional derivative is given as follows:

$$D^m e_2 = D^m x_2 - D^m x_{2d} \tag{12}$$

The fractional derivative of the new Lyapunov function is defined by:

$$D^m V_2 = e_1^T D^m e_1 + e_2^T D^m e_2 \tag{13}$$

The second virtual control law is taken as:

$$\begin{aligned} x_{3d} = p_3^{-1} & \left(p_2 x_2 + p_3 I_b + D^m x_{2d}\right. \\ & \left. + x_1.e_1 - c_2.e_2 - k_2 sign\left(e_2\right)\right) \end{aligned} \tag{14}$$

Therefore, a third error term is introduced:

$$x_3 = x_{3d} + e_3 \tag{15}$$

based on the third error term and using (14), the time derivative of $D^m V_2$ can be written as:

$$\begin{aligned} D^m V_2 = & -c_1 e_1^T.e_1 - e_1^T \left(k_1 sign(e_1) - x_4\right) - c_2 e_2^T.e_2 \\ & - e_2^T \left(k_2.sign\left(e_2\right)\right) + e_2^T p_3.e_3 \end{aligned} \tag{16}$$

Thus, the expression $D^m V_2$ is negative as long as $|x_4| < k_1$, $k_1 > 0$, $k_2 > 0$, $c_1 > 0$ and $c_2 > 0$, the residual term must be compensated in the third step.

3.3 Global control law

The design of the fractional feedback control U(t) results from (15)

$$D^m e_3 = D^m x_3 - D^m x_{3d} \tag{17}$$

The time derivative of the Lyapunov candidate function, which check the convergence of three errors e_1, e_2 and e_3 to zero, is :

$$D^m V_3 = e_1^T D^m e_1 + e_2^T D^m e_2 + e_3^T D^m e_3 \tag{18}$$

The control law which can ensure the asymptotic stability of the system, can be taken as:

$$\begin{aligned} U(t) = & \, p_4 \left(x_3 - I_b\right) + D^m x_{3d} - p_3.e_2 \\ & - c_3.e_3 - k_3.sign(e_3) \end{aligned} \tag{19}$$

Thus, the final expression of the Lyapunov candidate function is given as:

$$\begin{aligned} D^m V_3 = & -c_1 e_1^T e_1 - e_1^T \left(k_1 sign(e_1) - x_4\right) - c_2 e_2^T e_2 \\ & - e_2^T.k_2.sign(e_2) c_3.e_3^T.e_3 - e_3^T.k_3.sign(e_3) \end{aligned} \tag{20}$$

Finally, the control laws ensure the asymptotic stability of the system via a judicious choice of the parameters k_1, k_2, k_3,c_1, c_2 and c_3.

4. FRACTIONAL ADAPTIVE FUZZY BACKSTEPPING CONTROL METHOD FOR FEBMM

To overcome the problem of patient parameter knowledge, we propose to use adaptive fuzzy systems in the form $\xi_1^T \theta$ to calculate the control signal (Essounbouli et al. (2002b)). To design the new control laws, we will follow the same previous steps.

4.1 First virtual control law

The control law (9) becomes can be rewritten as:

$$x_{2d} = x_{2eq} + x_1^{-1} \left(c_1.e_1 + k_1.sign(e_1)\right) \tag{21}$$

where: $x_{2eq} = x_1^{-1} \left(-p_1(x_1 - G_b) - D^m Y_{ref}\right)$, which will be approximated by the fuzzy system: $\hat{x}_{2eq} = \xi_1^T \theta_1$.

ξ_1^T represents the regressive vector and θ_1 the adjustable parameter vector.
The new mathematical expression of the Lyapunov function is as follows:

$$V_1 = \frac{1}{2} e_1^T.e_1 + \frac{1}{2\gamma_1} \tilde{\theta}_1^T \tilde{\theta}_1 \tag{22}$$

Where γ_1 is the learning rate, $\tilde{\theta}_1 = \theta_1 - \theta_1^*$ is the estimation error and θ_1^* is the optimal value of θ_1.
Applying the fractional derivative to (22), we get the following expression:

$$\begin{aligned} D^m V_1 = & \, e_1^T \left(-c_1 e_1 - f_{e1} + x_1 e_2 + w_1 - \xi_1^T \tilde{\theta}_1\right) \\ & + \tfrac{1}{\gamma_1} \tilde{\theta}_1^T D^m \tilde{\theta}_1 \end{aligned} \tag{23}$$

Where $f_{e1} = k_1 sign(e_1) + \Delta_1$ and $w_1 = \hat{x}_{2d}^* - x_{2d}$ is the minimum approximation error..
Taking the following adaptation law:

$$D^m \theta_1 = \gamma_1 \xi_1 e_1 \tag{24}$$

Thus, an appropriate choice of c_1 allows to rearrange the equation (23) as follows:

$$D^m V_1 \leq -c_1 e_1^T e_1 + e_1^T x_1 e_2 \tag{25}$$

This mathematical equation includes a residual term $e_1^T x_1 e_2$, which will be compensated in the next step.

4.2 Second virtual control law

As indicated previously, a second adaptive fuzzy system is introduced:

$$\begin{aligned} x_{3d} &= p_3^{-1}\left(p_2 x_2 + p_3 I_b + D^m x_{2d} + x_1 e_1 \right. \\ &\left. -c_2 e_2 - k_2 sign(e_2)\right) \\ x_{3d} &= x_{3eq} + p_3^{1}\left(c_2 e_2 + k_2 sign(e_2)\right) \\ \hat{x}_{3eq_{eq}} &= \xi_2^T \theta_2 \end{aligned} \tag{26}$$

By performing a mathematical transformation similar to that of the previous step, the fractional derivative of the global Lyapunov function is given by:

$$D^m V_2 = D^m V_1 + e_2^T D^m e_2 + \frac{1}{\gamma_2}\tilde{\theta}_2^T D^m \tilde{\theta}_2 \tag{27}$$

In order to guarantee the asymptotic stability condition $D^m V_2 < 0$, we have:

$$D^m \theta_2 = \gamma_2 \xi_2 e_2 \tag{28}$$

and choosing an appropriate scalar value c_2, equation (27) becomes:

$$D^m V_2 < -c_1 e_1^T e_1 - c_2 e_2^T e_2 - p_3 e_2 \tag{29}$$

Thus, the subsystem stability depends on the parameter $p_3 e_2$, which will be compensated in next step.

4.3 Global control law

In accordance with the stability conditions of the feedback system and the approximation of the unknown parameters, the expression of the final control law is:

$$\begin{aligned} U(t) &= p_4\left(x_3 - I_b\right) + D^m x_{3d} \\ &+ p_3 e_2 - c_3 e_3 - k_3 sign(e_3) \\ &= U_{eq} - c_3 e_3 - k_3 sign(e_3) \\ \hat{U}_{eq} &= \xi_2^T \theta_3 \end{aligned} \tag{30}$$

Proceeding as before , the fractional derivative of the global Lyapunov function is:

$$D^m V_3 = D^m V_1 + D^m V_2 + e_3^T D^m e_3 + \tfrac{1}{\gamma_3}\tilde{\theta}_3^T D^m \tilde{\theta}_3 \tag{31}$$

where $w_3 = \hat{U}_{eq}^* - U_{eq}$.

To guarantee the stability that involves $D^m V_3 < 0$, we us the following adaptive law:

$$D^m \theta_3 = \gamma_3 \xi_3 e_3 \tag{32}$$

After some mathematical transformations and with an adequate selection of the parameters c_1, c_2 and c_3, we get this inequality:

$$D^m V_3 < -c_1 e_1^T e_1 - c_2 e_2^T e_2 - c_3 e_3^T e_3 \tag{33}$$

This mathematical approach of adaptation laws, deduced in the sense of Lyapunov, provides a demonstration of the global stability system.

5. SIMULATION AND RESULTS

In this section, we propose to evaluate the proposed approach via MATLAB/Simulink environment. These stimulation tests are based on a complete comparison between integer and fractional-order control schemes. Noted that the selected reference level for blood glucose is (80 mg/dl), reflecting an optimal value of blood glucose level. The model parameters are the same those given by (Ahmad et al. (2019), Nath et al. (2019)). Analysis of system response is obtained during meal intake assuming that the patient is initially in the state of hyperglycemia. The results highlighted the improvements in the considered control system (see Fig. 2). These enhancements concern the convergence time (from 0 to 50%), oscillatory response and steady state error (Table 1). The generated control signals are shown in Figure 3. The algorithm calculates the quantity of insulin required to avoid oscillations in the system response and reaches a maximum value that causes an instant decrease in glucose level. The improvement of the proposed control performance are also validated via a comparison with SMC which is another nonlinear method (see Figs. 4 and 5). Finally, the proposed control method presents better performances not only in terms of blood glucose regulation and insulin infusion rate, but also concerning the computation times online (see Fig. 6) which decreases by 68 %in this approach compared to its integer analog approach.

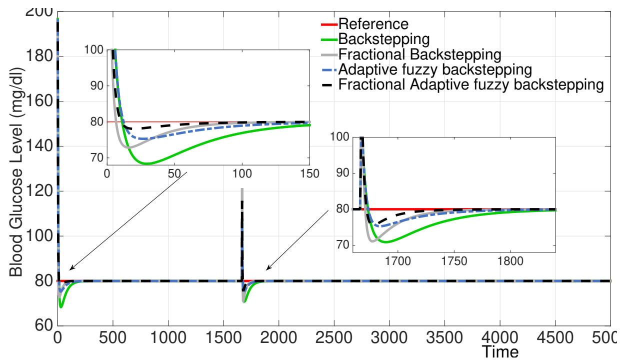

Fig. 2. Blood glucose response analysis with backstepping based approaches

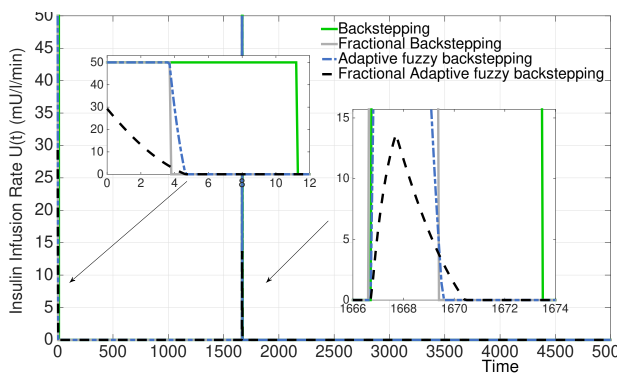

Fig. 3. Control signal of Backstepping based controllers

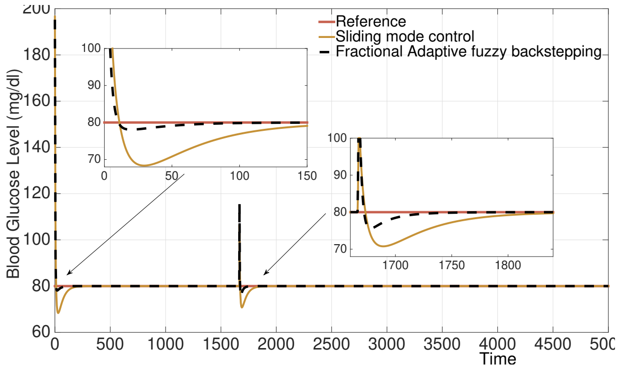

Fig. 4. Blood glucose analysis comparison by fractional adaptive fuzzy backstepping method and SMC approach

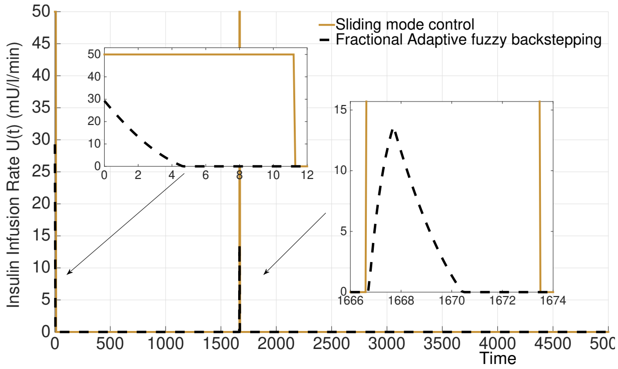

Fig. 5. Fractional adaptive fuzzy backstepping control signal compared to SMC approach

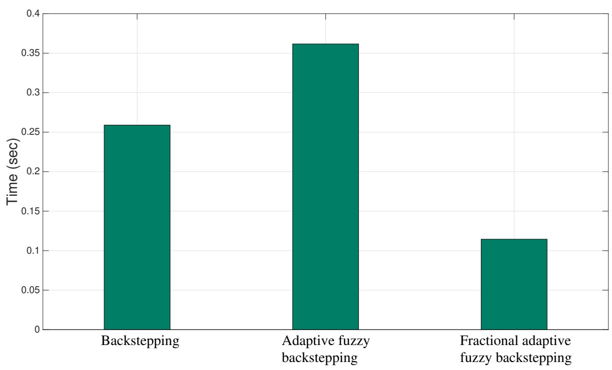

Fig. 6. Computation time

6. CONCLUSION

This paper proposes a non-linear control approach based on a fractional order adaptive fuzzy-backstepping controller using FEBMM for an automatic blood glucose regulation in Diabetes Type 1 patients. Adaptive Takagi-Sugeno fuzzy logic systems aim to get rid of the constraint related to the requirement of patients parameters knowledge. The fractional calculus leads to more accurateness in the control law. Lyapunov's theorem has been used to prove the asymptotic stability of the system. Simulation results have been performed, via MATLAB/Simulink, to illustrate the efficiency of fractional method during meal intake (faster convergence time, reduced steady state error and overshoots/undershoots) compared to integer controllers. As future work, a real-time implementation is suggested.

Table 1. Performance criteria values for the proposed controllers

		Blood Glucose
ISE	Backstepping	33873
	Fractional Backstepping	17758
	Fuzzy Type-1 Backstepping	26212
	Fractional Fuzzy Type-1 Backstepping	16992
	SMC	33948
IAE	Backstepping	1809.9
	Fractional Backstepping	836.92
	Fuzzy Type-1 Backstepping	980.02
	Fractional Fuzzy Type-1 Backstepping	442.38
	SMC	1816.6
ITAE	Backstepping	1.1564e+06
	Fractional Backstepping	6.0553e+05
	Fuzzy Type-1 Backstepping	5.4181e+05
	Fractional Adaptive Type-1 Backstepping	2.7593e+05
	SMC	1.168e+06

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